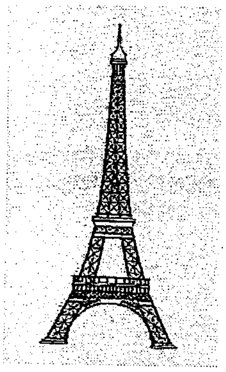
FROM

FREE FIELDS

TO

 dS

(via PARIS!)

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H.R.I., ALLAHABA

(Based on hep-th/0308184 + WORK IN PROGRESS)
" /0402063

TWO QUESTIONS

(1) HOW EXACTLY DOES A LARGE N FIELD THEORY REORGANISE ITSELF INTO A DUAL CLOSED STRING THEORY

(2) CAN WE SYSTEMATICALLY CONSTRUCT THE CLOSED STRING THEORY STARTING FROM THE FIELD THEORY?

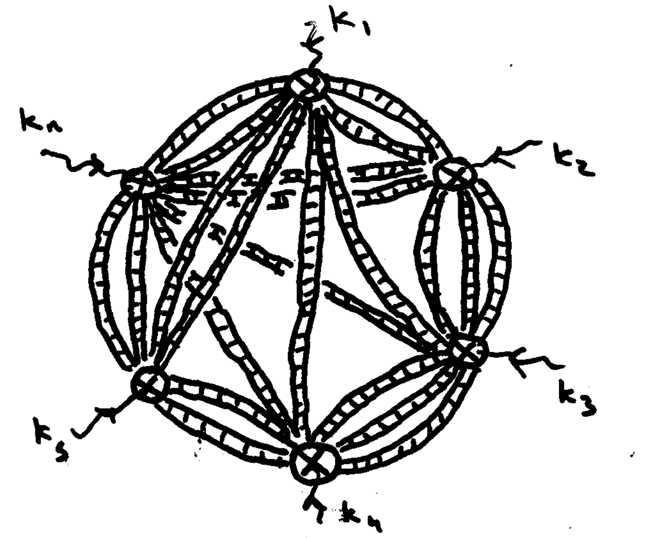
- ADDRESS THESE QUESTIONS IN THE CONTEXT OF FREE FIELD THEORIES

[But keeping in mind the extension to $\lambda \neq 0$.]

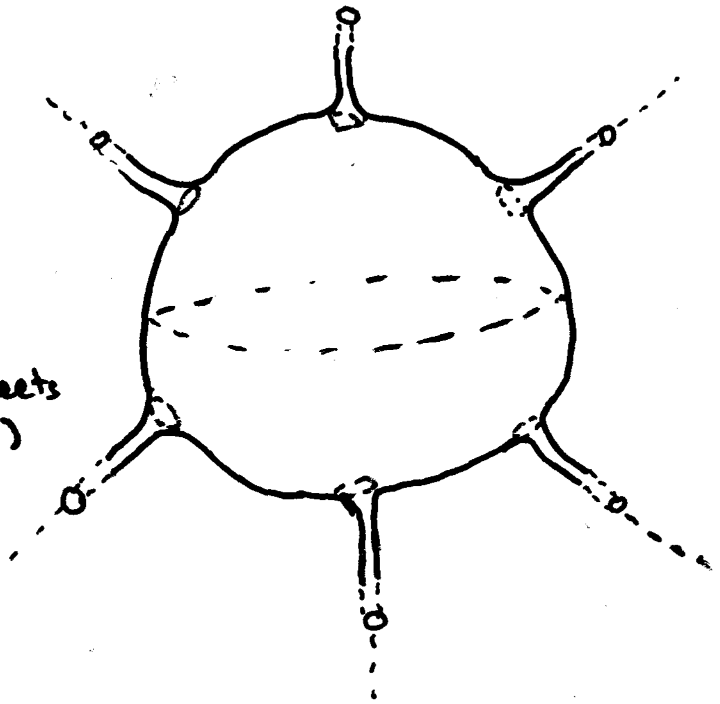
ONE AND A HALF ANSWERS

(1) $$G^{\{J_i\}}_{(g)}(k_1 \ldots k_n) = \Big\langle \prod_{i=1}^{n} \mathrm{Tr}\, \Phi^{J_i}(k_i) \Big\rangle_{\substack{\text{genus } g \\ \text{CONN.}}}$$

$$= \sum_{\substack{\text{graphs} \\ (\text{genus } g)}}$$



$$= \sum_{\substack{\text{worldsheets} \\ (\text{genus } g)}}$$



[DISTINCT FROM
MODULI SPACE O
OPEN STRING THEO
UNDERLYING THE
FIELD THEORY

[INTEGRAL OVER $\tilde{M}_{g,n} \cong M_{g,n} \times \mathbb{R}_+^n$

MODULI SPACE
OF GENUS g SURFACES
w/ n HOLES

MODULI S
OF GENUS g S
w/ n MARKED
POINT

(2) $G_{(g)}^{\{J_i\}}(k_1 \ldots k_n)$

$$= \int_{\mathcal{M}_{g,n} \times \mathbb{R}_+^n} [d\sigma]\, \rho^{\{J_i\}}(\sigma)\, e^{-\sum_{i,j=1}^{n} k_i \cdot k_j\, g_{ij}(\sigma)}$$

moduli

"universal" (independent of J_i)

- $\rho^{\{J_i\}}(\sigma)$ and $g_{ij}(\sigma)$ can be explicitly written.

- FORM OF INTEGRAND VERY REMINISCENT OF STRING THEORY (Contrast with $e^{\frac{1}{2}\sum_{i,j} k_i \cdot k_j \ln|z_i - z_j|}$ in Virasoro-Shapiro amplitude.)

- INTEGRAND PRESUMABLY THE <u>ANSWER</u> FOR A WORLD SHEET CORRELATOR

"$\displaystyle\int_{\mathcal{M}_{g,n}} \langle V_{J_1}(\xi_1) \ldots\ldots V_{J_n}(\xi_n) \rangle_{\substack{AdS\\WS}}$"

LOTS OF INFORMATION HERE TO RECONSTRUCT THE DUAL CLOSED STRING THEORY.

- TWO AND THREE PT. FUNCTIONS NATURALLY THOSE OF AdS

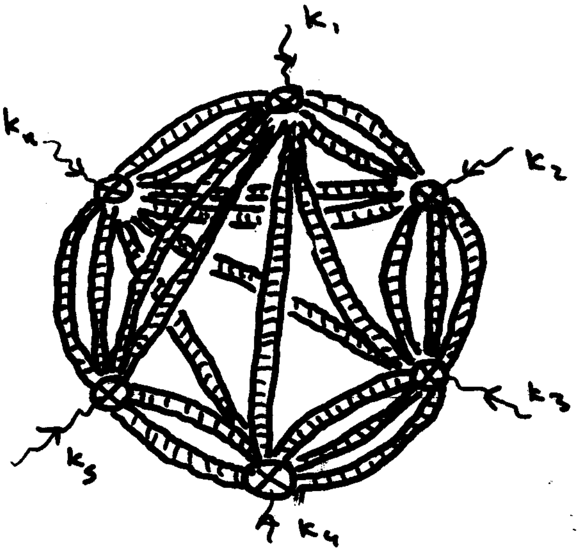
WORLDSHEET FOUR (or higher) POINT FUNCTIONS EXTRACTED FROM THIS PROCEDURE APPEAR TO BE CONSISTENT w/ WORLDSHEET OPE (follows from factorisation in Spacetime OPE). - WORK IN PROGRESS

SUGGESTS WE TAKE THESE EXPRESSIONS SERIOUSLY AS WORLDSHEET AMPLITUDES FOR RECONSTRUCTING THE CLOSED STRING THEORY

— IN FACT, OUR PROCEDURE IS A CONCRETE IMPLEMENTATION OF OPEN-CLOSED DUALITY.

Schwinger Parametrisation Of Field Theory Amplitudes

$$G^{\{J_i\}}_{(g)}(k_1 \ldots k_n) = \sum_{\substack{\text{graphs}\\(\text{genus } g)}}$$



Internal momenta

$$= \sum_{\substack{\text{graphs}\\(\text{genus } g)}} \int [d^d p] \int [d\tilde{\tau}]\, e^{-\tilde{P}(k,p,\tilde{\tau})}$$

Gaussian In (k, p)

$$\left[\text{Using } \frac{1}{p^2} = \int_0^\infty d\tilde{\tau}\, e^{-\tilde{\tau} p^2}\right]$$

e.g. Itzykson + Zuber]

$$= \sum_{\substack{\text{graphs}\\(\text{genus } g)}} \int_0^\infty \frac{\prod_i d\tilde{\tau}_i}{\Delta(\tilde{\tau})^{d/2}} \exp\left[-P(\tilde{\tau}, k)\right]$$

Quadratic in k

One Schwinger Parameter $\tilde{\tau}$ For Each Edge

$\left[\text{Total } \# = \frac{1}{2}\sum_i J_i\right]$

Thus, For E.G. A 6 Pt. Fn. Might Be Represented As An Integral Over 20,000 Parameters!

Explicit Expressions Can Be Written For $\Delta(\tilde{\tau})$ And $P(\tilde{\tau},k)$ In Terms Of Graph Structure.

[Nakanishi,
Symanzik,
Lam + Le Brun ...

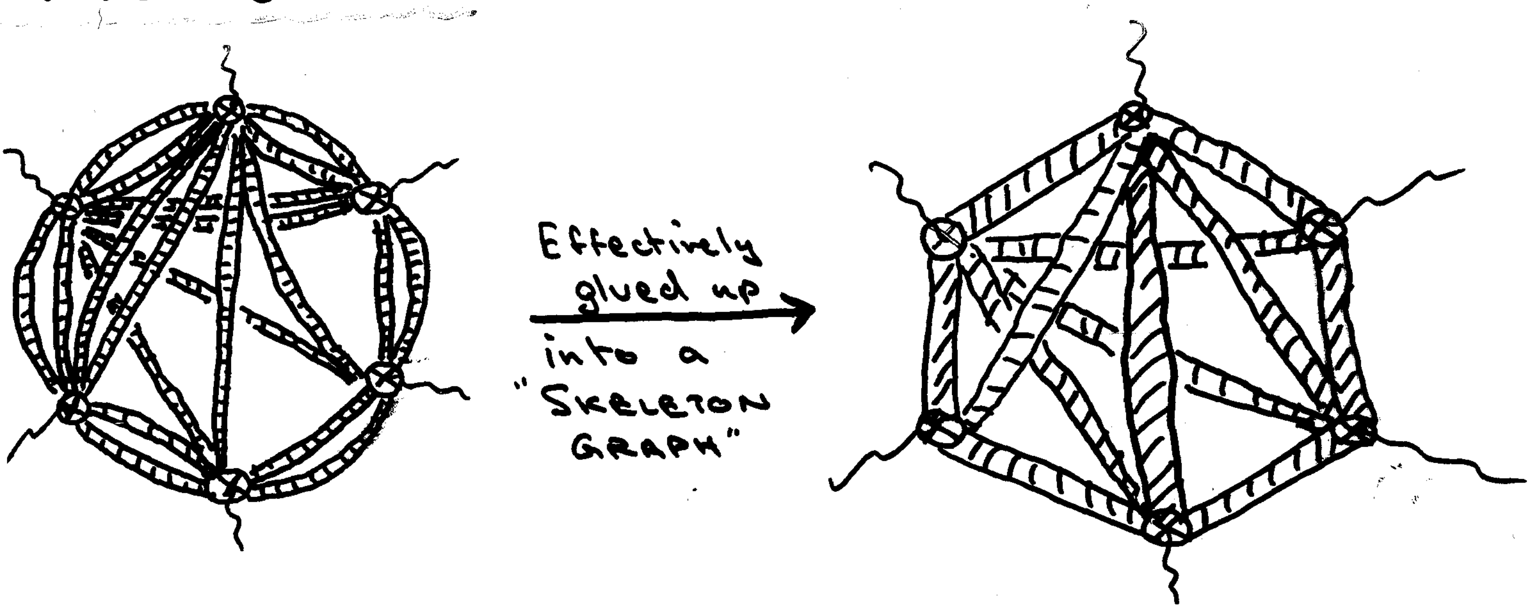
$$\Delta(\tilde{\tau}) = \sum_{T_1} \left(\prod^{\ell} \tilde{\tau} \right)$$

[T_1 = 1-tree: obtained by cutting graph at ℓ edges to get a connected tree. $\prod^{\ell}$ Over all parameters of cut edg

$$P(\tilde{\tau},k) = \frac{1}{\Delta(\tilde{\tau})} \sum \left(\prod^{\ell+1} \tilde{\tau} \right) \left(\sum k \right)^2$$

[T_2 = 2-tree: obtained by cutting graph at $\ell+1$ edges to get two disconnected trees. $\sum k$ = momentum flowing into either tree.]

SIMPLIFICATION:



CAN REPLACE PARALLEL EDGES (homotopically defm'bl

WITH SCHWINGER PARAMETERS $\tilde{\tau}_{r\mu_r}$ $(\mu_r = 1 \ldots m_r)$

BY A SINGLE EFFECTIVE SCHWINGER PARAMETER

$$\frac{1}{\tau_r} = \sum_{\mu_r=1}^{m_r} \frac{1}{\tilde{\tau}_{r\mu_r}}$$

m_r → MULTIPLICITY OF PARALLEL EDGES

POSSIBLE BECAUSE

$$P(\tilde{\tau}, k) = P_{Skel}(\tau, k)$$

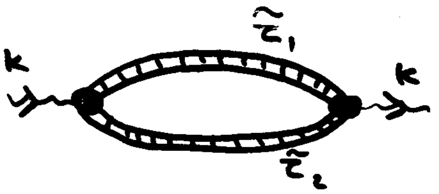
$$\Delta(\tilde{\tau}) = \left(\frac{\Pi\,\tilde{\tau}}{\Pi\,\tau}\right)\Delta_{Skel}(\tilde{\tau})$$

Note: RHS defined in terms of skeleton graph structure

Correspondence With Electrical Networks

Feynman Graph	$\longleftrightarrow$	Elec. Network
External Momenta (k_i)	$\longleftrightarrow$	Ext. Currents
Internal Momenta (p_i)	$\longleftrightarrow$	Int. Currents
Schwinger Moduli (τ)	$\longleftrightarrow$	Resistances
$\tilde{P}(k,p,\tau)$	$\longleftrightarrow$	Power Dissipated In Original Circuit
$P(\tau,k)$	$\longleftrightarrow$	Power Dissipated In Equiv. Circuit (after eliminating int. currents)

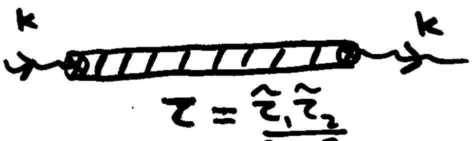
E.G.



$$= \int d^d p \int_0^\infty d\tilde{\tau}_1\, d\tilde{\tau}_2\, e^{-[\tilde{\tau}_1 p^2 + \tilde{\tau}_2 (k-p)^2]}$$

$$\tilde{P}(k,p,\tilde{\tau})$$

$\|$



$$= \int_0^\infty \frac{d\tilde{\tau}_1\, d\tilde{\tau}_2}{(\tilde{\tau}_1 + \tilde{\tau}_2)^{d/2}}\, e^{-\frac{\tilde{\tau}_1 \tilde{\tau}_2}{\tilde{\tau}_1 + \tilde{\tau}_2} k^2}$$

$$P(\tilde{\tau},k)$$

$\therefore$ Two Parallel Resistors $\tilde{\tau}_1$, $\tilde{\tau}_2$ Replaced By Single Effective Resistor.

Generalises To Multiple Parallel Edges

HENCE, FOR EACH GRAPH:

$$\int_0^\infty \frac{\prod_i d\tilde{\tau}_i}{\Delta(\tilde{\tau})^{d/2}} e^{-P(\tilde{\tau},k)} = C^{\{m_r\}} \int_0^\infty \prod_r \left(\frac{d\tau_r}{\tau_r^{(m_r-1)(\frac{d}{2}-1)}} \right) \frac{e^{-P_{skel}(\tau,k)}}{\Delta_{skel}(\tau)^{d/2}}$$

From Jacobian: $(\tilde{\tau} \to \tau)$

[Info. about J_i enters only here $\sum m_{r(i)} = J_i$]

Summing over graphs w/ diff. m_r

$$\Rightarrow G_{(g)}^{\{J_i\}}(k_1 \ldots k_n) = \sum_{\substack{\text{skeleton} \\ \text{graphs} \\ (\text{genus } g)}} \int_0^\infty \frac{\prod_r d\tau_r f^{\{J_i\}}(\tau)}{\Delta_{skel}(\tau)^{d/2}} e^{-P_{skel}(\tau,k)}$$

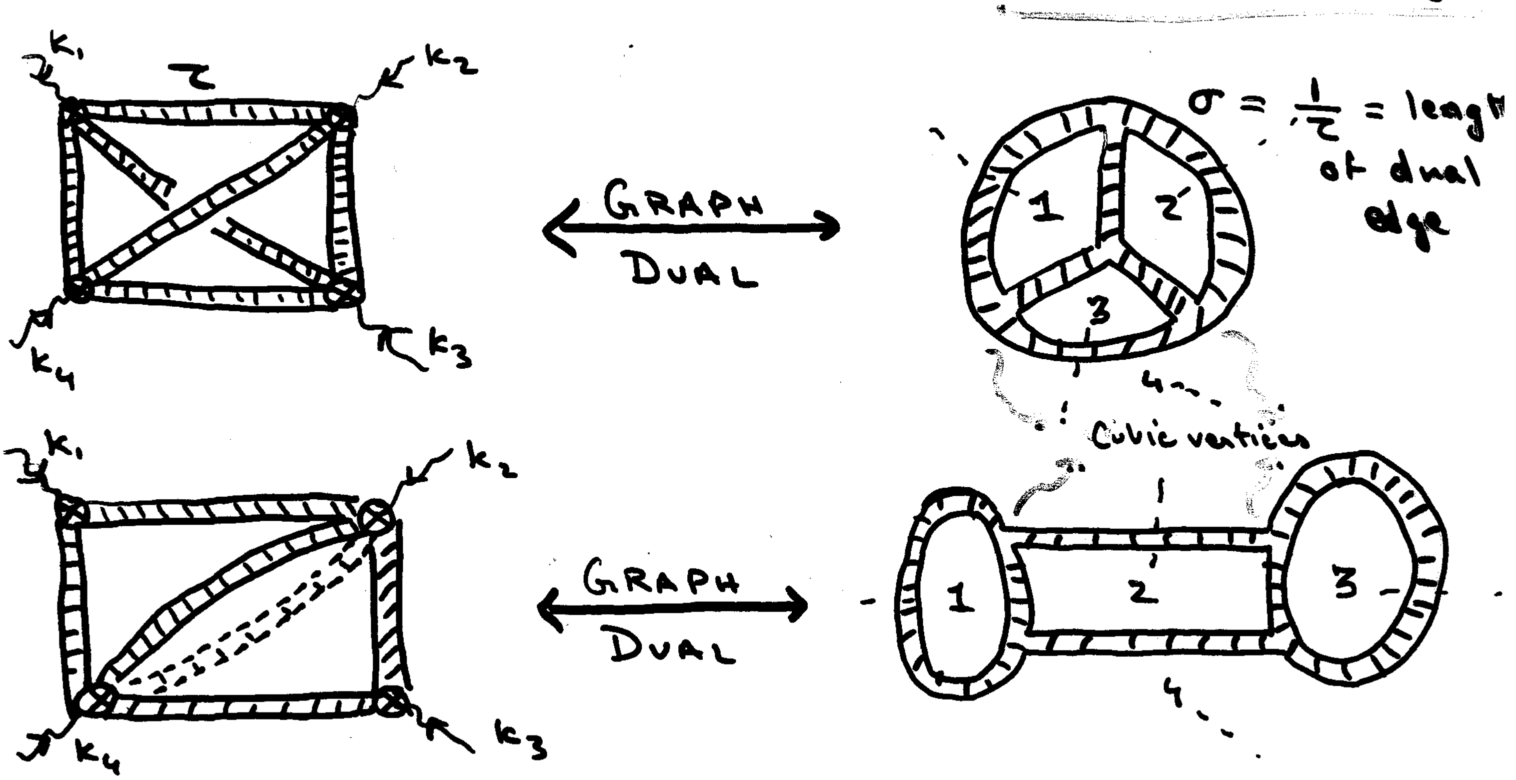
THUS SUM OVER MODULI SPACE OF SKELETON GRAPHS (lengths + connectivity).

GENERICALLY (for $J_i >$ min. # $\propto n$) SKEL. GRAPHS HAVE TRIANGULAR FACES. (all poss. connections consistent w/ genus)

[For e.g. planar skeleton graphs consist of triangulations of sphere w/ n vertices. $\Rightarrow 3(n-2)$ edges]

MODULI SPACE OF SKELETON GRAPHS (genus g w/ n marked vertices) $\equiv$ (via GRAPH DUALITY) MODULI SPACE OF RIEMANN SURFACES (genus g w/ n marked points + radius to each point)

$$= \boxed{\mathcal{M}_{g,n} \times \mathbb{R}_+^n \cong \hat{\mathcal{M}}_{g,n}}$$



DUAL GRAPHS GIVE CUBIC SFT CELL DECOMP. OF MODULI SPACE w/ $\sigma = \frac{1}{\tau}$ = length of strips.

[Cells match smoothly onto each other at boundaries $\sigma \to 0$.]

{Giddings, Martinec, Witten
⋮
Zwiebach

+ Parallel Math. Developments {Strebel, Penner, Kontsevich... }

$$G^{\{\tau_i\}}_{(g)}(k_1\ldots k_n) = \int\limits_{\mathcal{M}_{g,n}\times\mathbb{R}_+^n} [d\sigma]\,\rho^{\{\tau_i\}}(\sigma)\, e^{-\sum_{i,j}^n k_i\cdot k_j\, g_{ij}(\sigma)}$$

(after changing variables $\sigma = 1/\tau$.)

To Be Compared With Correlator In AdS String Theory

$$G^{\{\Delta_i\}}_{(g)}(k_1\ldots k_n) = \underbrace{\int_0^\infty \prod_{i=1}^n \left[dp_i\, p_i^{\Delta_i-\frac{d}{2}-1} e^{-\frac{k_i^2}{p_i}}\right]}_{\mathbb{R}_+^n} \int\limits_{\mathcal{M}_{g,n}} \Big\langle \prod_i^n t(\xi_i)^{\frac{\Delta_i}{2}} e^{-t(\xi_i)p_i}\, e^{ik_i\cdot X(\xi_i)}$$

parametrises ext. legs in AdS

[$t(\xi) \equiv x_0^2(\xi)$ and $\vec{X}(\xi)$ are worldsheet fields parametrising AdS.]

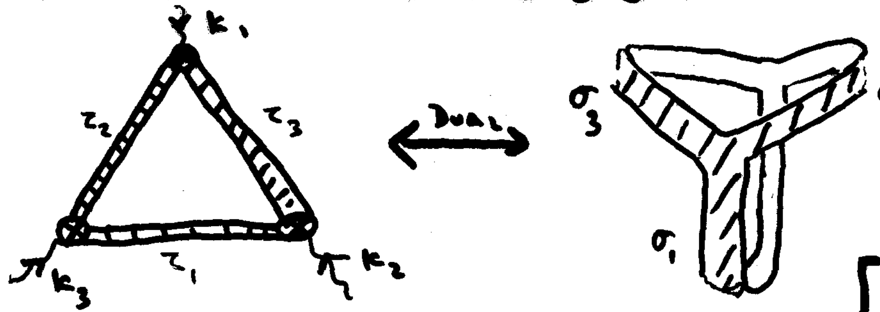
$$G^{\{\Delta_i\}}_{(g)}(x_1\ldots x_n) = \int\limits_{\mathcal{M}_{g,n}} \Big\langle \prod_{i=1}^n \frac{t(\xi_i)^{\Delta_i/2}}{\left[t(\xi_i)+(x_i-X(\xi_i))^2\right]^{\Delta_i}} \Big\rangle_{WS}$$

$$= \underbrace{\int_0^\infty \prod_{i=1}^n \left[dp_i\, p_i^{\Delta_i-1}\right]}_{\mathbb{R}_+^n} \int\limits_{\mathcal{M}_{g,n}} \Big\langle \prod_{i=1}^n t(\xi_i)^{\Delta_i/2} e^{-t(\xi_i)p_i - p_i(x_i-X(\xi_i))^2}$$

Fourier Transform And Obtain $G^{\{\Delta_i\}}_{(g)}(k_1\ldots k_n)$ As Above

Three Point Function

$$G_0^{\{J_i\}}(k_1,k_2,k_3) =$$



$$\left[\sigma_i = \frac{1}{\tau_i}\right]$$

$$= \int_0^\infty \frac{\prod_{r=1}^{3} d\sigma_r \, \sigma_r^{(m_r-1)(\frac{d}{2}-1)+\frac{d}{2}-2}}{\hat{\Delta}(\sigma)^{d/2}} \, e^{-\frac{1}{\hat{\Delta}(\sigma)}\left[\sigma_1 k_1^2 + \sigma_2 k_2^2 + \sigma_3 k_3^2\right]}$$

$$\left[\hat{\Delta}(\sigma) = \sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1\right]$$

Make Change Of Variables $\frac{1}{\rho_i} = \frac{\sigma_i}{\hat{\Delta}(\sigma)}$

[Going From Conductance Of Delta (Δ) To Star (Y)]

$$G_0^{\{J_i\}}(k_1,k_2,k_3) = \int_0^\infty \prod_{i=1}^{3} \left[d\rho_i \, \rho_i^{\Delta_i - \frac{d}{2} - 1} e^{-\frac{k_i^2}{\rho_i}}\right] \frac{1}{\left(\sum_k \rho_k\right)^{\sum_k \frac{\Delta_k}{2} - \frac{d}{2}}}$$

Compare With $G_0^{\{\Delta_i\}}(k_1,k_2,k_3)$ Of AdS

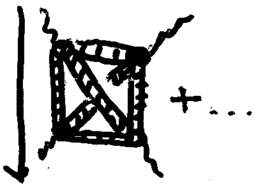
Exactly The Same Representation And Answer By Assuming Only Zero Modes Of $t(\xi)$ And $\vec{X}(\xi)$ Contribute To String Three Point Function.

FROM SPACETIME OPE TO WORLDSHEET OPE

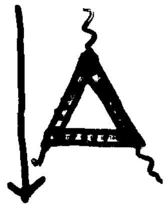
FOR HIGHER PT. FNS., WORLDSHEET INTERPRETATION REQUIRES THAT WORLDSHEET OPE HOLDS. HOW?

SPACETIME OPE:

$$\left\langle O_1(k_1)\, O_2(k_2)\, O_3(k_3)\, O_4(k_4) \right\rangle_{\substack{\text{planar}\\ \text{conn}}} \xrightarrow{|k_3-k_4|\to\infty} \sum_{l} C^{l}_{34}(k_3-k_4) \left\langle O_1(k)\, O_2(k)\, O_l(k_3 \ldots\right.$$



$+\ldots$



$$\int_0^\infty \prod_{i=1}^{6} d\tilde\sigma_i\, \rho_{(4)}[\hat\sigma]\, e^{-\frac{1}{\hat\Delta_4(\hat\sigma)}\left[\sum k_i^2 f_i(\hat\sigma) + (k_1+k_2)^2(\,) + \ldots\right]}$$

$$\xrightarrow[|k_3-k_4|\to\infty]{} \sum_{l} C^{l}_{34}(k_3-k_4) \int_0^\infty \prod_{i=1}^{3} d\sigma_i\, \rho_3[\sigma]\, e^{-\frac{1}{\Delta_3(\sigma)}\left[\sum k_i^2 \ldots\right.}$$

LAST LINE APPEARS TO FOLLOW INDEPENDENTLY FROM WORLDSHEET OPE.

As $|k_3-k_4| \to \infty$, LHS gets contributions from $\tau_{34} \to 0$ (). The Schwinger parametrisation collapses to that of 3-pt. fn. on RHS ()

[$\tau_{34} \to 0$ corresponds to going to bdy. of moduli sp. $\sigma_{34} \to \infty$]

$\therefore$ WORLDSHEET OPE AS $\tau_{34} \to 0$ SEEMS CONSISTENT w/ SPACETIME OPE (and thus factorisability).

WHAT NOW?

- World sheet OPE (from spacetime OPE?) as characterisation of closed string theory. How to extract useful info. from this? Reconstruct world sheet action?
- Procedure generalises to pert. theory in λ (introduces extra "internal" vertices $\Rightarrow$ additional punctures.)

$\therefore$ Spacetime pert. theory $\Rightarrow$ World sheet pert. theory? Changes background? Role of SUSY? Dimension?

- Explore connection w/ cubic string field theory? Is it telling us something? Relation to open-closed string duality in tachyon condensation (e.g. Gaiotto-Rastelli)?
- Change of variables from "SFT moduli" σ_r to "CFT moduli" (ξ_i, μ_a)?
- Emergence of stringy moduli space somewhat subtle. For fixed n-pt. fn., need $J_i > c)n$ for bulk of mod. sp. $\mathcal{M}_{g,n}$. To see $\mathcal{M}_{g,n}$, $\therefore$ need $J_i \to \infty$. Relation to BMN limit, semiclassical strings?