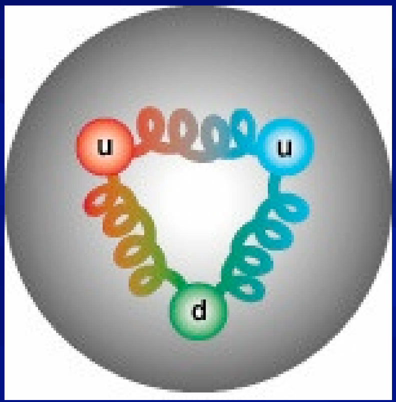
# N=4 Super-Yang-Mills Theory QCD and Collider Physics




Lance Dixon, SLAC

Strings 2004, Paris, 30 June 2004

# Why N=4 super-Yang-Mills theory?

- Unique, given the gauge group
- Maximal supersymmetry (sans gravity)
- 4-d conformal field theory
- Connected to gravity and string theory via AdS/CFT, holography, weak/strong coupling duality
- How is simplicity of strong coupling limit (in dual picture) reflected in structure of weak coupling expansion?
- How to study?
  - Anomalous dimensions of composite, gauge invariant operators (Thursday afternoon)
  - Regulated scattering amplitudes for plane-wave elementary field excitations (today)
- Perturbative N=4 SYM amplitudes share many properties with regime of QCD probed at colliders: “theoretical playground”
- As components of QCD amplitudes, N=4 SYM scattering amplitudes can be considered:
- “the simplest pieces” -- in terms of which types of loop integrals appear
- “the most complicated pieces” – contain highest “degree of transcendentality” of polylogarithms that appear

# Why QCD and Collider Physics?

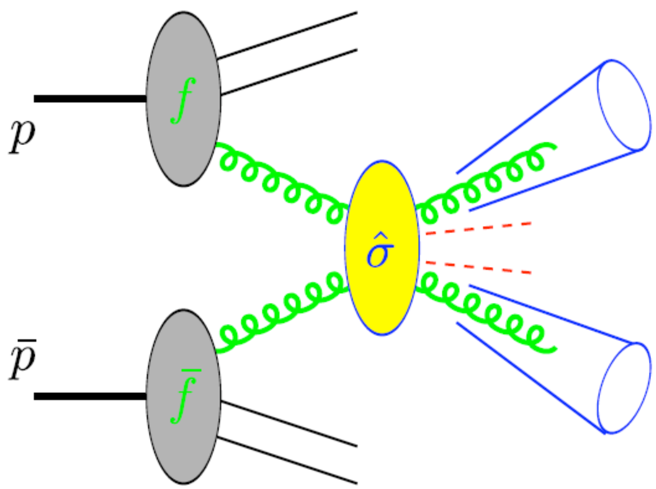
- Energy frontier is at hadron colliders
  - –Tevatron now, LHC in 2007
- New physics contends with Standard Model backgrounds
- All physics processes at hadron colliders –signals & backgrounds - are QCD processes
- Leading-order (LO) in QCD only qualitative
- NLO begins to be quantitative
- Errors only reliably estimated at NNLO -- need 2-loop amplitudes and integrals

# QCD Factorization at Colliders

In "hard" (short distance) process, soft, nonperturbative structure of proton factors out, confined to measurable parton distribution functions $f_a(x,\mu_F)$ – matrix elements of leading-twist operators, *e.g.:*

$$\int_0^1 dx\, x^j\, q(x,\mu) \sim \langle p\,|[\bar{q}\gamma^+\partial_+^j q]_{(\mu)}\,|\,p\rangle$$

$$\sigma_{p\bar{p}\to jjX} = \sum_{a,b}\int dx_1 dx_2\; f_a(x_1;\mu_F)\,\bar{f}_b(x_2;\mu_F) \times \hat{\sigma}_{ab\to jjX}(sx_1x_2;\mu_F,\mu_R,\alpha_s(\mu_R))$$



# Inputs for NNLO Predictions

- Experimental value for $\alpha_s(\mu_R)$ and its evolution

- 3-loop β-function Tarasov et al. (1980); Larin & Vermaseren (1993)

- Experimental values for $f_a(x;\mu_F)$ and their evolution

- 3-loop leading-twist anomalous dimensions

Moch, Vermaseren & Vogt (2004)

- NNLO terms in expansion of partonic cross section

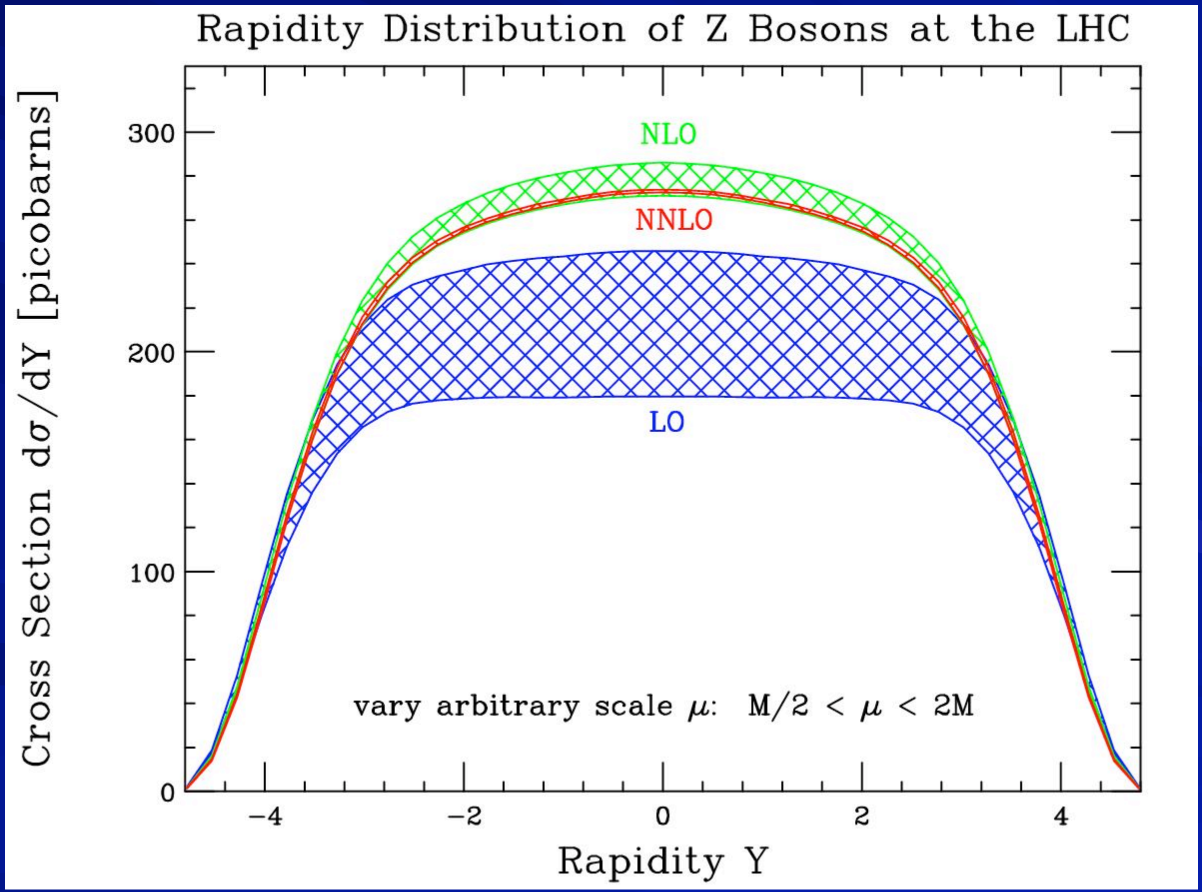
$$\hat{\sigma}_{ab\to jjX} = \alpha_s^2 (A + \alpha_s\, B + \alpha_s^2\, C)$$

– $C$ only now becoming available for selected processes

# Example of Error Reduction at NNLO

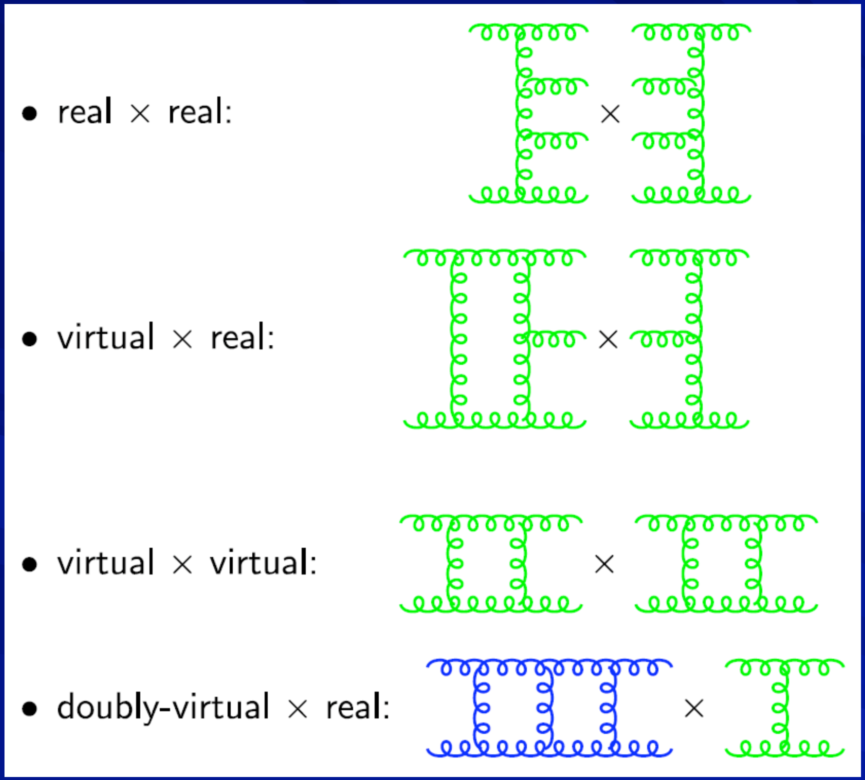
- 100 million Ws, 10 million Zs per year at LHC.

Use them to constrain underlying pdfs in same experiments where you look for new physics (partonic luminosity monitor)




Anastasiou, LD, Melnikov, Petriello (2003)

# Ingredients for NNLO $\hat{\sigma}_{ab\to jjX}$




- Severe infrared divergences
- Regulate with $D=4-2\varepsilon$ (breaking conformal invariance) get $1/\varepsilon^4$, ..., $1/\varepsilon$ poles
- Poles cancel in sum, for short-distance process, after removing collinear singularities associated with pdfs

# N=4 SYM as testing ground for pQCD

- N=4 SYM equivalent to QCD at tree level
- Progressively more removed at each higher loop order
- Though more fields in N=4 SYM, symmetries enormous simplify the calculation when done “the right way” (unitarity)
- Infrared behavior very similar:
  - –purely soft divergences identical (due to gluons)
  - –collinear divergences less difficult than soft ones (less color tangling)
  - –for example, at one loop:

$$|\mathcal{M}_n^{(1)}\rangle = \boldsymbol{I}^{(1)}(\epsilon)\, |\mathcal{M}_n^{(0)}\rangle + |\mathcal{M}_n^{(1)\mathrm{fin}}\rangle$$

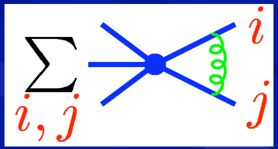
where

$$\boldsymbol{I}^{(1)}(\epsilon) = \frac{1}{2}\frac{e^{-\epsilon\psi(1)}}{\Gamma(1-\epsilon)} \sum_{i=1}^{n} \sum_{j\neq i}^{n} \boldsymbol{T}_i \cdot \boldsymbol{T}_j \left[\frac{1}{\epsilon^2} + \frac{\gamma_i}{\boldsymbol{T}_i^2}\frac{1}{\epsilon}\right] \left(\frac{\mu^2}{-s_{ij}}\right)^{\epsilon}$$

$$\gamma_i = \beta^{(1)} = 0 \quad \text{for } \mathrm{N} = 4\,\mathrm{SYM}$$

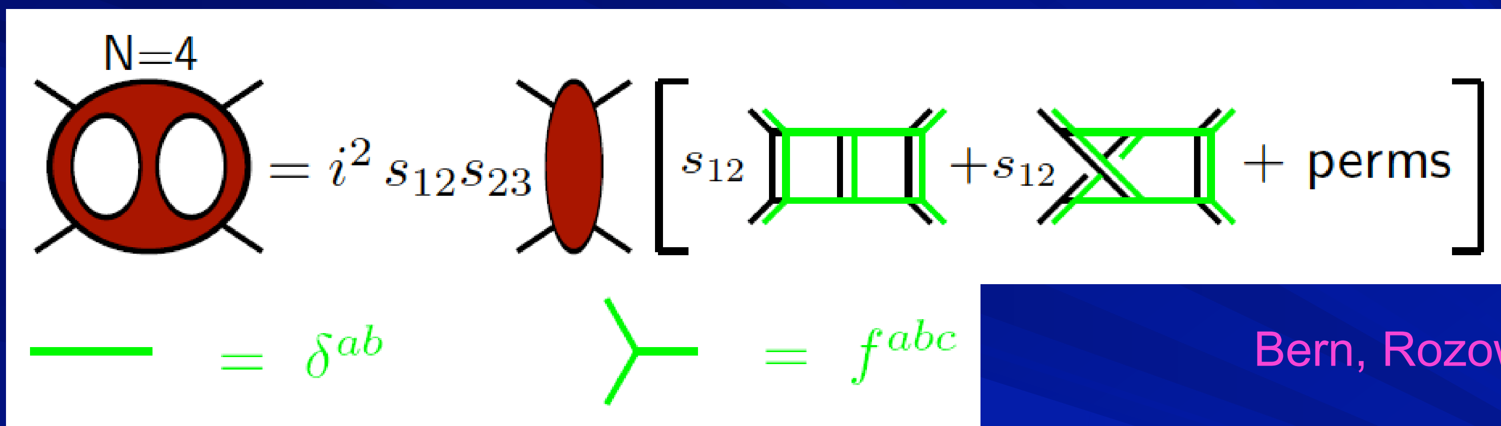
soft

collinear



# N=4 SYM as testing ground (cont.)

- For example, *gg -> gg* scattering at 2-loops computed in N=4 SYM in 1997 (in terms of loop integrals):




Bern, Rozowsky, Yan

- Analogous computation in QCD not completed until 2001

Glover, Oleari, Tejeda-Yeomans (2001); Bern, De Freitas, LD (2002)

# Generic form of result

- For all-massless kinematics (e.g. for $gg \to gg$, $q\bar{q} \to q\bar{q}$, etc.), all you need is polylogarithms, $\mathrm{Li}_n(x)$, $n = 2, 3, 4$,

$$\mathrm{Li}_n(x) = \sum_{i=1}^{\infty} \frac{x^i}{i^n} = \int_0^x \frac{dt}{t} \mathrm{Li}_{n-1}(t), \qquad \mathrm{Li}_2(x) = -\int_0^x \frac{dt}{t} \ln(1-t)$$

where $-x \in \{s/t, t/s, s/u, u/s, t/u, u/t\}$.

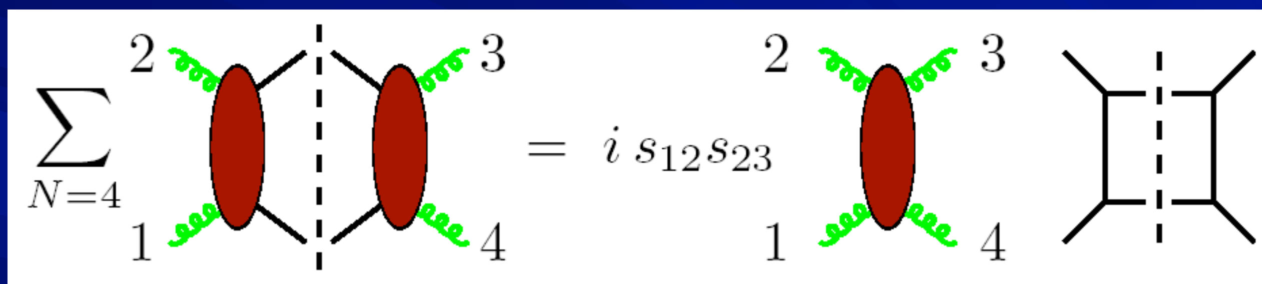
- Define "transcendentality": 1 for $\ln(x)$ and $\pi$
  $n$ for $\mathrm{Li}_n(x)$ and $\zeta_n = \mathrm{Li}_n(1)$

- N=4 SYM results always homogeneous, with maximum transcendentality:

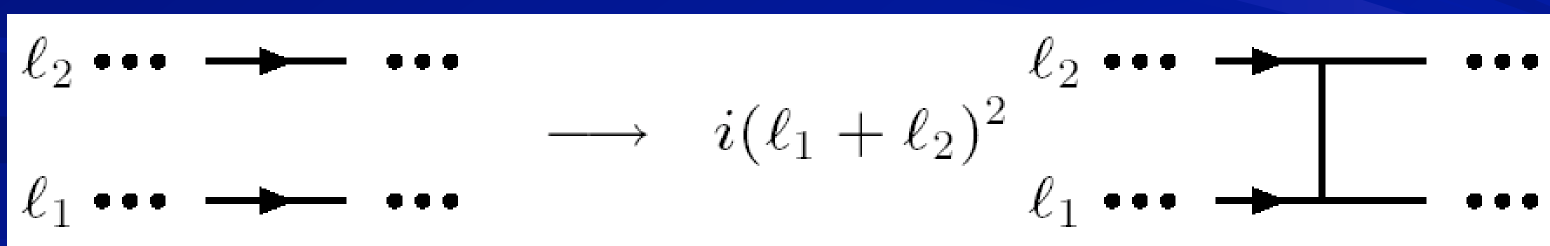
$$1/\varepsilon^4\, f_0(x) + 1/\varepsilon^3\, f_1(x) + 1/\varepsilon^2\, f_2(x) + 1/\varepsilon\, f_3(x) + f_4(x) + \ldots$$

# Higher-loop N=4 SYM

- Many contributions can be deduced from a simple property of the 2-particle cuts  Bern, Rozowsky, Yan

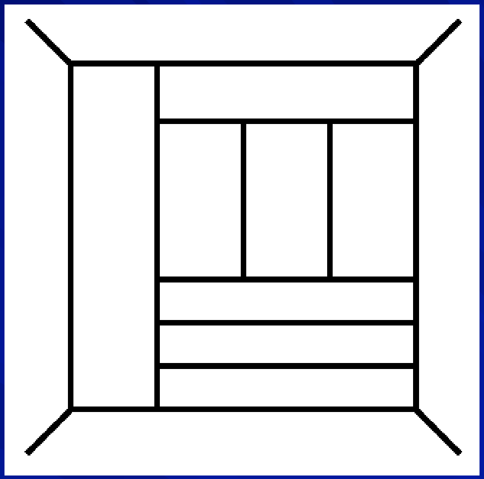



- Leads to "rung rule" for computing diagrams which can be built by  iterating 2-particle cuts



# Higher-loop N=4 SYM (cont.)

- Such diagrams called "iterated 2-particle cut-constructible"



- Better terminology might be (planar) "Mondrian diagrams"

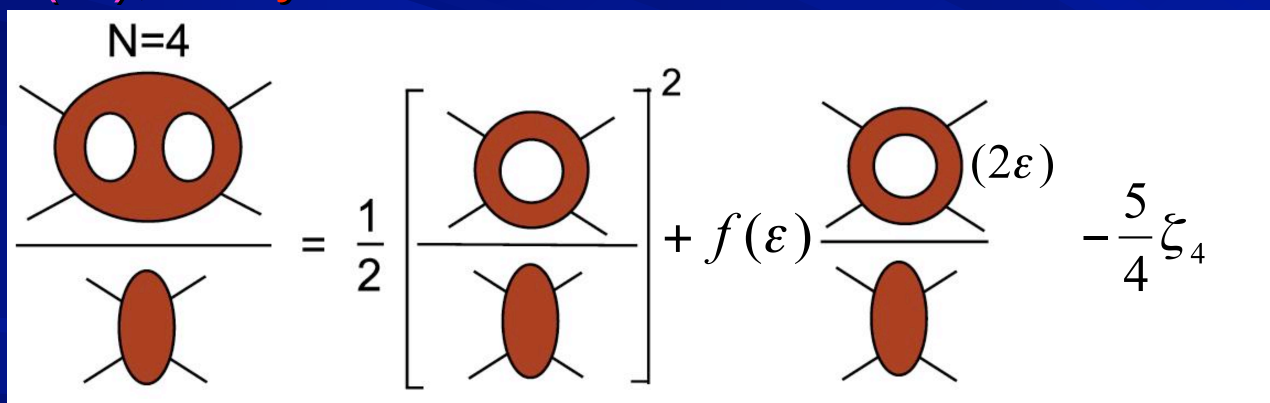
- I.e. those diagrams which could be painted by Mondrian (on the surface of a cow)


# Iterative Property of Planar N=4 SYM Amplitudes

- Two-loop 4-point N=4 SYM amplitude available in 1997. Could have been expanded in $\varepsilon$ by 1999, but no-one bothered to until recently.

  Smirnov; Tausk

- Planar (leading-color) terms, up to and including finite parts, $O(\varepsilon^0)$, obey




$$f(\varepsilon) = -(\zeta_2 + \varepsilon\zeta_3 + \varepsilon^2\zeta_4 + \ldots)$$

Anastasiou, Bern, LD, Kosower, hep-th/0309040

- In QCD, $O(\varepsilon^0)$ $\zeta_4$ term replaced by ~10-20 page expression!

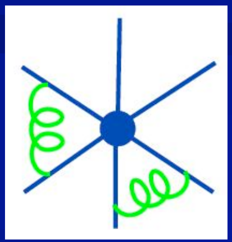
# Iterative Property (cont.)

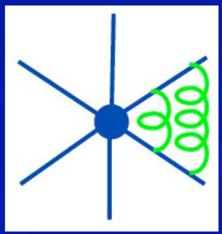
- Though "miraculous" at $O(\varepsilon^0)$, relation "well-understood" for singular $1/\varepsilon^n$ terms: Kinoshita-Lee-Nauenberg (1960s); Catani (1998)

$$|\mathcal{M}_n^{(2)}\rangle = \boldsymbol{I}^{(1)}(\epsilon)\,|\mathcal{M}_n^{(1)}\rangle + \boldsymbol{I}^{(2)}(\epsilon)\,|\mathcal{M}_n^{(0)}\rangle + |\mathcal{M}_n^{(2)\mathrm{fin}}\rangle$$

$$\boldsymbol{I}^{(2)}_{\mathrm{R.S.}}(\epsilon) = -\frac{1}{2}\boldsymbol{I}^{(1)}(\epsilon)\left(\boldsymbol{I}^{(1)}(\epsilon) + \frac{4\pi\beta_0}{\epsilon}\right) + \frac{e^{\epsilon\psi(1)}\Gamma(1-2\epsilon)}{\Gamma(1-\epsilon)}\left(\frac{2\pi\beta_0}{\epsilon} + K\right)\boldsymbol{I}^{(1)}(2\epsilon) + \boldsymbol{H}^{(2)}_{\mathrm{R.S.}}(\epsilon)$$

$\boldsymbol{I}^{(2)}(\epsilon)$ diagrammatically:





# Iterative Property (cont.)

- Relation also well-understood in $s >> t$ *limit* (Regge/BFKL) where ln($s/t$) exponentiates.

Balitsky, Fadin, Kuraev, Lipatov (1976)

- For general $s/t$, requires degree-4 polylogarithm identities
- No analogous relation at $O(\varepsilon^1)$ -- special to $D=4$
- Also no analogous relation for subleading-color terms – special to planar limit
- Evidence of identical relation for $n > 4$ external legs (at least for some helicity configurations) from collinear limits. (In fact, this is how relation was uncovered.)

# Collinear Limits

- Useful for understanding analytic structure of complicated multi-point amplitudes, at tree and loop level. (2-loop analysis in N=4 SYM again predated QCD)

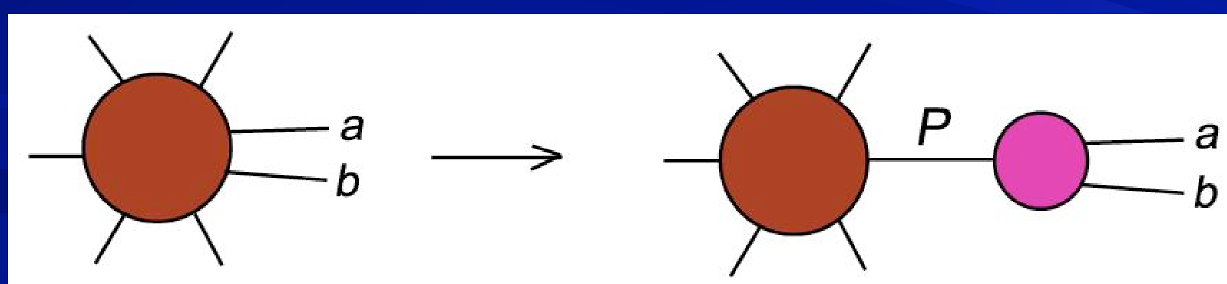
- Can use to compute leading-twist anomalous dimensions, not in "moment space", but directly in "*x*-space" – Altarelli-Parisi kernels

Kosower, Uwer

- Tree amplitude behavior:

  Stringy derivation: Mangano, Parke

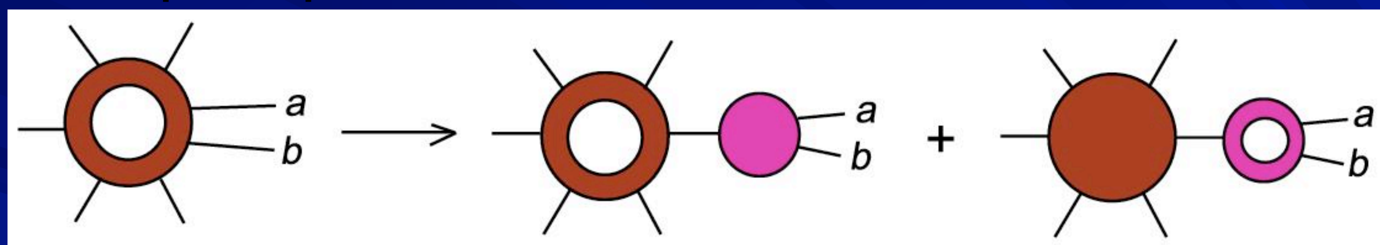
$$k_a \to z k_P \qquad k_b \to (1-z) k_P$$




$M_n^{(0)}$ $\qquad$ $M_{n-1}^{(0)}$ $\qquad$ $\mathrm{Split}^{(0)}(\lambda_P, \lambda_a, \lambda_b)$

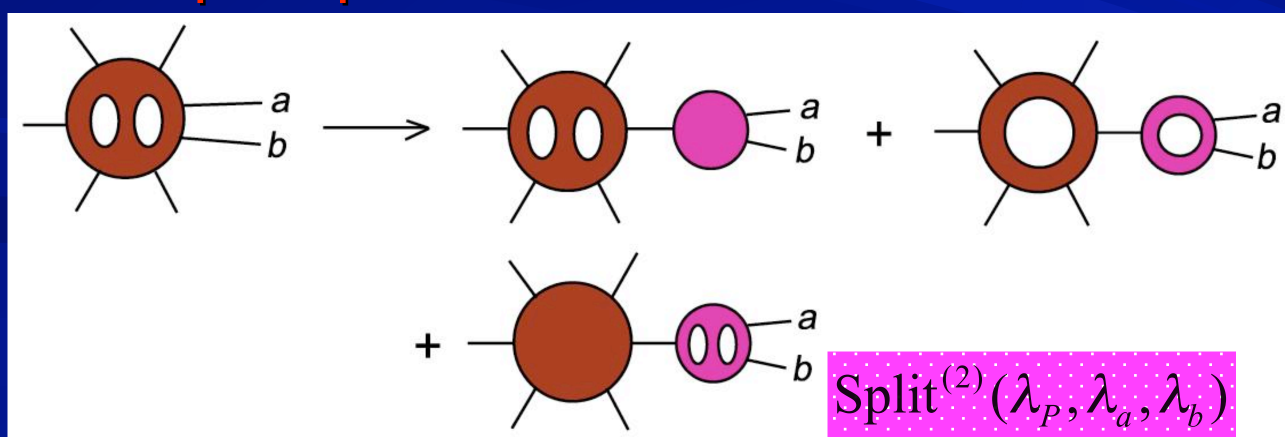
# Loop-level Collinear Limits

- One-loop amplitude behavior:




$M_n^{(1)}$ $M_{n-1}^{(1)}$ $\mathrm{Split}^{(1)}(\lambda_P, \lambda_a, \lambda_b)$

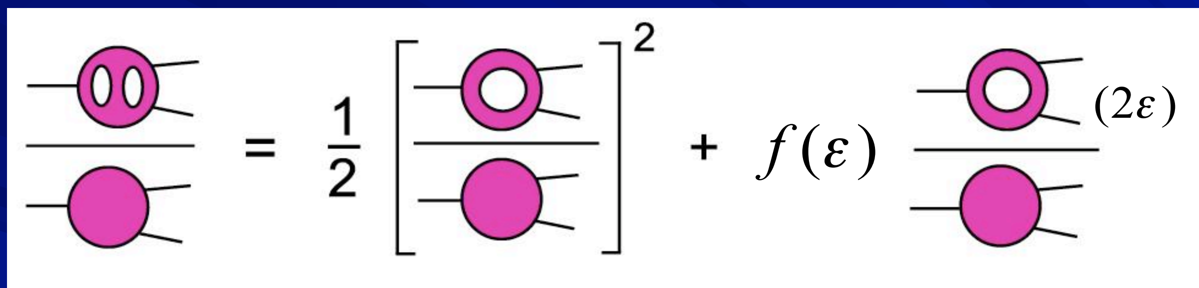
- Two-loop amplitude behavior:



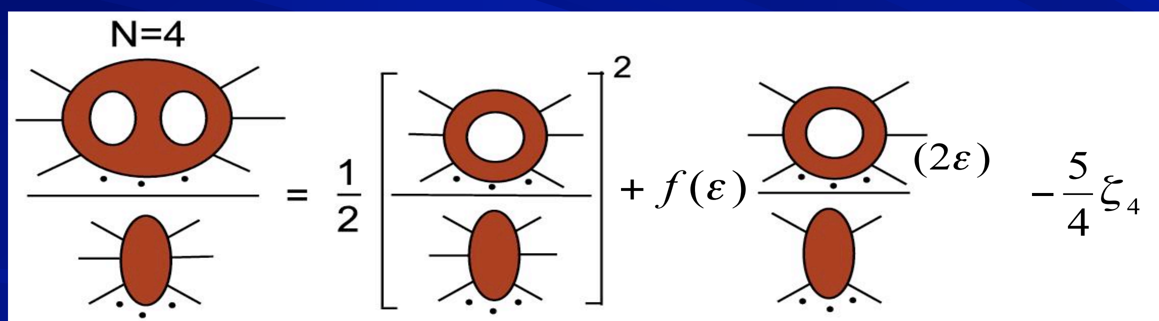

$\mathrm{Split}^{(2)}(\lambda_P, \lambda_a, \lambda_b)$

# Iterative Behavior for Splitting Amplitudes

- Two-loop splitting amplitude obeys:




which is consistent with the *n*-point amplitude ansatz



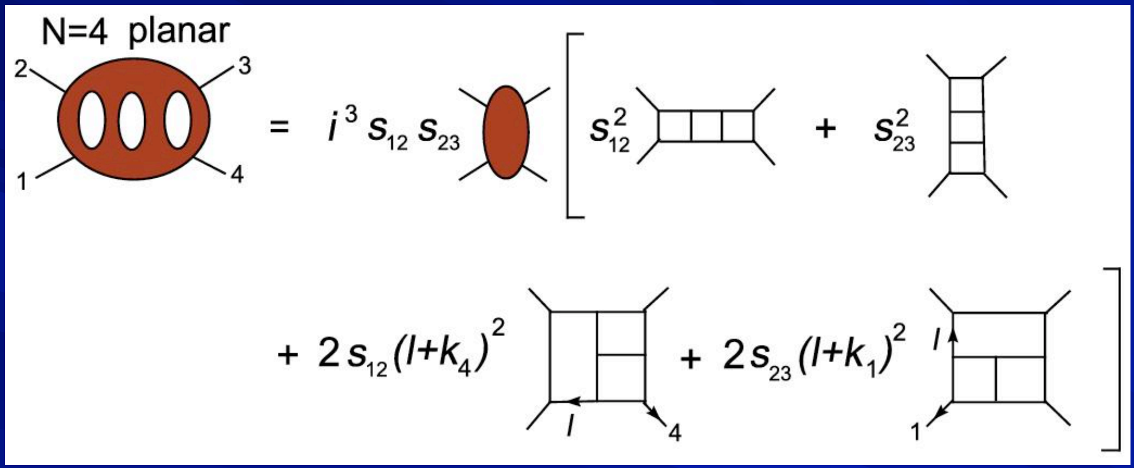

Seems likely to hold for at least MHV amplitudes.
Would be interesting to examine twistor-string diff. eqns.

Witten, hep-th/0312171; Cachazo, Svrcek, Witten, hep-th/0406177

# Test at Three Loops Soon?

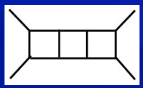
- 3-loop planar N=4 amplitude also dates from 1997

(3-particle cuts checked later)

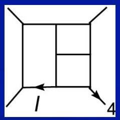



Bern, Rozowsky, Yan (1997)

- Integrals now being computed:



Smirnov, hep-ph/0305142




Smirnov, in progress

- *$1/\varepsilon^6$* to *$1/\varepsilon^2$* poles predictable   Sterman, Tejeda-Yeomans, hep-ph/0210130
  - Works to order computed so far -- *$1/\varepsilon^3$*

# Significance of Iterative Behavior?

- From need for strongly-coupled series to sum up to "something simple" in weakly-coupled supergravity?
- Hints of iterative structure in 4-point correlation functions of chiral primary (BPS) composite operators. Exact structure not clear yet.

  Eden et al., hep-th/9906051, hep-th/0003096,
  Arutyunov et al., hep-th/0103230

- Much recent progress computing anomalous dimensions of non-BPS operators (e.g. "SU(2) sector"), using integrability of dilatation operator

  Minahan, Zarembo, hep-th/0212208
  Beisert et al., hep-th/0303060, 0308074, 0405001

- Piece of anom. dim. matrix even summed to all orders in terms of hypergeometric functions

  Ryzhov, Tseytlin, hep-th/0404215

- Quantities we consider – $D=4-2\varepsilon$ scattering amplitudes of color non-singlet states -- are quite different from the above, yet there should be some underlying connection

# Aside: Anomalous Dimensions

- Set of NNLO leading-twist anomalous dimensions in QCD recently computed (7-year effort)

Moch, Vermaseren & Vogt (2004)

- Universal NNLO anomalous dimensions in N=4 SYM extracted from this result (in 1 month)

Kotikov, Lipatov, Onishchenko, Velizhanin, hep-th/0404092

- Non-trivial because N=4 SYM contains scalars while QCD does not
- However, KLOV noticed (from BFKL limit, NLO case) that the "most complicated" terms of "maximum transcendentality" in the QCD computation always coincide with N=4 SYM result. (We can now verify this for the virtual pieces at NNLO.)
- Observation allowed extraction of N=4 SYM result
- For *j* = *4* agrees with predictions based on integrability

Beisert, Kristjansen, Staudacher, hep-th/0303060

# Conclusions & Outlook

- N=4 super-Yang-Mills theory an excellent testing ground for computing QCD scattering amplitudes needed for precise theoretical predictions at hadron colliders.

- We can even learn something about the structure of N=4 SYM itself in the process, although there is clearly much more to be understood:
- Is there any AdS/CFT "dictionary" for color non-singlet states?
- Can we recover composite operator correlations from any limits of multi-point scattering amplitudes?
- Is there a better way to IR regulate?
  - $S^3$ radius?
  - twistor space?
  - coherent states?