Basic Results in Vacuum Statistics

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Abstract

Based on hep-th/0303194, hep-th/0405279 and

- hep-th/0307049 with Sujay Ashok
- math.CV/0402326 with Bernard Shiffman and Steve Zelditch (Johns Hopkins)
- hep-th/0404116 and to appear with Frederik Denef
- hep-th/0404257 with F. Denef and Bogdan Florea.

1. Predictions from string theory

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In recent years, even more dramatic possibilities have been suggested, which would lead to new, distinctive particles or phenomena: large extra dimensions (KK modes), a low fundamental string scale (massive string modes), or rapidly varying warp factors (modes bound to branes, or conformal subsectors).

Any of these could lead to dramatic discoveries. But should we expect string/M theory to lead to any of these possibilities? Would not discovering them be evidence against string theory?

At Strings 2003, I discussed a statistical approach to these and other questions of string phenomenology. Over the last year, our group at Rutgers, and the Stanford group, have made major progress in developing this approach, with

- Explicit proposals for vacua with all moduli stabilized along lines of KKLT (work with Denef and Florea).
- Detailed results for distributions of these vacua (with Shiffman, Zelditch and Denef)
- Preliminary results on the statistics of
  - supersymmetry breaking scales (MRD, hep-th/0405279; see also Susskind, hep-th/0405189 and hep-ph/0406197)
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Much work will be needed to bring this about. But we may be close to making some predictions: those which use just the most generic features of string/M theory compactification, namely the existence of many hidden sectors.

2. Hidden sectors

Before string theory, and during the "first superstring revolution," most thinking on unified theories assumed that internal consistency of the theory would single out the matter content we see in the real world.

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If we live in a "typical" string compactification, it seems very likely that there are many hidden sectors, not directly visible to observation or experiment.

Should we care? Does this lead to any general predictions?

3. A new limit

Hidden sectors may or may not lead to new particles or forces. But what they do generically lead to is a multiplicity of vacua, because of symmetry breaking, choice of vev of additional scalar fields, or other discrete choices.

Let us say a hidden sector allows c distinct vacua or "phases." If there are N hidden sectors, the multiplicity of vacua will go as

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While the many hidden sectors certainly make the detailed study of string compactification more complicated, we should consider the idea that they lead to simplifications as well.

Thus we might ask, what can we say about the case of a large number N of hidden sectors ? Clearly there will be a large multiplicity of vacua.

We only live in one vacuum. However, as pointed out by Brown and Teitelboim; Banks, Dine and Seiberg (and no doubt many others), vacuum multiplicity can help in solving the cosmological constant problem. In an ensemble of $\mathcal{N}_{vac}$ vacua with roughly uniformly distributed c.c. Λ, one expects that vacua will exist with Λ as small as $M_{pl}^4/\mathcal{N}_{vac}$.

To obtain the observed small nonzero c.c. $\Lambda \sim 10^{-122} M_{pl}^4$, one requires $\mathcal{N}_{vac} > 10^{120}$ or so.

Now, assuming different phases have different vacuum energies, adding the energies from different hidden sectors can produce roughly uniform distributions. In fact, the necessary $\mathcal{N}_{vac}$ can easily be fit with $\mathcal{N}_{vac} \sim c^N$ and the parameters $c \sim 10$, $N \sim 100 - 500$ one expects from string theory, as first pointed out by Bousso and Polchinski.

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One might regard fitting the observed small nonzero c.c. in **any** otherwise acceptable vacuum as solving the problem, or one might appeal to an anthropic argument such as that of Weinberg to select this vacuum. In the absence of other candidate solutions to the problem, we might even turn this around and call these ideas **evidence** for the hypothesis that we are in a compactification with many hidden sectors.

4. Supersymmetry breaking

So can we go further with these ideas? Another quantity which can get additive contributions from different sectors is the scale of supersymmetry breaking. Let us call this M^2_{susy} (we will define it more carefully below).

We recall the classic arguments for low energy supersymmetry from naturalness. The electroweak scale m_{EW} is far below the other scales in nature M_{pl} and M_{GUT}. According to one definition of naturalness, this is only to be expected if a symmetry is restored in the limit $m_{EW} \to 0$. This is not true if m_{EW} is controlled by a scalar (Higgs) mass m_H, but can be true if the Higgs has a supersymmetric partner (we then restore a chiral symmetry).

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A more general definition of naturalness requires the theory to be stable under radiative corrections, so that the small quantity does not require fine tuning. Again, low energy supersymmetry can accomplish this. Many theories have been constructed in which

$$M_H^2 \sim c M^2_{susy},$$

with $c \sim 1/10$ without fine tuning. Present data typically requires $c < 1/100$, which requires a small fine tuning (the "little hierarchy problem.")

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With the development of string compactification, it has become increasingly clear that there is a large multiplicity of vacua. The vacua differ not only in the cosmological constant, but in every possible way: gauge group, matter content, couplings, etc. What should we do in this situation?

The "obvious" thing to do at present is to make the following definition:

> An effective field theory (or specific coupling, or observable) T_1 is more natural in string theory than T_2, if the number of phenomenologically acceptable vacua leading to T_1 is larger than the number leading to T_2. (Douglas, 0303194; Susskind, 0406197)

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In any case, the unambiguously defined information provided by string/M theory is the number of vacua and the distribution of resulting EFT's. For example, we could define

$$\begin{aligned} d\mu[M_H^2, M_{susy}^2, \Lambda] &= \rho(M_H^2, M_{susy}^2, \Lambda) dM_H^2\ dM_{susy}^2\ d\Lambda \\ &= \sum_{T_i} \delta(M_{susy}^2 - M_{susy}^2|_{T_i}) \delta(M_H^2 - M_H^2|_{T_i}) \delta(\Lambda - \Lambda|_{T_i}) \end{aligned}$$

a distribution which counts vacua with given c.c., susy breaking scale and Higgs mass, and study the function

$$\rho(10^4\ \text{GeV}^2, M_{susy}^2, \Lambda \sim 0).$$

5. Statistical selection

Is this definition of "stringy naturalness" good for anything? Suppose property X (say low scale susy) is realized in 10^{40} phenomenologically acceptable vacua, while $\bar{X}$ (say high scale susy) is realized in 10^{20} such vacua. We should **not** conclude that string theory predicts low scale susy.

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Suppose the total number of relevant vacua were $\mathcal{N}_{vac} \sim 10^{100}$ or so. Although this may seem large, it is not enough to reproduce all couplings and the c.c. by fine tuning, even given supersymmetry.

In this case, given any structure in the distribution, it seems quite likely that some regions would contain no vacua, and predictions would emerge. On the other hand, $\mathcal{N}_{vac} \sim 10^{1000}$ would probably not lead to predictions, unless the distribution were very sharply peaked.

The reason we can hope for predictivity even with 10^{100} vacua is of course that we are asking that the correct vacuum actually realizes the observed c.c., a very selective condition. Now literally showing that no vacuum reproduces the observed small c.c. is far beyond any computation we could conceivably do. But tractable approximations to the true distribution of vacua can estimate how much unexplained fine tuning is required to achieve the desired EFT.

If we computed such an approximate distribution and found that low scale susy was realized in 10^{10} acceptable vacua, while high scale susy was realized in 10^{-10} vacua, we would be saying that high scale susy could work only with an additional 10^{-10} fine tuning, **not** provided by the multiplicity of vacua. On the other hand, low scale susy could work without it. This would give us fairly good grounds for predicting low scale susy.

6. Absolute numbers

The basic estimate for numbers of flux vacua (Ashok and Douglas) is

$$\mathcal{N}_{vac} \sim \frac{(2\pi L)^{K/2}}{(K/2)!}[c_n]$$

where K is the number of distinct fluxes ($K = 2b_3$ for IIb on CY_3) and L is a "tadpole charge" ($L = \chi/24$ in terms of the related CY_4). The "geometric factor" $[c_n]$ does not change this much, while other multiplicities are probably subdominant to this one.

Typical $K \sim 100-400$ and $L \sim 500-5000$, leading to $\mathcal{N}_{vac} \sim 10^{500}$. This is probably too large for statistical selectivity to work.

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Such considerations might drastically cut the number of vacua. While of course we would need to incorporate these effects in the distribution, it is not inconceivable that to a good approximation these effects are statistically independent of the properties of the distribution which concern us, so that the statistics we are computing now are the relevant ones.

7. Stringy naturalness

The upshot of the previous discussion is that in this picture, either string theory is not predictive because there are too many vacua, or else the key to making predictions is to count vacua and find their distributions, in other words that we should live in a natural vacuum according to the definition above.

While we admittedly do not know all the consistency conditions on vacua, and might need to take additional cosmological considerations into account, we believe the problem with such considerations taken into account will not look so different formally (and perhaps even physically) from the problem without them, and thus we proceed.

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Thus, we need to establish that vacua satisfying the various requirements exist, and estimate their distribution. We now discuss results on these two problems, and finally return to the question of the distribution of supersymmetry breaking scales.

8. Constructing KKLT vacua

The problem of stabilizing all moduli in a concrete way in string compactification has been studied for almost 20 years. One of the early approaches (Dine *et al*, Derendinger *et al*, Krasnikov, Dixon *et al*, ...) was to derive an effective Lagrangian by KK reduction, find a limit in which nonperturbative effects are small, and add sufficiently many nonperturbative corrections to produce a generic effective potential. Such a generic potential, depending on all moduli, will have isolated minima.

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A big step forward was the development of flux compactification (Strominger, Polchinski, Becker^2, Sethi *et al*, Giddings *et al*). Since the energy of fluxes in the compactification manifold depends on moduli, turning on flux allows stabilizing a large subset of moduli at the classical level.

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Acharya has proposed that in G_2 compactification, all metric moduli could be stabilized by fluxes. However it is not yet known how to make explicit computations in this framework.

The most computable class of flux compactifications at present is that of Giddings, Kachru and Polchinski, in IIb orientifold compactification, because one can appeal to the highly developed theory of Calabi-Yau moduli spaces and periods (Candelas *et al.*, Strominger, ...).

However the IIb flux superpotential does not depend on Kähler moduli, nor does it depend on brane or bundle moduli. Now one can argue that the brane/bundle moduli parameterize compact moduli spaces (*e.g.* consider the D3 brane), and thus they will be stabilized by a generic effective potential.

However, for the Kähler moduli, we need to show that the minimum is not at infinite volume, or deep in the stringy regime (in which case we lose control). Thus we need a fairly explicit expression for their effective potential.

Kachru *et al.* 0301240 (KKLT) proposed to combine the flux superpotential with

- a nonperturbative superpotential produced by D3 instantons and/or D7 world-volume gauge theory effects. These depend on Kähler moduli and can in principle fix them.
- energy from an anti-D3 brane, which would break supersymmetry and lift the c.c. to a small positive value.

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The main problem is that most brane gauge theories in compactifications which cancel tadpoles, have too much matter to generate superpotentials. One needs systematic techniques to determine this matter content, or compute instanton corrections, and find the examples which work.

In Denef, MRD and Florea 0404257, we found the first working examples. Our construction relies heavily on the analysis of instanton corrections in F theory due to Witten, Donagi and especially A. Grassi, math.AG/9704008.

Their starting point was to compactify M theory on a Calabi-Yau fourfold X. This leads to a 3D theory with four supercharges, related to F theory and IIb if X is T^2-fibered, by taking the limit $\mathrm{vol}(T^2) \to 0$. The complex modulus of the T^2 becomes the dilaton-axion varying on the base B.

$$\begin{tikzcd} T^2 \arrow[r] & X \arrow[d, "\pi"] \\ & B. \end{tikzcd}$$

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$$\begin{array}{ccc} T^2 & \longrightarrow & X \\ & & \downarrow{\scriptstyle \pi} \\ & & B. \end{array}$$

In M theory, an M5 brane wrapped on a divisor D (essentially, a hypersurface), will produce a nonperturbative superpotential, if D has arithmetic genus one:

$$1 = \chi(\mathcal{O}_D) = h^{0,0} - h^{0,1} + h^{0,2} - h^{0,3}.$$

Each of these complex cohomology groups leads to two fermion zero modes; an instanton contributes to W if there are exactly two. A known subset of these (vertical divisors) survive in F theory.

Thus, we looked for F theory compactifications on an elliptically fibered fourfold X with enough divisors of arithmetic genus one, so that a superpotential

$$W = W_{flux} + \sum_i b_i e^{-\vec{t}\cdot D_i}$$

will lead to non-trivial solutions to $DW = 0$ by balancing the exponentials against the dependence coming from the Kähler potential.

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In the math literature, there is a very general relation between divisors of a.g. one, and contractions of manifolds. It implies that no model with one Kähler modulus can stabilize Kähler moduli (see also Robbins and Sethi, 0405011), at least by using a.g. one divisors.

There may be ways beyond the a.g. one condition: Witten suggested that $\chi(D) > 1$ might work as well (without providing examples); furthermore it is possible that flux lifts additional matter and relaxes some of these constraints (Kachru, Trivedi *et al.*, to appear). Granting this, working one parameter models appear to exist (Denef, MRD, Florea, and Kachru, work in progress).

In any case, there is no problem if the CY threefold has more than one Kähler modulus, as in the vast majority of cases.

Using the very complete study of divisors of a. g. one of A. Grassi, math.AG/9704008, we have found 6 models with toric Fano threefold base which can stabilize all Kähler moduli, and could be analyzed in detail using existing techniques.

The simplest, $\mathcal{F}_{18}$, has 89 complex structure moduli. According to the AD counting formula, it should have roughly $\epsilon \times 10^{307}$ flux vacua with all moduli stabilized, where

$$\epsilon = g_s \times \frac{|W|^2}{m_s^6}\Big|_{max}.$$

Models which stabilize all Kähler moduli are not generic, because a.g. one divisors are not. However, they are not uncommon either; there are 29 out of 92 with Fano base, and probably many more with $\mathbb{P}^1$ fibered base. This last class of model should be simpler, in part because they have heterotic duals, but analyzing them requires better working out the D7 world-volume theories.

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We expect one can add antibranes or D breaking as in the KKLT discussion to get de Sitter vacua, but have not yet analyzed this.

9. Flux vacua

We recall the "flux superpotential" of Gukov, Taylor, Vafa and Witten in IIb string on CY,

$$W = \int \Omega(z) \wedge \left(F^{(3)} + \tau H^{(3)}\right).$$

The simplest example is to consider a rigid CY, *i.e.* with $b^{2,1} = 0$ (for example, the orbifold $T^6/\mathbb{Z}_3$). Then the only modulus is the dilaton τ, with Kähler potential $K = -\log \mathrm{Im}\ \tau$.

The flux superpotential reduces to

$$W = A\tau + B; \qquad A = a_1 + \Pi a_2; B = b_1 + \Pi b_2$$

with $\Pi = \int_{\Sigma_2} \Omega^{(3)} / \int_{\Sigma_1} \Omega^{(3)}$, a constant determined by CY geometry.

9. Flux vacua

We recall the "flux superpotential" of Gukov, Taylor, Vafa and Witten in IIb string on CY,

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Now it is easy to solve the equation $DW = 0$:

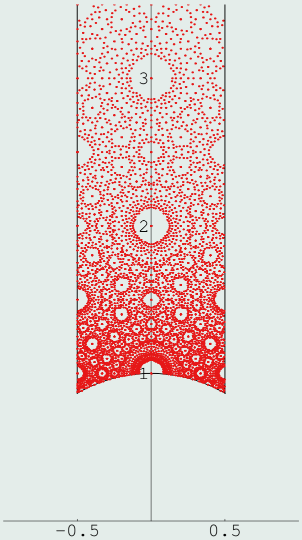
$$\begin{aligned} DW &= \frac{\partial W}{\partial \tau} - \frac{1}{\tau - \bar\tau} W \\ &= \frac{-A\bar\tau - B}{\tau - \bar\tau} \end{aligned}$$

so $DW = 0$ at

$$\bar\tau = -\frac{B}{A}$$

where $\bar\tau$ is the complex conjugate.

Here is the resulting set of flux vacua for $L = 150$ and $\Pi = i$:



This graph was obtained by enumerating one solution of $a_1 b_2 - a_2 b_1 = L$ in each $SL(2, \mathbb{Z})$ orbit, taking the solution $\tau = -(b_1 - ib_2)/(a_1 - ia_2)$ and mapping it back to the fundamental region.

The total number of vacua is $N = 2\sigma(L)$, where $\sigma(L)$ is the sum of the divisors of L. Its large L asymptotics are $N \sim \pi^2 L/6$.

A similar enumeration for a Calabi-Yau with n complex structure moduli, would produce a similar plot in $n+1$ complex dimensions, the distribution of flux vacua. It could (in principle) be mapped into the distribution of possible values of coupling constants in a physical theory.

The intricate distribution we just described has some simple properties. For example, one can get exact results for the large L asymptotics, by computing a continuous distribution $\rho(z, \tau; L)$, whose integral over a region R in moduli space reproduces the asymptotic number of vacua which stabilize moduli in the region R, for large L,

$$\int_R dz d\tau \ \rho(z, \tau; \ L) \sim_{L \to \infty} N(R).$$

For a region of radius r, the continuous approximation should become good for $L >> K/r^2$. For example, if we consider a circle of radius r around $\tau = 2i$, we match on to the constant density distribution for $r > \sqrt{K/L}$.

Another one complex structure modulus example with $r < \sqrt{K/L}$ was discussed by Girvayets, Kachru, Tripathy 0404243.

Explicit formulas for these densities can be found, in terms of the geometry of the moduli space $\mathcal{C}$. The simplest such result (with Ashok) computes the index density of vacua:

$$\rho_I(z,\tau) = \frac{(2\pi L)^{b_3}}{b_3!\pi^{n+1}} \det(-R - \omega \cdot 1)$$

where ω is the Kähler form and R is the matrix of curvature two-forms. Integrating this over a fundamental region of the moduli space produces an estimate for the total number of flux vacua. For example, for T^6 we found $I \sim 4 \cdot 10^{21}$ for $L = 32$. Since $r \sim 1$ in the bulk of moduli space, the condition $L > K/r^2$ for the validity of this estimate should be satisfied, but it is worth looking for subtleties here!

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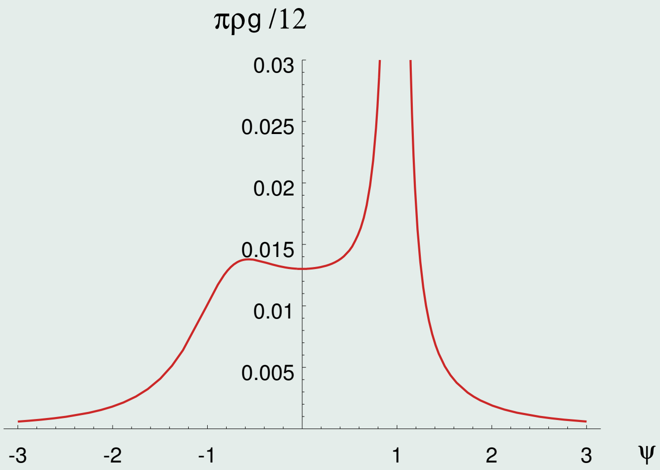
More examples: the "mirror quintic," with a one parameter moduli space $\mathcal{M}_c(\tilde{Q})$ (with Denef). The integral

$$\frac{1}{\pi^2}\int_{\mathcal{C}} \det(-R-\omega) = \frac{1}{12}\chi(\mathcal{M}_c(\tilde{Q})) = \frac{1}{60}.$$

This density is "topological" and there are mathematical techniques for integrating it over general CY moduli spaces (Z. Lu and MRD, work in progress). Good estimates for the index should become available for a large class of CY's over the coming years.

10. Distributions of flux vacua

Let us look at the details of the distribution of flux vacua on the mirror quintic ($K = 4$ and $n = 1$), as a function of complex structure modulus:



Note the divergence at $\psi = 1$. This is the conifold point, with a dual gauge theory interpretation. It arises because the curvature $R \sim \partial\bar{\partial} \log \log |\psi - 1|^2$ diverges there. The divergence is integrable, but a finite fraction of all the flux vacua sit near it.

Quantitatively, for the mirror quintic,

- About 3% of vacua sit near the conifold point, with an induced scale $|\psi - 1| < 10^{-3}$.
- About 1% of vacua have $|\psi - 1| < 10^{-10}$. More generally, the density and number of vacua with $S \equiv \psi - 1$ goes as

$$\rho_{vac} \sim \frac{d^2 S}{|S \log S|^2}; \qquad \mathcal{N}_{vac}|_{S<S_*} \sim \frac{1}{|\log S_*|}.$$

  Writing $S = e^{-1/g^2}$, this is $\rho \sim d^2 g/|g|^2$.

- About 36% of vacua are in the "large complex structure limit," defined as $\mathrm{Im}\ t > 2$ with $5\psi = e^{2\pi i t/5}$. Here $\rho \sim d^2 t/(\mathrm{Im}\ t)^2$.

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Vacua close to conifold degenerations are interesting for model building, as they provide a natural mechanism for generating large scale hierarchies (by dual gauge theory, or in supergravity as in Randall and Sundrum, etc.). We have found that such vacua are common, but are by no means the majority of vacua.

Suppose we go on to break supersymmetry by adding an anti D3-brane, or by other D term effects. The previous analysis applies (since we have not changed the F terms), but now it is necessary that the mass matrix at the critical point is positive.

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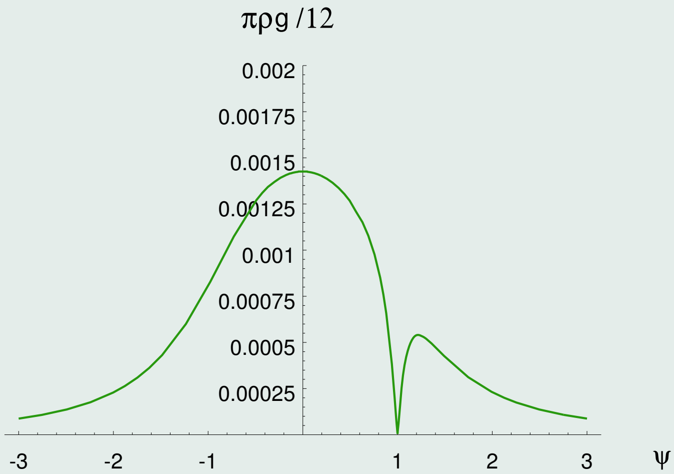
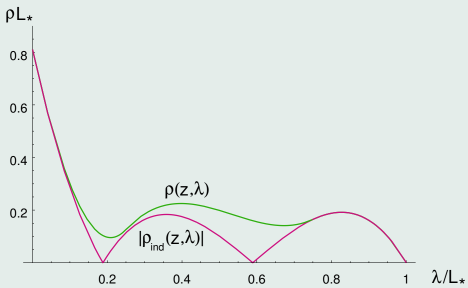
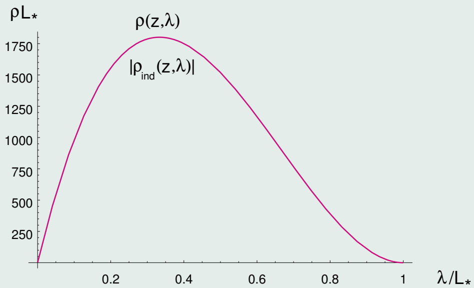
The distribution of tachyon-free D breaking vacua is



In fact, most D breaking vacua near the conifold point have tachyons (for one modulus CY's), so we get suppression, not enhancement. This is not hard to understand in detail; the mechanism is a sort of "seesaw" mixing between modulus and dilaton, which seems special to one parameter models.

Here is the distribution of (negative) AdS cosmological constants $\hat{\Lambda} = 3e^K|W|^2$, both at generic points (left) and near the conifold point (right). Note that at generic points it is fairly uniform, all the way to the string scale. On the other hand, imposing small c.c. competes with the enhancement of vacua near the conifold point.





The left hand graph compares the total number of vacua (green) with the index (red). The difference measures the number of Kähler stabilized vacua, vacua which exist because of the structure of the Kähler potential, not the superpotential.

11. Large complex structure/volume

Another simple universal property: given $n >> 1$ moduli, the number of vacua falls off rapidly at large complex structure, or in a IIa mirror picture at large volume V, as

$$\int_{V>V_0} \rho \sim V_0^{-n/3}.$$

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To see this, first note that $R \sim \omega$ in this regime, so the distribution of vacua is determined by the volume form derived from the metric on the space of metrics,

$$\langle \delta g_{ij}, \delta g_{kl} \rangle = \frac{1}{V} \int_{CY} \sqrt{g} g^{ik} g^{jl} \delta g_{ij} \delta g_{kl}$$

The $1/V$ factor (which compensates the $\sqrt{g}$) comes from the standard derivation of the kinetic term in KK reduction on CY.

Because of the inverse factors of the metric, this falls off with volume as $V^{-1/3}$. Since the volume form is $\sqrt{G}$, this factor appears for each modulus.

For large n, $\mathcal{N} \sim V^{-n/3}$ is a drastic falloff, and (as we saw explicitly in an example in DDF) typically there are no vacua in this regime.

A possible physical application of this: we know how to stabilize complex structure moduli using fluxes in IIb. Suppose we can use T-duality to get a corresponding class of models in IIa with stabilized Kähler moduli. Then, the mirror interpretation of this result is the number of vacua which stabilize the volume of the compact dimensions at a given value.

Clearly large volume is highly unnatural in this construction, but do **any** vacua reach the $V \sim 10^{30}$ of the "large extra dimensions" scenario?

Writing $V \sim R^6$, we find a number of vacua

$$N \sim \frac{(2\pi L)^K R^{-K}}{K!}$$

so large K disfavors large volume in this case, and the maximum volume one expects is of order

$$V \sim \left(\frac{2\pi L}{K}\right)^6 .$$

where the parameter $L = \chi/24$ in F theory compactification on fourfolds.

In fact the maximal value known for L is $L = 75852$, but this comes with a large K as well. On the other hand, the effective K might be reduced by imposing discrete symmetries.

12. Susy breaking

D breaking vacua (with $DW = 0$) are described by the earlier results, just we require the vacua to be tachyon free and have near zero c.c. With Denef, we have analyzed F breaking flux vacua in orientifolds in some detail. These satisfy

$$0 = \partial_I V = Z_I \bar{Y}^0 + \mathcal{F}_{IJK} \bar{Y}^J \bar{Z}^K - 2Y_I \bar{X} \qquad (1)$$

where $X = |W|$, $Y_I = D_I W$, $Z_I = D_0 D_I W$ and $\mathcal{F}_{IJK}$ are the special geometry cubic couplings.

The simplest way to satisfy this is to have $W = D^2 W = 0$, giving an "anti-supersymmetric branch" of vacua, whose flux is anti-self-dual. These are probably not physical as they have large positive c.c. The more interesting F breaking vacua are the "mixed branches," in which the parameters $Y = DW$ satisfy relations allowing $X = |W| \neq 0$ (the matrix implicit in (1) is reduced in rank).

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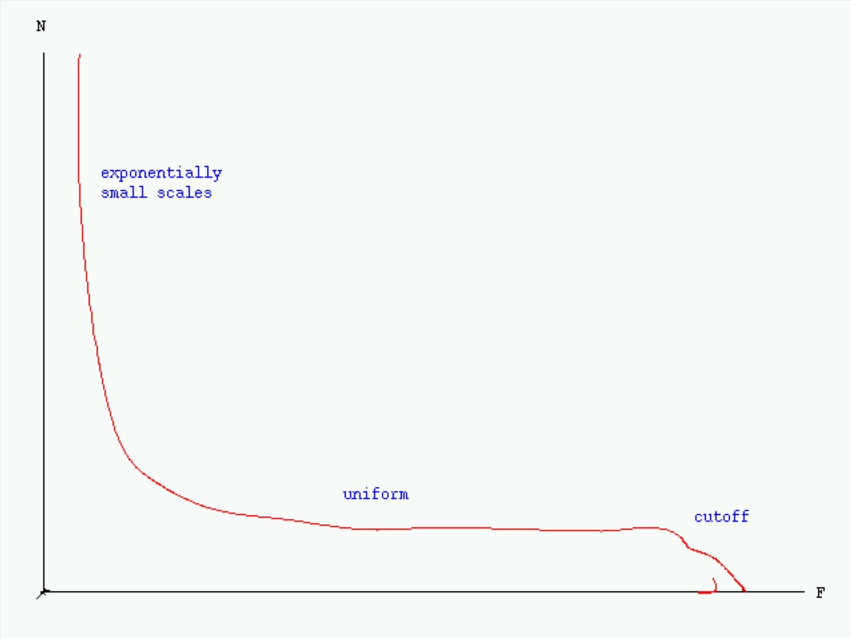
Such mixed breaking F vacua, in which W can be non-zero, appear generic, but (in the limits we considered – large complex structure and conifold) all with $\Lambda = V < 0$. We think this is because stabilizing de Sitter vacua requires two points of inflection in the potential, and in these limits the potential is too simple to obtain this.

In other regimes or with multiple moduli, it seems likely that the traditional scenarios (*e. g.* susy breaking at hierarchically small scales) are fairly generic. While this remains to be verified in detail, the likely picture of the distribution of individual supersymmetry breaking parameters, by analogy to previous results, is the sum of a uniform component with a fairly high cutoff, probably M^2_{str}/n_{moduli}, and a component at hierarchically small scales.



Predictions from . . .
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Let us work with this ansatz, and see what we can say about the distribution of supersymmetry breaking scales in models which work. First, we allow models with a high scale of supersymmetry breaking, granting that in a fraction M_H^2/M_{susy}^2, statistical fine tuning will produce the correct Higgs mass. Furthermore, since as $M_{susy} \to 0$ we have $\Lambda \to 0$, one might guess that

$$d\mu[M_H^2, M_{susy}^2, \Lambda] \sim \frac{dM_H^2}{M_{susy}^2} \; \frac{d\Lambda}{M_{susy}^4} d\mu[M_{susy}^2].$$

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However, as we saw earlier, the distribution of the parameter

$$\hat{\Lambda} = 3e^K |W|^2$$

in flux vacua (not $|W|$) is fairly uniform, from zero all the way to the string scale. This means that an arbitrary supersymmetry breaking contribution to the vacuum energy,

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Thus, the distribution goes as $d\Lambda/M_{str}^4$, and the need to get small c.c. **does not favor a particular scale of susy breaking** in these models.

Thus, our corrected guess is

$$d\mu[M_H^2, M_{susy}^2, \Lambda] \sim \frac{dM_H^2}{M_{susy}^2}\ \frac{d\Lambda}{M_{str}^4} d\mu[M_{susy}^2].$$

The physics behind this is that the superpotential W is a sum of contributions from the many sectors. This includes supersymmetric hidden sectors, so there is **no reason** W should be correlated to the scale of supersymmetry breaking, and **no reason** the cutoff on the W distribution should be correlated to the scale of supersymmetry breaking.

Such a sum over randomly chosen complex numbers will tend to produce a distribution d^2W, uniform out to the cutoff scale. For fluxes this is the string scale, and this is plausible for supersymmetric sectors more generally. Finally, writing $W = e^{i\theta}|W|$, we have

$$d^2W = \frac{1}{2}d\theta d(|W|^2) = \frac{1}{2}d\theta d(|W|^2)$$

leading very generally to the uniform distribution in $\hat{\Lambda} = 3|W|^2$.

Finally, since this distribution is uniform and uncorrelated with F, D, solving the constraint $\Lambda = |F|^2 + |D|^2 - \hat{\Lambda}$ for $\hat{\Lambda}$ introduces no dependence on M_{susy}.

We still need to estimate $d\mu[M^2_{susy}]$. The first definition of M^2_{susy} we might try is

$$M^4_{susy} = \sum_i |F_i|^2 + \sum_\alpha D^2_\alpha .$$

With this definition, if several F or D terms are present, with any significant uniform component in their distribution, the overall distribution heavily favors high scale breaking (DD 0404116 and MRD 0405279; this observation was made independently in Susskind, hep-th/0405189).

For example, convolving uniform distributions gives

$$\begin{aligned} \rho(M^2_{susy}) &= \int \prod_{i=1}^{n_F} d^2F \ \prod_{\alpha=1}^{n_D} dD \ dM^4_{susy} \ \delta(M^4_{susy} - \sum |F|^2 - \sum D^2) \\ &\sim (M^2_{susy})^{2n_F+n_D-1} dM^2_{susy} \end{aligned}$$

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Combining this with the factor m^2_H/M^2_{susy}, we find that high scale susy breaking is favored if $2n_F + n_D > 2$, a condition surely satisfied by almost all string models.

Now the distribution we suggested earlier was not uniform: there was a second component counting susy breaking vacua at hierarchically small scales. This can modelled by the distribution

$$d\mu[F] = \prod_{i=1}^{n_F} d^2F_i \left(c\delta^2(F_i) + (1-c)d^2F_i\right)$$

in which the delta function represents all "low scale" vacua. Then, for small c (recall $c \sim .03$ for the mirror quintic),

$$\rho(M^2_{susy}) = \int d\mu[F]\ dM^4_{susy}\ \delta(M^4_{susy} - \sum |F|^2)$$

still heavily favors high scale breaking.

One can compute this convolution of distributions. A rough description of the result is that low scale breaking requires **all** $n \equiv n_F$ breaking parameters to come from the $\delta^2(F_i)$ part of the distribution, while if **any** sector sees the uniform component, high scale breaking will result. This leads to the rough estimate that a fraction c^n of vacua will have low scale breaking, while the rest have high scale breaking.

Given that a fraction c^n of vacua have low scale breaking, while the rest have high scale breaking, high scale breaking will be favored if

$$\frac{M_H^2}{M_{high}^2} > c^n$$

where M_{high} is the cutoff or peak of the distribution of high scale breaking vacua. Assuming this is $M_{high} \sim M_s \sim 10^{16}$ GeV would lead to the criterion

$$n_F > \log_{1/c} 10^{28} \sim 20$$

granting $c \sim .03$ as above. Since the vast majority of CY's have more than 20 moduli, and we need many moduli to tune the c.c., this argument seems to predict **high scale supersymmetry breaking**.

What are the physical consequences of this? The most direct is the gravitino mass, which is

$$M_{3/2}^2 = M_{susy}^4/M_{pl}^2.$$

Thus one would generally predict a **high gravitino mass** thanks to **supersymmetry breaking in hidden sectors**.

In general, this is not the parameter which controls the hierarchy or most of the observable consequences of susy breaking. However, since gravitino loop effects generally produce soft masses of order $M \sim M_{3/2}^2/M_{pl}$, for $M_{susy} \geq 10^{15}$ GeV or so, these effects will produce soft masses $M > 10$ TeV.

Thus, if $M_{high} \geq 10^{15}$ GeV (well below M_s given string scale compact dimensions), we start to have the gist of an argument predicting that **we will not see superpartners at LHC.**

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So will we see superpartners at LHC ? If we believe in the joint distribution we just discussed of F breaking parameters of otherwise acceptable vacua, then apparently not.

With Denef, we are working on better computing and characterizing this distribution. So far, we have identified several loopholes in the above argument.

1. Since the equations $V' = 0$ for nonsupersymmetric flux vacua are quadratic in the fluxes, they have several branches of solutions, characterized by the dimension $2m$ of the subspace of breaking parameters F_i. The resulting distribution looks more like

$$d\mu[F] \sim L^K \sum_m \left(\frac{F^2}{L}\right)^m d^2F.$$

A simple model distribution illustrating the probable effect of this is to add a **third** component, which is literally $\delta^2(F_i)$, allowing for the possibility that supersymmetry is unbroken in some directions:

$$d\mu[F] = \prod_{i=1}^{n_F} d^2F_i \left((\frac{1}{2} + c)\delta^2(F_i) + (\frac{1}{2} - c)d^2F_i \right)$$

This changes the numbers: now

$$n_F > \log_2 10^{28} \sim 100$$

favors high scale breaking, but not the general conclusion: most CY's still have $b_{2,1} > 100$.

2. Some CY fourfolds actually have $\chi \leq 0$ and one expects high scale vacua to dominate from the start. However these are in the minority.

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Suppose we trust the LM_s^4 cutoff on supersymmetric vacua (leading to numbers like L^K), but believe that supersymmetry breaking vacua are typically unstable when $F, D > M_s^2$. Then every factor F^2/L in

$$d\mu[F] \sim L^K \sum_m \left(\frac{F^2}{L}\right)^m d^2F.$$

turns into a factor $1/L$, and we find

$$\frac{N(\text{high scale vacua})}{N(\text{low scale vacua})} \sim \left(1+\frac{1}{L}\right)^{n_F}.$$

Since $L \sim n_F$ at best, this is very unlikely to match the 10^{28} we gain from low scale supersymmetry, and we predict **low scale susy**.

Many points would have to be nailed down to get a convincing argument one way or the other. Perhaps the two which emerged most clearly from our discussion are to push through the formal computations of distributions of nonsupersymmetric vacua in a variety of constructions, and to understand what cuts off the high end of the distribution of supersymmetry breaking scales

To name a few more issues, it might be that physics we neglected also puts a lower cutoff on the maximal flux for supersymmetric vacua, it might be that the μ problem is hard to solve, there might be large new classes of nonsupersymmetric vacua, etc. Even if the majority of string/M theory vacua predicted high scale susy, one might try to argue that the initial conditions biased the distribution, etc.

In any case, we explained a simple observation, namely the existence of vacua with high breaking scales in hidden sectors, and the large multiplicity of hidden sectors, which in principle might lead to so many high scale models that high scale supersymmetry breaking becomes the natural outcome of string/M theory compactification.

13. Conclusions

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We have gone some distance in justifying and developing the statistical approach to string compactification:

We have specific IIb orientifold compactifications in which all Kähler moduli are stabilized, and vacuum counting estimates which suggest that all moduli can be stabilized. So far, it appears that such vacua are not generic, but they are not uncommon either: about a third of our sample of F theory models with Fano threefold base should work.

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We have explicit results for distributions of flux vacua of many types: supersymmetric, non-supersymmetric, tachyon-free. They display a lot of structure, with suggestive phenomenological implications:

- Large uniform components of the vacuum distribution.
- Enhanced numbers of vacua near conifold points.
- Correlations with the cosmological constant.
- Falloff in numbers at large volume and large complex structure.

There are intuitive arguments for some of the most basic properties. For example, the $-3|W|^2$ contribution to the supergravity potential is uniformly distributed with a large (at least string scale) cutoff, because of contributions from supersymmetric hidden sectors. Thus, the need to tune the c.c. does not much influence the final numbers.

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We start to see the possibility of making real world predictions:

- Large extra dimensions are heavily disfavored with the present stabilization mechanisms.
- Hierarchically small scales (gauge theoretic or warp factor) are relatively common.
- Uniform distributions involving many susy breaking parameters, favor high scales of supersymmetry breaking.
- This may imply that the gravitino and even superpartner masses should be high, thanks to supersymmetry breaking in hidden sectors.

While various assumptions entered into the arguments we gave, the only essential ones are that

- Our present pictures of string compactification are representative of the real world possibilities.
- The absolute number of relevant string/M theory compactifications is not too high.

With further work, all the other assumptions can be justified and/or corrected, because they were simply shortcuts in the project of characterizing the actual distribution of vacua.

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Since interesting results already follow from general properties of the theory, and we now have evidence that the detailed distribution of string/M theory vacua has many simple properties, we are optimistic that a reasonably convincing picture of supersymmetry breaking and other predictions can be developed in time for Strings 2008 at CERN.

> Tout ce qui est simple est faux, mais tout ce qui ne l'est pas est inutilisable.
> – Paul Valéry