The Deconfinement and Hagedorn Phase Transitions in Weakly Coupled Large N Gauge Theories

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Outline

- Review of the deconfinement and Hagedorn phase transitions and their relation in large N gauge theories. Motivations for studying weakly coupled theories.
- The partition function of free large N SU(N) Yang-Mills theories at finite volume as a unitary matrix model. Solution of the matrix model and the free Yang-Mills phase diagram.
- Generalization to weakly coupled theories and possible extrapolations to strong coupling.

Confinement and deconfinement

- Confinement (no charged finite-energy states) – an experimental property of QCD, believed to happen in many asymptotically-free gauge theories.
- Observed on lattice but hard to understand theoretically despite many models.
- Related to strong coupling at low energies, so expected to disappear at high temperatures (compared to the QCD scale ~170 MeV) where the coupling is weak, in a deconfinement phase transition. (Also high densities)
- In general, hard to compute theoretically even the order of this phase transition (except pure SU(3)).

- Order parameters for deconfinement :
- θ At finite temperature, Wilson line around periodic time direction $\mathsf{P} = \frac{1}{N} tr(P \exp(-\oint A))$ (in Euclidean path integral) vanishes in confined phase but not in deconfined phase (free energy of external quark). Charged under Z_N global symmetry (for SU(N)) of "large gauge transformations".
- θ Another order parameter appears in the large N limit : in confined phase the free energy $F(T) \sim \mathsf{O}(1)$ (glueballs, mesons) while in deconfined phase $F(T) \sim \mathsf{O}(N^2)$ (quark-gluon plasma). So, we can use as an order parameter

$$\lim_{N\to\infty} F_{SU(N)}(T)/N^2$$

which also vanishes in the confined phase.

- Can we possibly study this transition perturbatively ? Not in infinite space, but maybe at finite small volume $1/R >> \Lambda_{QCD}$ with weak coupling at IR cutoff (if no zero modes, e.g. S^3), if transition persists to small volume.
- At finite volume phase transitions are generally smoothed out (and correspondingly thermal Wilson line always vanishes by Gauss' law).

  However, can still have phase transitions at large N. These can be characterized by our second order parameter, or by $<|P|^2>$, even though they are smoothed out for any finite N.
- Does the large N deconfinement transition persist to small R ? This can be answered perturbatively, and can compute the order of the transition !

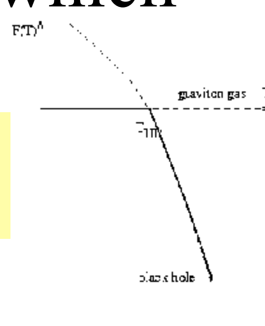
Deconfinement = Hagedorn ?

- In the 't Hooft large N limit with fixed $\lambda = g_{YM}^2 N$, gauge theories are believed to be equivalent to string theories with $g_s \sim 1/N$.
- The confining phase is naturally  described by strings (Regge trajectories, etc.), while the deconfined phase seems "non-stringy". This is qualitatively similar to the Hagedorn transition which could also occur in weakly coupled string theories (Atick+Witten), at least in curved space; also there the order parameter winds around the temporal circle. Could these transitions be related ? There are not many known examples to study this issue…

- The best understood example is in the AdS/CFT correspondence between type IIB string theory on $AdS_5 \times S^5$ and the N=4 supersymmetric SU(N) Yang-Mills theory on S^3 . In this case we can analyze the strongly coupled gauge theory compactified on a 3-sphere of radius R using string theory at small curvature :

θ In the microcanonical ensemble, at large energies (but small compared to $1/g_s$) there is Hagedorn behavior with $T_H \propto \lambda^{1/4}/R$.

θ In the canonical ensemble there is (Witten) a deconfinement-like phase transition (according to the order parameters) which is the Hawking-Page transition, happening at $T_{HP} = 3/2\pi R = 0.477465/R$.



- So, no apparent relation at strong coupling… How about weak coupling ? Naively no hope of stringiness…

The partition function of free Yang-Mills theory

(also derived by Sundborg, hep-th/9908001)

- Free Yang-Mills theory at finite volume is non-trivial, because we still have the Gauss law constraint – the total charge must vanish, all states are singlets of the gauge group. Could this cause “confinement” ?
- To compute the partition function we need to count the number of inequivalent ways to put particles in some representation R, in such a way as to get a singlet of energy E. This is a combinatorical problem whose only inputs are the easily computable :

$$z_B(T) = \sum_{bosonic-one-particle-states} \exp(-E/T)$$

$$z_F(T) = \sum_{fermionic-one-particle-states} \exp(-E/T)$$

- We have two ways to compute the exact partition function :

♣ We can sum over all states in the Fock space of our particles, and then impose a projection onto singlets.

♣ We can explicitly compute the Euclidean path integral of the free gauge theory (at one-loop) with periodic time.

- Both computations lead to the same result, which is a group integral :

$$Z(T) = \int [dU] \exp\{\sum_R \sum_{n=1}^{\infty} \frac{1}{n}[z_B^R(\frac{T}{n}) - (-1)^n z_F^R(\frac{T}{n})] tr_R(U^n)\}$$

In the path integral, U is the holonomy of the gauge field along the periodic time direction (averaged over space), which is the only zero mode (all other fields appear quadratically and can be integrated out). So, tr(U) is the naïve order parameter for deconfinement.

Solution of the unitary matrix model

- As usual, to solve the matrix model we change variables to the eigenvalues of U, $\{e^{i\alpha_i}\}; i = 1, \ldots, N; -\pi \le \alpha_i \le \pi$, and to the eigenvalue distribution $\rho(\alpha)$ which becomes continuous in the large N limit.
- For all fields in adjoint representation :

$$Z(T) = \int [dU] \exp\{\sum_{n=1}^{\infty} \frac{1}{n}[z_B(\frac{T}{n}) - (-1)^n z_F(\frac{T}{n})] \cdot tr(U^n) tr((U^+)^n)\} =$$

$$= \int [d\alpha_i] \exp\{-\sum_{i<j} V(\alpha_i - \alpha_j)\}$$

$$V(\theta) = -\ln(\sin(\theta/2)) - \sum_{n=1}^{\infty} \frac{1}{n}[z_B(\frac{T}{n}) - (-1)^n z_F(\frac{T}{n})] \cos(n\theta)$$

with a repulsive force coming from the measure, and an attractive force growing with the temperature.

- This matrix model can be solved exactly, by similar methods to those used to study the Gross-Witten phase transition of two dimensional lattice gauge theories.
- The analysis of the low-temperature phase is simplest in the (constrained) variables

$$\rho_n \equiv \int \rho(\alpha)\exp(in\alpha)d\alpha = tr(U^n)/N,$$

in which the "effective action" becomes

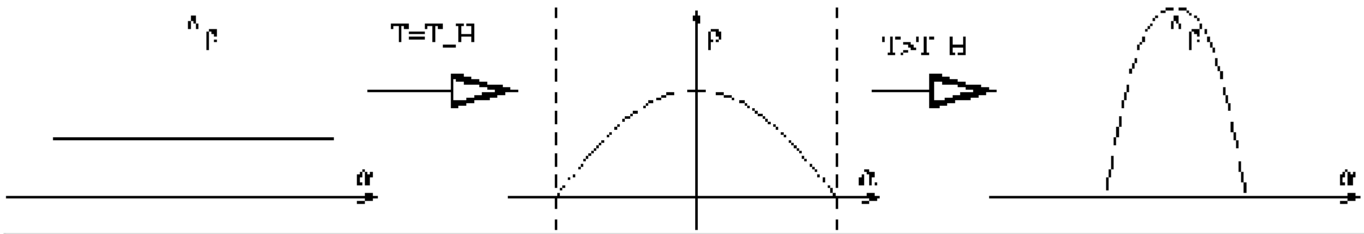
$$S = N^2 \sum_{n=1}^{\infty} |\rho_n|^2 \frac{1}{n}[1 - z_B(\frac{T}{n}) + (-1)^n z_F(\frac{T}{n})].$$

At low temperatures all coefficients are positive, the large N saddle point is a constant distribution $\rho_n = 0$, and the quadratic integration gives :

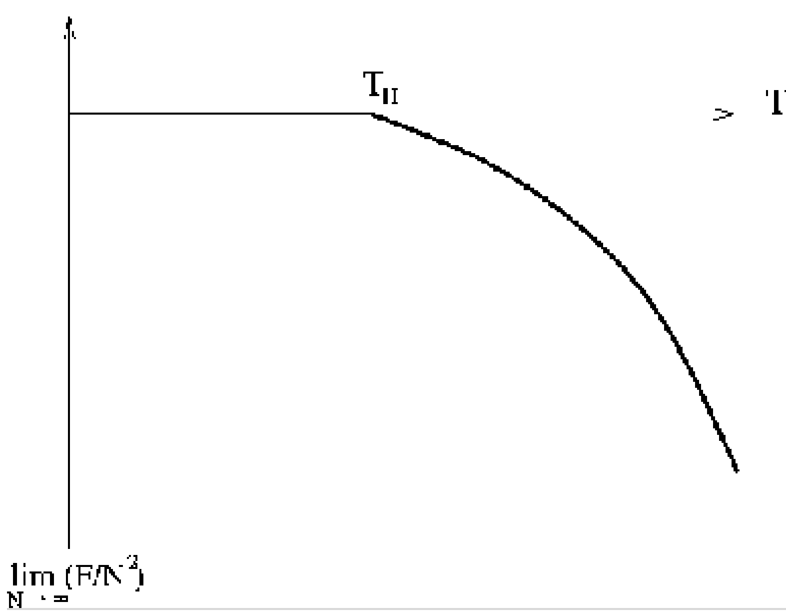
$$Z(T) = \prod_{n=1}^{\infty} \frac{1}{1 - z_B(\frac{T}{n}) + (-1)^n z_F(\frac{T}{n})}$$

- This breaks down at the temperature where $z_B(T_H) + z_F(T_H) = 1$ at which ρ_1 becomes tachyonic. Near this temperature we have (at large N) a Hagedorn-like behavior $Z(T) \sim 1/(T_H - T)$, corresponding to a density of states $\rho(E) \sim \exp(E/T_H)$.

  So, free large N Yang-Mills theories have a Hagedorn spectrum ! (can also be seen directly by counting states)
- In the free N=4 SYM theory on S^3 (for example) we find $T_H = -1/R\ln(7-4\sqrt{3}) = 0.379663/R$.
- Above the Hagedorn temperature, ρ_1 (which is precisely the temporal Wilson line) becomes tachyonic and condenses, and at higher temperatures the higher modes of the distribution condense as well :



- We find a (weakly) first-order phase transition at the Hagedorn temperature, which is a deconfinement transition according to our two order parameters ! For a given field content we can compute all thermodynamical quantities (though the high-temperature expressions are complicated). For example, the free energy :



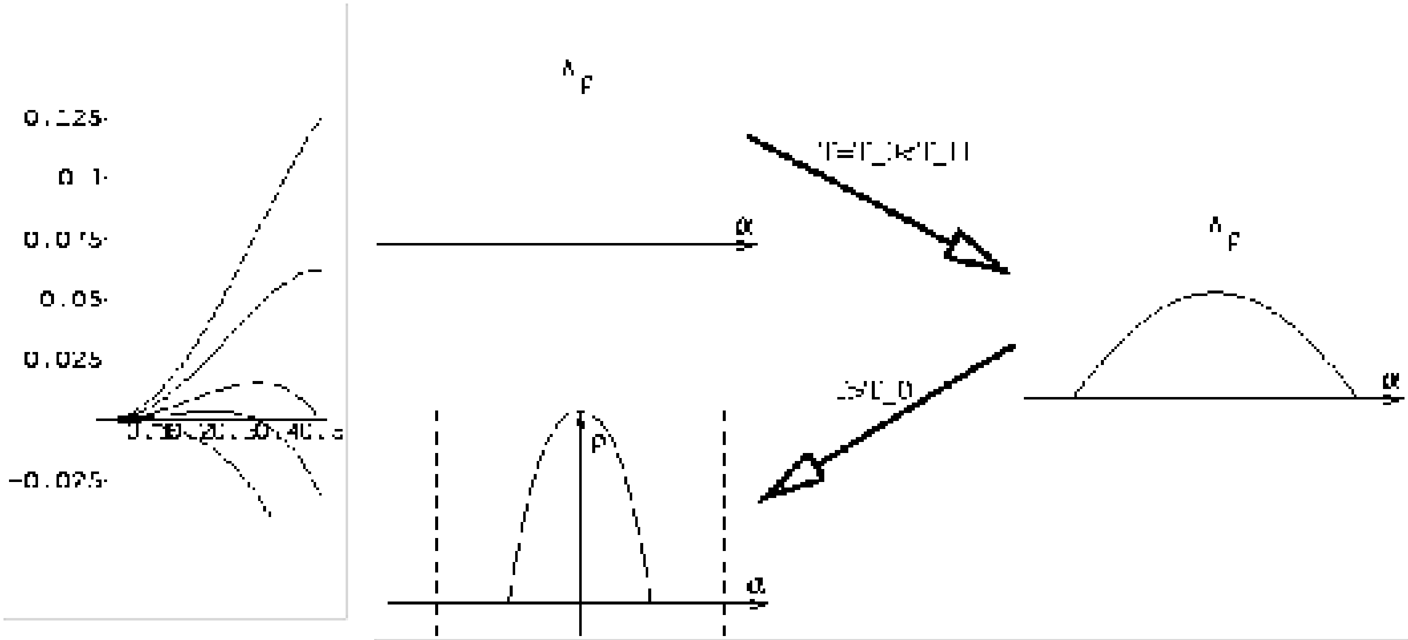
Weak coupling behavior

- At weak coupling our matrix model is still valid, but the action gets perturbative corrections. At k-loop order these take the form $\lambda^{k-1} tr(U^{n_1}) tr(U^{n_2}) \cdots tr(U^{n_{k+1}})$ with computable coefficients.
- The behavior near the phase transition is dominated by the light mode $\rho_1 \sim tr(U)/N$, and the effective Lagrangian for this mode near the transition takes the form

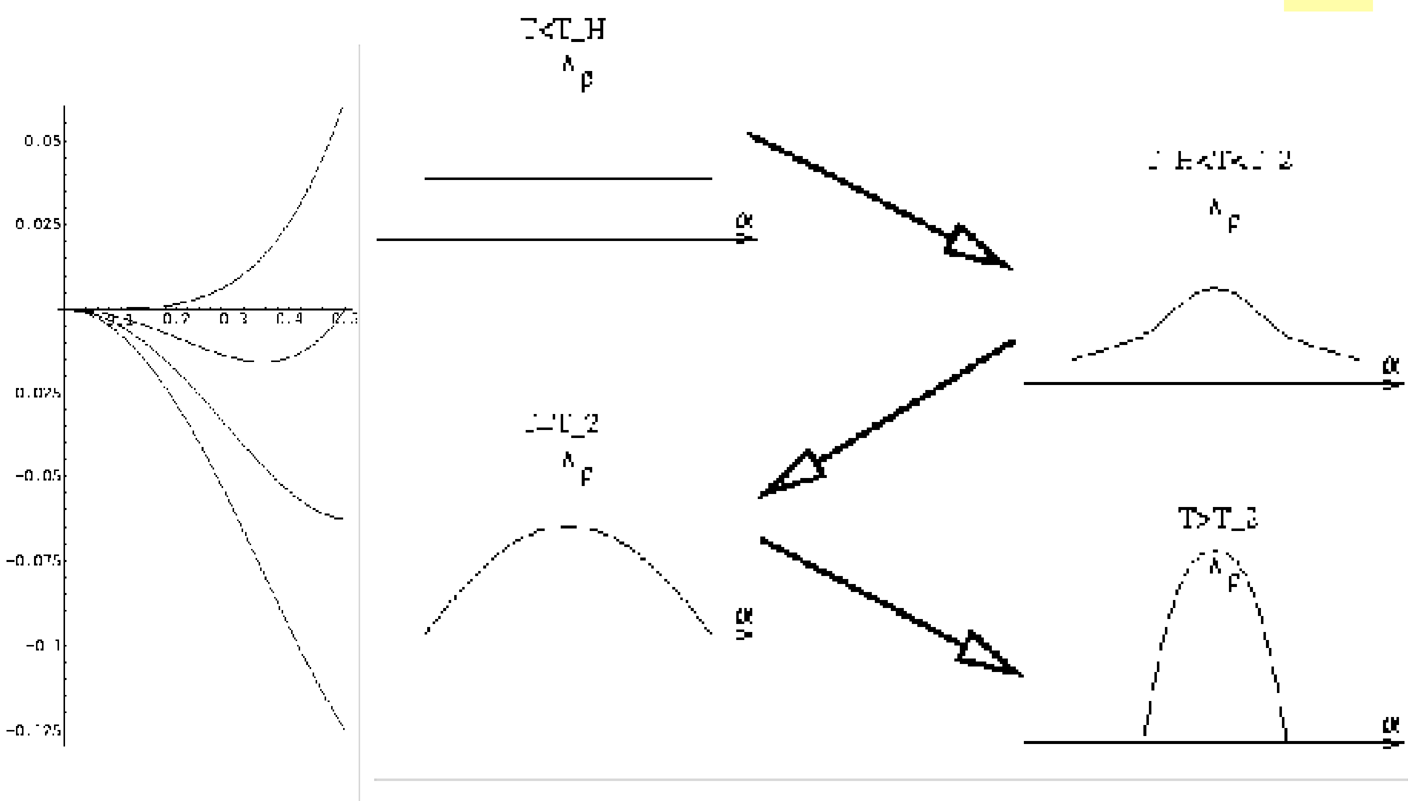
$$S_{eff} = N^2[a(T_H - T)|\rho_1|^2 + b\lambda^2|\rho_1|^4],$$

where a and b can be computed from two-loop and three-loop diagrams in the theory.
- As usual, the order of the phase transition depends on the sign of b :

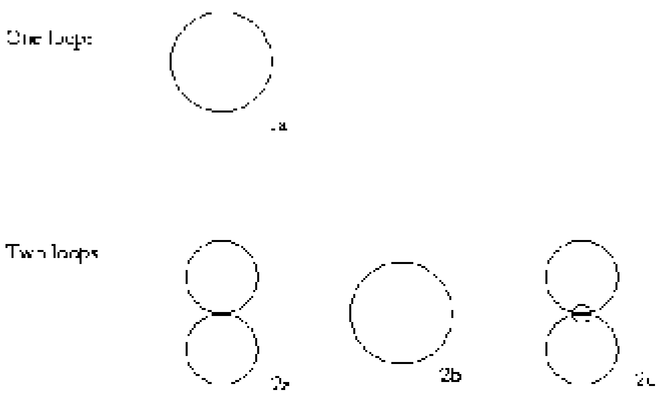
b<0 : first order transition below T_H



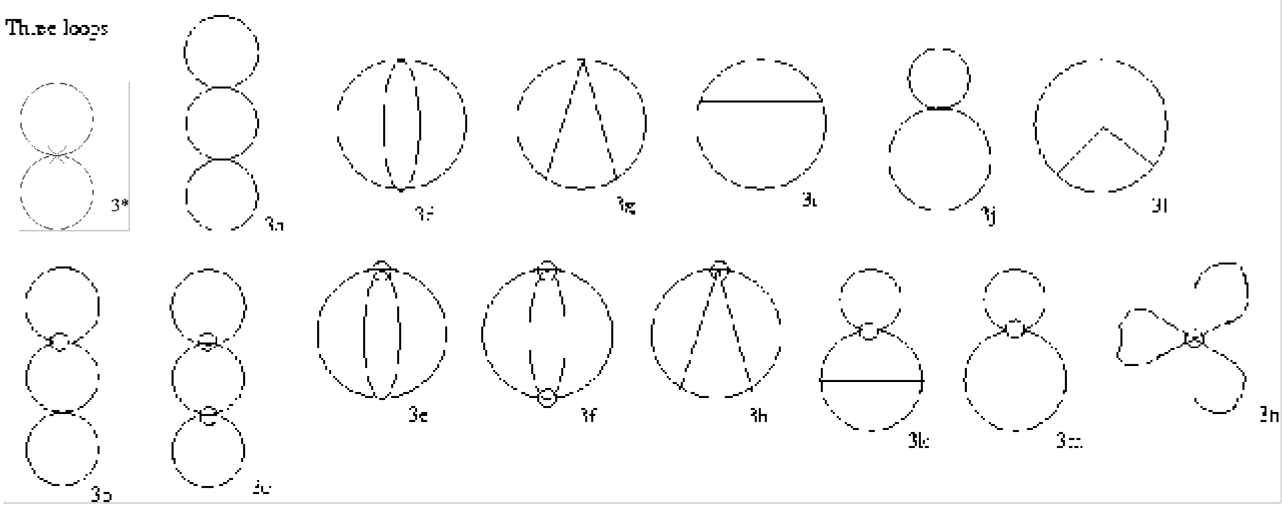
b>0 : two continuous transitions, first at T_H



- We must sum all relevant 3-loop vacuum diagrams in order to know the order of the deconfining phase transition, and whether it occurs at or below the Hagedorn temperature.
- In pure Yang-Mills theory on S^3, in a convenient choice of gauge (and after integrating out some fields), the following diagrams contribute :



•So far we have summed 14 of the 15 3-loop diagrams; one counter-term remains…



Summary

- Weakly coupled large N gauge theories on compact spaces :

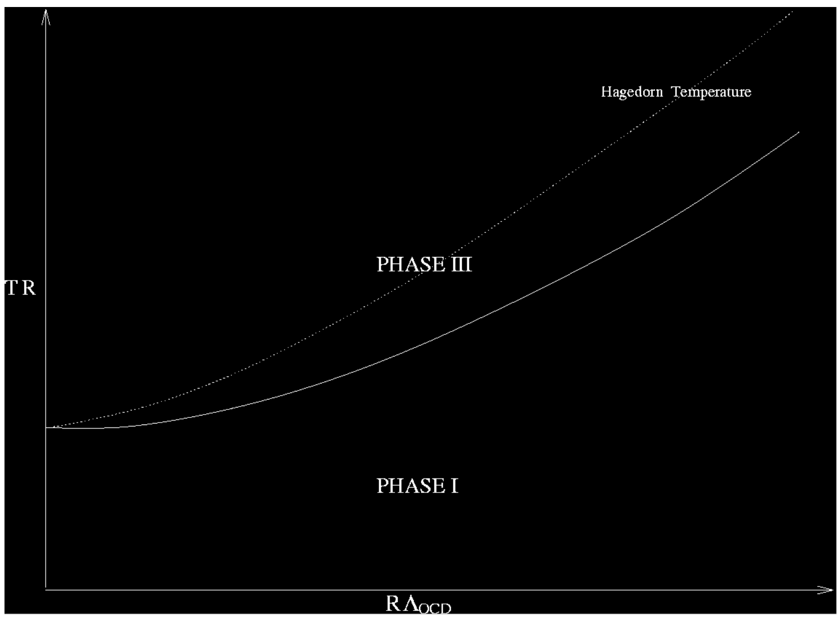
* Exhibit a Hagedorn spectrum (for finite energies in the large N limit),
* Have a deconfinement phase transition at at a temperature inversely related to the size of the space.

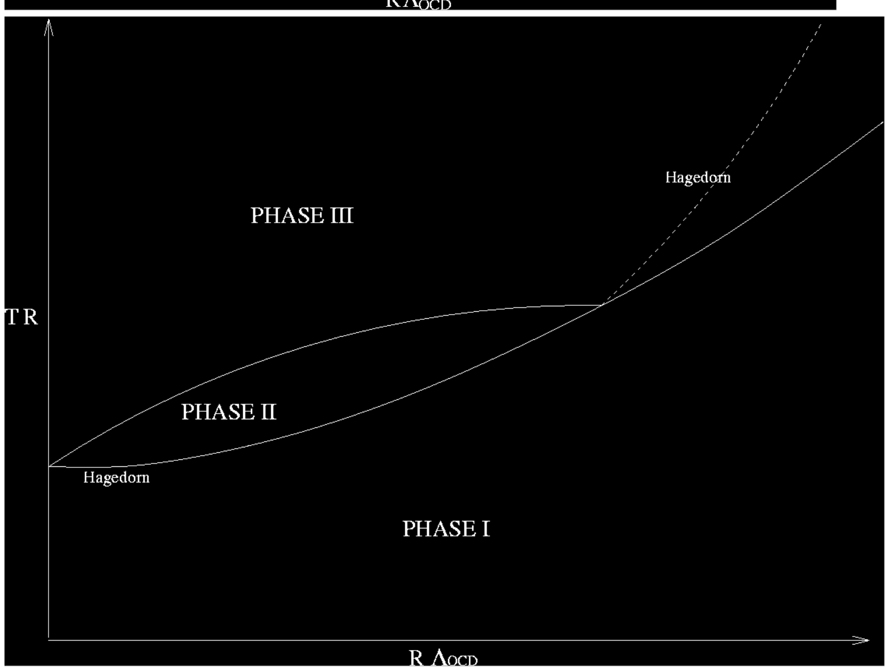
- The deconfinement transition is either :

* A first order transition below the Hagedorn temperature, or
* A second order transition at the Hagedorn temperature, followed by another third order phase transition.

- The properties of the transition (and of the stringy spectrum) can be computed in perturbation theory, through a unitary matrix model.

- The details of confinement here are very different from strong coupling (in pure YM and in N=4 SYM), but the behavior of the order parameters is the same, so they could be continuously connected (by increasing the radius / coupling) :





- Future directions :
- Compute the value of b (= the order of the transition) in several interesting cases (pure YM, N=4 SYM). Both signs are possible ! When adding a large number of fundamentals the transition is always smooth due to a potential for single eigenvalues (Schnitzer).
- Try to understand the free string theory corresponding to the free large N (supersymmetric) Yang-Mills theory. We know the exact spectrum of single-string and multi-string states ! Zero size AdS ? Not tensionless strings…
- Understand better the "intermediate phase" which sometimes appears. Is this related to "small black holes" ?
- Study integrability properties of the large N limit. (Info about anom. dim.)

- Generalize to cases with zero modes (on the compact space). Currently working on torus (with Wiseman). Here there is a more intricate phase structure involving also spatial holonomies. We find similar phase transitions for the spatial holonomies, which at strong coupling become (via the AdS/CFT correspondence) the Gregory-Laflamme transition !

- Can we extrapolate from weak coupling to strong coupling, and get a good model for "realistic" deconfinement ? Are the regimes continuously related ?