VARIATIONS ON
THE WARPED DEFORMED CONIFOLD

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Based on

S. Gubser, C. Herzog, IRK,

hep-th/0405282

THE CONIFOLD

Non-compact Calabi-Yau space

$$z_1^2+z_2^2+z_3^2+z_4^2=0$$

It is a cone over $T^{11}=\dfrac{SU(2)\times SU(2)}{U(1)}$

with metric $dr^2+r^2 ds^2_{T^{11}}$,

$$ds^2_{T^{11}}=\frac{1}{9}\left(d\psi+\cos\theta_1\,d\varphi_1+\cos\theta_2\,d\varphi_2\right)^2$$
$$+\frac{1}{6}\sum_{i=1}^{2}\left(d\theta_i^2+\sin^2\theta_i\,d\varphi_i^2\right).$$

T^{11} is an S^1 bundle over $S^2\times S^2$.

Topologically, it is $\sim S^2\times S^3$.

Geometric symmetries of the conifold: $U(1)_R$: $z_k\to e^{i\alpha}z_k$;

$O(4) \sim SO(4) \times Z_2$ that rotates the z's.

An example of AdS/CFT is obtained by placing N D3-branes at the apex of the conifold

The 10-d metric is

$$h^{-\frac{1}{2}}(r)\, dx_{\parallel}^2 + h^{\frac{1}{2}}(r)\left(dr^2 + r^2 dS_{T^{11}}^2\right);$$

$$dx_{\parallel}^2 \equiv -(dx^0)^2 + (dx^1)^2 + (dx^2)^2 + (dx^3)^2.$$

In the near-horizon limit

$$h(r) = \frac{L^4}{r^4} \Rightarrow \text{find } AdS_5 \times T^{11}.$$

Type IIB string on $AdS_5 \times T^{11}$ is dual to the $\mathcal{N}=1$ SCFT on the N D3-branes

I.K., Witten
Morrison, Plesser

$SU(N) \times SU(N)$ gauge theory coupled to

A_1, A_2 in $(N, \bar{N})$

B_1, B_2 in $(\bar{N}, N)$

with superpotential

$$\epsilon^{ij}\epsilon^{kl}\, \mathrm{Tr}\,(A_i B_k A_j B_l)$$

The SCFT has global symmetry

$$U(1)_R \times U(1)_B \times SO(4) \times \mathbb{Z}_2 \times \mathbb{Z}_2$$

The A's and B's have R-charge $\frac{1}{2}$.

$$SO(4) \sim \frac{SU(2) \times SU(2)}{\mathbb{Z}_2}.$$

The $SU(2)$'s rotate A's and B's respectively

$U(1)_B: A_k \to e^{i\beta} A_k \; ; \; B_k \to e^{-i\beta} B_k$

The massless gauge field in AdS_5 dual to the $U(1)_B$ global symmetry of the CFT is $\mathcal{A}_\mu$ coming from

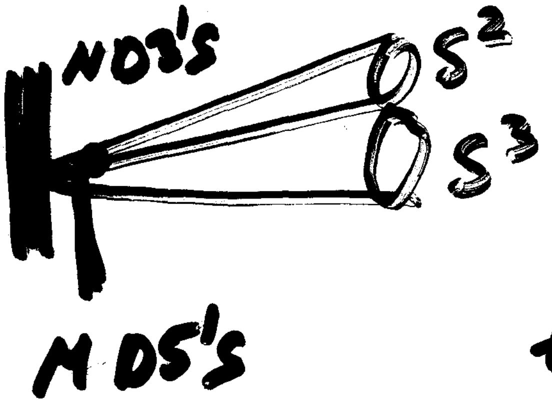
$\delta C_4 \sim \mathcal{A} \wedge \omega_3$

(ω_3 is the harmonic 3-form on T^{11})

The discrete symmetry important for us is the Z_2 that interchanges the A's and the B's and implements charge conjugation $N \leftrightarrow \overline{N}$ in each gauge group. In the string theory this Z_2 interchanges (θ_1, φ_1) and (θ_2, φ_2) of T^{11} accompanied by the center of the $SL(2,Z)$ S-duality symmetry of IIB strings.

THE WARPED DEFORMED CONIFOLD

Add M D5-branes wrapped over the S^2 at the apex.



This adds M units of RR 3-form flux through the S^3, which causes a geometric transition to the deformed conifold

$$z_1^2+z_2^2+z_3^2+z_4^2=\epsilon^2$$

IK, Strassler
Vafa
Maldacena, Nunez

The warped 10-d metric is

$$h^{-\frac{1}{2}}(\tau)\, dx_{||}^2 + h^{\frac{1}{2}}(\tau)\, ds_6^2$$

The CY metric of the deformed conifold is known explicitly

$$ds_6^2 = \frac{\epsilon^{4/3} K(\tau)}{2} \left[\frac{1}{3K^3}\left(d\tau^2 + g_5^2\right) \right.$$

$$\left. + \cosh^2\left(\frac{\tau}{2}\right)\left(g_3^2 + g_4^2\right) + \sinh^2\left(\frac{\tau}{2}\right)\left(g_1^2 + g_2^2\right) \right]$$

$$g^1 = \frac{e^1 - e^3}{\sqrt{2}} ; \quad g^2 = \frac{e^2 - e^4}{\sqrt{2}} ;$$

$$g^3 = \frac{e^1 + e^3}{\sqrt{2}} , \quad g^4 = \frac{e^2 + e^4}{\sqrt{2}} ; \quad g^5 = e^5 ;$$

$$e^5 = d\psi + \cos\theta_1 \, d\varphi_1 + \cos\theta_2 \, d\varphi_2 ;$$

$$e^1 = -\sin\theta_1 \, d\varphi_1 ; \quad e^2 = d\theta_1 ;$$

$$e^3 = \cos\psi \sin\theta_2 \, d\varphi_2 - \sin\psi \, d\theta_2 ;$$

$$e^4 = \sin\psi \sin\theta_2 \, d\varphi_2 + \cos\psi \, d\theta_2 .$$

$$K(\tau) = \frac{\left(\sin(2\tau) - 2\tau\right)^{1/3}}{2^{1/3} \sinh\tau} .$$

At $\tau = 0$ the S^2 shrinks, and a round S^3 is left.

The warp factor is monotonic and reaches a finite value at $\tau = 0$:

$$h(\tau) = (g_s M \alpha')^2 \, 2^{2/3} \, \epsilon^{-8/3} \int_\tau^\infty dx \, \frac{x \coth x - 1}{sh^2 x} \left(sh(2x) - 2x \right)^{1/3}$$

Holographic calculation of Wilson loops gives AREA LAW.

The background also includes complex 3-form of type (2,1) and the self-dual 5-form field strength.

The dual gauge theory is the cascading $SU(N+M) \times SU(N)$ coupled to A_1, A_2 in $(N+M, \bar{N})$, B_1, B_2 in $(\overline{N+M}, N)$.

Now the $U(1)_R$ is broken by the chiral anomaly to Z_{2M} in the UV.

The holographic dual of this is understood

Ik, Ouyang, Witten.

Furthermore, in the IR

$Z_{2M} \to Z_2$ by the gluino condensate.

The remaining Z_2 symmetry is

$z_k \to -z_k$ symmetry of the deformed conifold.

The $SO(4)$ rotational symmetry is unbroken. There is also a Z_2 that extends ~~it~~ to $O(4)$: $z_4 \to -z_4$, accompanied by $G_3 \to -G_3$ (the $-I$ of $SL(2,Z)$)

In the gauge theory this is $A_k \leftrightarrow B_k$ accompanied by charge conjugation.

What is the fate of the $U(1)_B$?
It is not anomalous in the UV.
In the IR it is spontaneously broken by expectation values of baryonic operators.

IK
Strassler

In the $SU(2M) \times SU(M)$ gauge theory at the bottom of the cascade, there is superpotential

$$X[\det N - B\tilde{B} - \Lambda^{4M}]$$

$N \sim AB$ is the meson matrix.
Baryon operators exist because the $SU(2M)$ gauge theory effectively has $2M$ flavors:

$$B \sim A_1^M A_2^M \,; \quad \tilde{B} \sim B_1^M B_2^M$$

10 On the BARYONIC BRANCH Argyres Plesser Seiberg

$X=0$; $N=0$; $\mathcal{B}\tilde{\mathcal{B}}=-\Lambda^{4M}$

the SO(4) global symmetry is unbroken => this is a good candidate for the dual of the warped deformed conifold. IK, Strassler

But where is the pseudoscalar Goldstone boson of the spontaneously broken $U(1)_B$? Aharony.

A seemingly unrelated old puzzle is: What is the gauge theory dual of the D-string at $\tau=0$?

Just as the F-string, dual to the confining string, the D-string has

a non-vanishing tension at $\tau = 0$
What is it in the gauge theory?

The two puzzles have a common solution! Gubser, Herzog, IK
The D-string is dual to the axionic string in the gauge theory.
The axion ≡ pseudoscalar Goldstone boson of the spontaneously broken $U(1)_B$ symmetry.
This normalizable mode around the warped deformed conifold background, i.e. a massless glueball, comes from the RR-sector.

THE VARIATIONS

$$\delta F_3 = *_4 da - d\left[f_2(\tau)\, da \wedge g^5\right]$$

$$\delta F_5 = \left(*_4 da - \frac{\epsilon^{4/3} h(\tau)}{6K^2(\tau)}\, da \wedge d\tau \wedge g^5\right) \wedge B_2$$

$$B_2 = \frac{g_s M \alpha' (\tau \coth\tau - 1)}{2\, \mathrm{sh}\,\tau}\left[\mathrm{sh}^2\left(\frac{\tau}{2}\right) g^1 \wedge g^2 + \mathrm{ch}^2\left(\frac{\tau}{2}\right) g^3 \wedge g^4\right]$$

is the background NS-NS field, and

$\delta B_2 = 0$.

Linearized SUGRA equations require $a(x^0, x^1, x^2, x^3)$ to be harmonic:

$$d *_4 da = 0.$$

This is the 4-d axion mode.

The equation for f_2 has a normalizable solution

$$f_2 \sim \frac{1}{K^2\, \mathrm{sh}^2\tau} \int_0^\tau h(x)\, \mathrm{sh}^2 x\, dx$$

For large τ, $f_2 \sim \tau e^{-2\tau/3}$

For small τ, $f_2 \sim \tau$

The form of δF_5 shows that da is the longitudinal component of the vector field dual to the baryon current. Hence, the 4-d effective action contains

$$\frac{1}{f_a} \int d^4x \, J^\mu \partial_\mu a$$

On the baryonic branch

$$\mathcal{B} = i \xi \Lambda^{2M}; \quad \tilde{\mathcal{B}} = \frac{i}{\xi} \Lambda^{2M},$$

a enters as the phase of ξ.

SUSY also requires the existence of a modulus field, "a saxion".

Just like the pseudoscalar mode, this scalar breaks the Z_2 symmetry of the background ($(\theta_1, \varphi_1) \leftrightarrow (\theta_2, \varphi_2)$, together with $G_3 \to -G_3$).

$\delta B_2 = \chi(\tau) d g^5$;

$\delta G_{13} = \delta G_{24} = m(\tau)$

$g^1 g^3 + g^3 g^1 + g^2 g^4 + g^4 g^2 = \sin\theta_1 d\theta_1 d\varphi_1 - \sin\theta_2 d\theta_2 d\varphi_2$

are the same metric components turned on in a SMALL RESOLUTION of the conifold. Pando Zayas, Tseytlin

Solving the SUGRA eqns. we find

$m(\tau) \sim h^{1/2}(\tau) (\tau \coth\tau - 1)$

and $\chi(\tau)$ is determined by $m(\tau)$.

An ansatz for Z_2 breaking of the deformed conifold beyond the linear order was written down by Papadopoulos and Tseytlin, hep-th/0012034. Our linearized solution agrees with that found from their ansatz.
In the ~~limit~~ of ~~large~~ Z_2 breaking PT obtain the Maldacena-Nunez solution.

The interpolating solutions,

THE RESOLVED WARPED DEFORMED CONIFOLDS,

are not yet known explicitly.
They should be dual to baryonic branch of the cascading theory, with [illegible]

COMPACTIFICATION

The warped deformed conifold throat may be embedded into a CY compactification with fluxes. GKP

Then global symmetries of the gauge theory become gauged.
On the baryonic branch, we then find SUSY Higgs Mechanism for the $U(1)_B$. The pseudoscalar gets "eaten" and becomes part of massive vector field; the scalar Higgs is degenerate with π — they belong to a $\mathcal{N}=1$ massive vector supermultiplet.
Now we expect D-strings to be dual to the Abrikosov-Nielsen-Olesen

strings.

Such strings are not expected to be BPS. Since there are K units of NS-NS 3-form flux, K parallel D-strings can "break" on a wrapped D3-brane.

Hence, D-string charge should take values in $\mathbb{Z}_K$.

Copeland
Myers
Polchinski

CONCLUSIONS

The warped deformed conifold background of type IIB string theory is dual to the cascading gauge theory on the baryonic branch in the $\mathbb{Z}_2$ symmetric case $|\mathcal{B}| = |\tilde{\mathcal{B}}|$

Transformation $\mathcal{B} \to (1+\alpha)\mathcal{B}$; $\tilde{\mathcal{B}} \to \frac{\tilde{\mathcal{B}}}{1+\alpha}$

breaks the $\mathbb{Z}_2$; its SUGRA dual is understood to linear order in α.

Finding exact SUGRA solutions dual to the entire baryonic branch,

THE RESOLVED WARPED DEFORMED CONIFOLDS,

remains a challenge.