

*Effective actions of intersecting brane worlds, fluxes and
gaugino condensation*

Dieter Lüst (HU-Berlin, MPI-München)



Work in collaboration with P. Mayr, R. Richter and S. Stieberger, hep-th/0404134

S. Reffert, S. Stieberger, hep-th/0406092

G.L. Cardoso, G. Curio, G. Dall'Agata, hep-th/0406118

Review: hep-th/0401156

Strings 2004, Paris

I) Introduction

Goal of superstring theory:

Embedding of the Standard Model into a unified description of gravitational and gauge forces.

Consider orientifold (type I) models with D-branes:

- gravity in the 10 D bulk $\leftrightarrow$ closed strings
- gauge & matter fields localized at the D-brane (intersections) $\leftrightarrow$ open strings

(At least) two scenarios of how supersymmetry breaking can be realized:

- (i) Closed string sector preserves $\mathcal{N} = 1$ SUSY,
open string sectors break SUSY $\implies$ large extra dimensions.
- (ii) Open string sectors preserve $\mathcal{N} = 1$ SUSY,
closed string sector breaks SUSY $\implies$ soft SUSY breaking terms in the effective action of the open string matter fields.

Concrete scenario: SUSY is “spontaneously” broken in the closed string sector by internal background fluxes of closed string field strength fields $\langle G_{ijk} \rangle \neq 0$.

I) Introduction

Further plan of the talk:

- II) Set up: Type IIA intersecting D6-brane orientifold models and their IIB mirror D9/D5 resp. D3/D7 descriptions
- III) Scattering amplitudes of gauge, matter (ϕ) and moduli fields (M) from (intersecting) D-branes $\implies$ low energy supergravity action action:
Kählerpotential $K(\phi, \bar{\phi}, M, \bar{M})$, gauge kinetic function $f(M)$.
- IV) Adding internal 3-form fluxes in orientifolds with D3/D7-branes:
superpotential $\hat{W}(M) \implies$ soft SUSY breaking.
- V) (3,0) G-flux and the dynamical formation of gaugino condensation

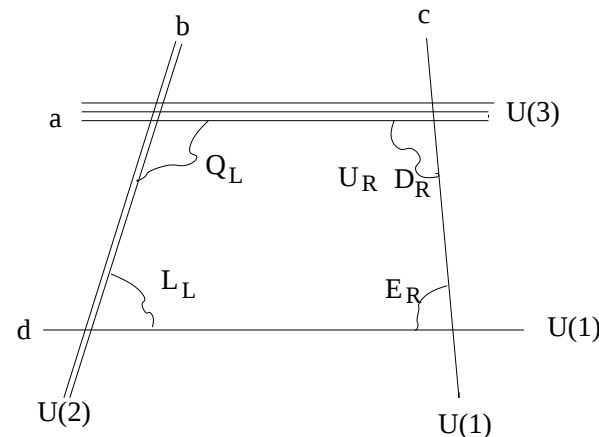
II) IIA Intersecting D6-Brane Models and their IIB mirrors

Type IIA orientifold models with intersecting D6-branes:

M. Berkooz, M. Douglas, R. Leigh (1996); R. Blumenhagen, L. Görlich, B. Körs, D.L. (2000); G. Aldazabal, S. Franco, L. Ibanez, R.

Rabadan, A. Uranga (2000); M. Cvetič, G. Shiu, A. Uranga (2001)

Local picture of the Standard Model: realization by k stacks of intersecting N_a D6_{*a*}-branes ($a = 1, \dots, k$) – **chiral fermions are localized at brane intersections:**



$\mathcal{N} = 1$ space-time supersymmetry is preserved if all stacks of D-branes preserve the following angle conditions among each other:

$$\theta_{ab}^1 + \theta_{ab}^2 + \theta_{ab}^3 = 0 \mod \pi$$

II) IIA Intersecting D6-Brane Models and their IIB mirrors

Compact type IIA orientifolds with intersecting D6-branes:

Space-time: $(\mathbb{R}^{1,3} \times \mathcal{M}^6)/(\Omega \times \mathcal{R})$, complex structure moduli U_i .

$D6_a$ -branes are wrapped around supersymmetric 3-cycles π_a, π'_a inside $\mathcal{M}^6$.

- Cancellation of internal Ramond charges on compact space (Gauss law)

$$\sum_a N_a (\pi_a + \pi'_a) - 4\pi_{O6} = 0.$$

- NS tadpoles – cancellation of internal brane tensions (stability problem).

D-term scalar potential

$$\mathcal{V}_D(\phi_4, U_i) = \tau_6 \frac{e^{-\phi_4}}{\sqrt{\text{Vol}(\mathcal{M}^6)}} \left(\sum_{a,a'} N_a \text{Vol}_{\pi_a}(U_i) - 4\text{Vol}_{\pi_{O6}}(U_i) \right)$$

Absence of NS-tadpoles: $\partial \mathcal{V}_D / \partial \phi_4 = \partial \mathcal{V}_D / \partial U_i = 0$. This is automatically ensured if all D6-branes mutually satisfy the $\mathcal{N} = 1$ angle condition.

Note: $\mathcal{V}_D$ fixes the IIA bulk complex structure moduli U_i .

II) IIA Intersecting D6-Brane Models and their IIB mirrors

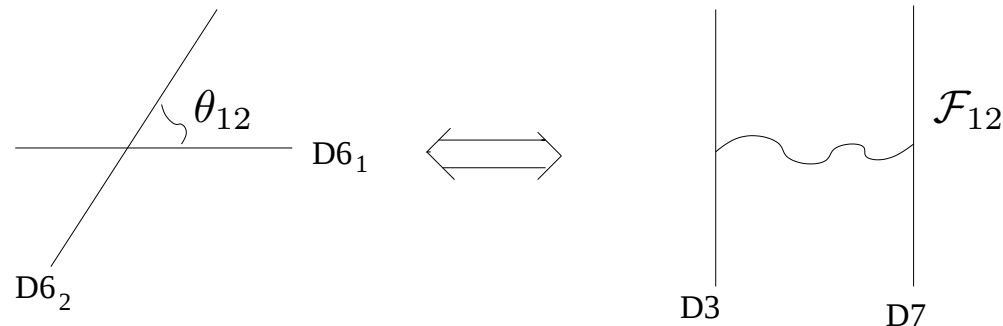
Type IIB mirror orientifolds via T-duality transformations in 3 toroidal directions:

→ mirror space $\mathcal{W}^6$ with Kähler moduli T_i .

C. Bachas (1995)

- $D5$ - and $D9_a$ -branes (plus $O9/O5$ -planes) – wrapped around 2-cycles resp. $\mathcal{W}^6$.
- $D7_a$ - and $D3$ -branes (plus $O3/O7$ -planes) – wrapped around 4-cycles.

The intersection angle θ_{ab} between two D6-branes is mapped to open string 2-form flux $\mathcal{F}_{ab}$ through 2-cycles on the $D9_a$ resp. $D7_a$ -brane world volumes:



Now chiral matter originates from open strings between $D5 - D9_a$, $D9_a - D9_b$ resp. $D3 - D7_a$, $D7_a - D7_b$.

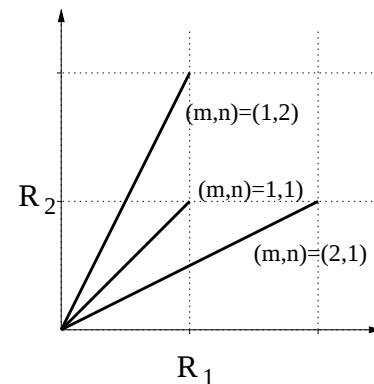
Note: $\mathcal{V}_D$ now fixes the IIB bulk Kähler moduli T_i .

II) IIA Intersecting D6-Brane Models and their IIB mirrors

Explicit model: $\mathbb{Z}_2 \times \mathbb{Z}_2$ -orientifold ($\mathcal{M}^6 = (T^2)^3 / (\mathbb{Z}_2 \times \mathbb{Z}_2)$):

Type IIA: 3 complex structure moduli $U_j = R_j^2 / R_j^1 e^{i\alpha^j}$.

D6-branes are wrapped around the product of three 1-cycles with wrapping numbers (m_j, n_j) :



$$\tan(\pi\theta_j) = -\frac{U_1^j}{U_2^j} + \frac{m_j}{n_j} \frac{|U^j|^2}{U_2^j}$$

T-dual (IIB) picture: D5/D9- or D3/D7-branes with background $\mathcal{F}^j$ -flux through T_j^2 :

$$\mathcal{F}_j = \tan(\pi\theta_j) = \frac{1}{\Im(T^j)} (b^j + 2\pi\alpha' F^j), \quad F^j = \frac{1}{2\pi\alpha'} \frac{m^j}{n^j}.$$

3 Kähler moduli: $T^j = b^j + iR_1^j R_2^j \sin \alpha^j$.

III) Effective low-energy action (open-closed string amplitudes)

In $\mathcal{N} = 1$ supersymmetric models the low-energy supergravity action of the massless gauge A_μ^I and matter fields Φ_a is determined by three moduli-dependent functions:

Cremmer, Ferrara, Girardello, Van Proeyen (1983)

Gauge kinetic function: $f_{IJ}(M) W_\alpha^I W_\alpha^J$

Superpotential: $W = \hat{W}(M) + W_{abc}(M) \Phi_a \Phi_b \Phi_c + \dots$

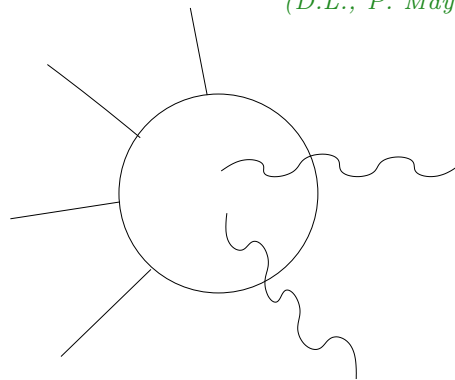
Kähler potential: $K = \hat{K}(M, \bar{M}) + K(M, \bar{M})_{a\bar{b}} \Phi_a \bar{\Phi}_b + \dots$

$\Rightarrow$ Yukawa couplings: $Y_{abc} = e^{\hat{K}/2} \sqrt{K_{a\bar{d}}^{-1} K_{b\bar{e}}^{-1} K_{c\bar{f}}^{-1}} W_{def}$

Compute f, K, W by calculating string tree level scattering amplitudes on the disk:

(D.L., P. Mayr, R. Richter, S. Stieberger, hep-th/0404134)

N_o open and N_c closed strings:



III) Effective low-energy action (open-closed string amplitudes)

The boundary of the disk is attached to the D-brane world volume – this is conformally equivalent to the upper half plane $\mathbb{H}_+$. $\implies$ Correlators on $\mathbb{H}_+$:

$$\begin{aligned}\langle X^a(z_1)X^b(z_2) \rangle &= -g^{ab} \log(z_1 - z_2), \quad \langle X^a(z_1)X^b(\bar{z}_2) \rangle = -D^{ab} \log(z_1 - \bar{z}_2), \\ \langle \psi^a(z_1)\psi^b(z_2) \rangle &= \frac{g^{ab}}{z_1 - z_2}, \quad \langle \psi^a(z_1)\bar{\psi}^b(\bar{z}_2) \rangle = \frac{D^{ab}}{z_1 - \bar{z}_2}.\end{aligned}$$

D^{ab} depend on open string boundary conditions:

$$D = -g^{-1} + 2(g + \mathcal{F})^{-1}.$$

Closed string vertex operator: $V_c(\bar{z}, z) = \bar{V}_c(\bar{z})V_c(z).$

Open string vertex operator: $V_o(z) = V(z)$ at $z = \bar{z}.$

$\implies$ Doubling “trick”: Closed string vertex operators have two momenta $k_{||}$ and $k_{\perp}$ resp. $k = q$ and $k = Dq$ ($k_{||} = \frac{1}{2}(q + Dq)$).

III) Effective low-energy action (open-closed string amplitudes)

Mixed open/closed string disk amplitudes are generically rather different from corresponding closed (heterotic) string amplitudes:

Consider

$$\mathcal{A}_{N_c+N_o} \sim \langle V_c^1(\bar{z}_1, z_1) \dots V_c^{N_c}(\bar{z}_{N_c}, z_{N_c}) V_o^1(w_1) \dots V_o^{N_o}(w_{N_o}) \rangle :$$

- It involves the integration over $2N_c + N_o - 3$ real positions z_i
 $\Rightarrow$ nontrivial momentum dependence.

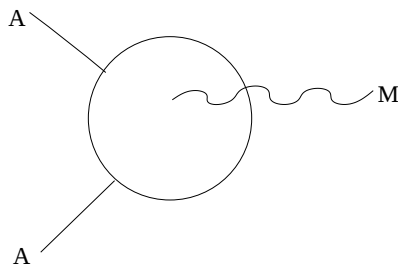
E.g. $N_c = 1$, $N_o = 2$: One z -integration, $\mathcal{A}_3$ is of $\mathcal{O}(k^2)$.

- Correlators between holomorphic and anti-holomorphic operators in general contribute
 $\Rightarrow$ Less restrictions from internal charge conservation.

$\Rightarrow$ These kind of mixed amplitudes are not just the square roots of the corresponding closed (heterotic) string matter amplitudes!

III) Effective low-energy action (open-closed string amplitudes)

(i) Gauge kinetic function:



$$\alpha' \rightarrow 0 \implies \frac{\partial g_I^{-2}}{\partial M^i}$$

$$\mathcal{A}_3 = \langle A^{a_1} A^{a_2} T^j \rangle = \frac{i\bar{D}^j}{2T_2^j} \left[\frac{1}{2}(p_1 p_2)(\xi_1 \xi_2) - \frac{1}{2}(p_1 \xi_2)(p_2 \xi_1) \right] t \frac{\Gamma(-2t)}{\Gamma(1-t)^2}, \quad t = p_1 p_3.$$

$$\mathcal{N} = 1 \text{ SUSY condition: } \frac{f^k}{T_k^2} = -\frac{f^l}{T_l^2} \quad (f^j = b^j + 2\pi\alpha' F^j).$$

Gauge kinetic functions:

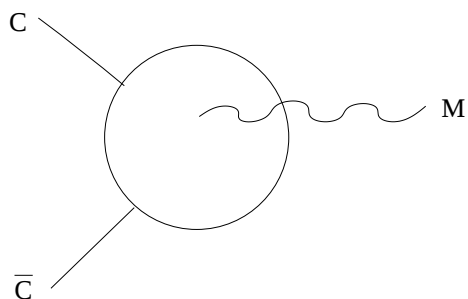
$$f_{D3} = S,$$

$$f_{D7_i} = |n^k n^l| (\mathcal{T}^i - f^k f^l S).$$

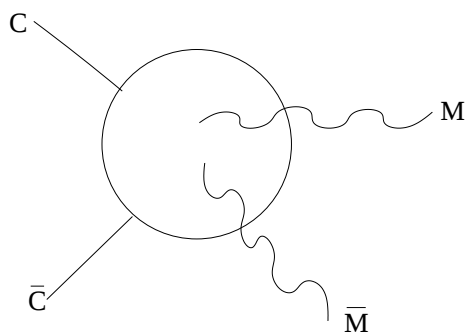
$$\Re \mathcal{T}^i = e^{-\phi_4} \sqrt{\frac{T_k^2 T_l^2}{T_i^2}}, \quad \Re S = e^{-\phi_4} \frac{1}{\sqrt{T_1^2 T_2^2 T_3^2}}.$$

III) Effective low-energy action (open-closed string amplitudes)

(ii) Matter field Kähler potential:



$$\alpha' \rightarrow 0 \implies \frac{\partial K_{C^a \bar{C}^b}}{\partial M^i}$$



$$\alpha' \rightarrow 0 \implies K_{C^a \bar{C}^b} K_{M^i \bar{M}^j} \frac{ts}{u} + s R_{C^a \bar{C}^b M^i \bar{M}^j}$$

L. Dixon, V. Kaplunovsky, J. Louis (1990)

III) Effective low-energy action (open-closed string amplitudes)

C_θ : twisted matter field: located at intersection of two $D6$ -branes at angle θ resp. between $D3 - D7_a$, $D7_a - D7_b$ resp. $D5 - D9_a$, $D9_a - D9_b$ branes with fluxes $\mathcal{F}_\theta$:

$$\mathcal{A}_3 = \langle C^\theta T^j \bar{C}^{-\theta} \rangle \sim -e^{-i\pi\theta^j} \frac{\Gamma(-2t) [\Gamma(-t + \theta^j) + (1 - \theta^j) \Gamma(-1 - t + \theta^j)]}{\Gamma(-1 - t + \theta^j) \Gamma(-t + \theta^j) \Gamma(1 - t - \theta^j)}$$

From that we obtain the following differential equation:

$$\frac{\partial K_{C_\theta \bar{C}_\theta}}{\partial T_j} \sim e^{-\pi i \theta_j} \sin(\pi \theta_j) [2\gamma_E + \psi(\theta_j) + \psi(1 - \theta_j)] K_{C_\theta \bar{C}_\theta}$$

Twisted (IIB) matter field Kähler metric:

$$K_{C_{7_a} \bar{C}_{7_b}}^{IIB} \sim \delta_{ab} e^{\gamma_E} \sum_{j=1}^3 \theta_j^a \prod_{j=1}^3 \sqrt{\frac{\Gamma(\theta_j^a)}{\Gamma(1 - \theta_j^a)}}, \quad \mathcal{F}_j^a(T_j) = \tan(\pi \theta_j^a(T_j)).$$

Further moduli dependences: 4-point amplitude $\mathcal{A}_4 = \langle C^\theta U^m \bar{U}^m \bar{C}^{-\theta} \rangle$:

$$K_{C_{7_a} \bar{C}_{7_b}}^{IIB} \sim \delta_{ab} e^{\gamma_E} \sum_{j=1}^3 \theta_j^a \prod_{j=1}^3 \sqrt{\frac{\Gamma(\theta_j^a)}{\Gamma(1 - \theta_j^a)}} \prod_{j=1}^3 (U_j + \bar{U}_j)^{-\theta_j^a}.$$

IV) 3-form fluxes and soft SUSY breaking

- Turn on non-vanishing bulk 3-form flux in type IIB orientifolds:

$$\frac{1}{(2\pi)^2\alpha'} \int_{C_3} G_3 \neq 0, \quad G_3 = F_3 - SH_3, \quad N_{flux} = \frac{1}{(2\pi)^4\alpha'^2} \int H_3 \wedge F_3.$$

- Use the previous open string effective action for matter and gauge fields

⇒ Soft SUSY breaking parameters for D3/D7-brane system with chiral fermions

D.L., S. Reffert, S. Stieberger, hep-th/0406092

Some previous results:

- D3-branes and supersymmetric 3-form fluxes

S. Kachru, M. Schulz, S. Trivedi, hep-th/0201028

- D9(D6)-branes and supersymmetric 3-form fluxes

R. Blumenhagen, D.L., T. Taylor, hep-th/03031016; J. Cascales, A. Uranga, hep-th/0303024; K. Behrndt, M. Cvetič, hep-th/0308024

- Soft terms for D3-branes and 3-form fluxes

P. Camara, L. Ibanez, A. Uranga, hep-th/0311241; M. Grana, T. Grimm, H. Jockers, J. Louis, hep-th/0312232

- Soft terms for intersecting D6-branes

B. Körs, P. Nath, hep-th/0309167

IV) 3-form fluxes and soft SUSY breaking

Effects of the 3-form fluxes:

- Modified Ramond tadpole condition:

$$N_{flux} + N_{D_3} + \frac{2}{(2\pi)^4 \alpha'^2} \sum N_a m_a^i m_a^j \int \mathcal{F} \wedge \mathcal{F} = 32.$$

- Modified D-term scalar potential:

$$\mathcal{V}_D(T^i) = \mathcal{V}_{D3/D7} + \mathcal{V}_{O3/O7} - e^{-\phi_{10}} T_3 N_{flux} \quad (\mathcal{V}_D \text{ fixes Kaehler moduli!})$$

- New superpotential:

Gukov, Vafa, Witten (1999); Taylor, Vafa (1999); Mayr (2000)

$$\hat{W}(S, U^i) = \frac{1}{(2\pi)^2 \alpha'} \int G_3 \wedge \Omega.$$

Associated scalar potential:

$$\hat{\mathcal{V}}_F(S, U^i) = \frac{1}{(2\pi)^7 \alpha'^4} \int G^{IASD} \wedge \star_6 \bar{G}^{IASD}, \quad \star_6 G^{(A)ISD} = \pm G^{(A)ISD}.$$

$\hat{\mathcal{V}}_F$ fixes complex structure moduli U^i and dilaton S .

IV) 3-form fluxes and soft SUSY breaking

$\hat{\mathcal{V}}_F$ is of the standard supergravity form:

$$\hat{\mathcal{V}}_F = \hat{K}_{ij} F^i \bar{F}^j - 3e^{\hat{K}} |\hat{W}|^2.$$

Minimization of $\hat{\mathcal{V}}_F$ ($\frac{\partial \hat{\mathcal{V}}_F}{\partial S} = \frac{\partial \hat{\mathcal{V}}_F}{\partial U^i} = 0$):

(i) Supersymmetric minima:

G_3 is a primitive ISD (2,1)-form: $G_{2,1} \neq 0$.

(ii) SUSY breaking F-terms:

– G_3 is a ISD (0,3)-form: $G_{0,3} \neq 0 \quad \Rightarrow \quad F^{T^i} = \hat{W} \frac{\partial \hat{K}}{\partial T^i} \neq 0.$

(Kähler SUSY breaking!)

– G_3 is a IASD (3,0)-form: $G_{3,0} \neq 0 \quad \Rightarrow \quad F^S = \hat{W} \frac{\partial \hat{K}}{\partial S} + \frac{\partial \hat{W}}{\partial S} \neq 0.$

(Dilaton SUSY breaking!)

– G_3 is a IASD (1,2)-form: $G_{1,2} \neq 0 \quad \Rightarrow \quad F^{U^i} = \hat{W} \frac{\partial \hat{K}}{\partial U^i} + \frac{\partial \hat{W}}{\partial U^i} \neq 0.$

(Complex structure SUSY breaking!)

IV) 3-form fluxes and soft SUSY breaking

Soft SUSY breaking terms for gauge and chiral matter sector on D3/D7-branes:

Effective scalar potential:

$$V^{\text{eff}} = K^{C_a \bar{C}_b} |\partial_a W^{(\text{eff})}|^2 + m_{a\bar{b}, \text{soft}}^2 C_a \bar{C}_b + \frac{1}{3} A_{abc} C_a C_b C_c + \text{h.c.},$$

$$W^{(\text{eff})} = \frac{1}{3} e^{\hat{K}/2} Y_{abc} C_a C_b C_c.$$

Soft supersymmetry breaking terms:

$$m_{a\bar{b}, \text{soft}}^2 = (|m_{3/2}|^2 + \hat{\mathcal{V}}) K_{C_a \bar{C}_b} - F^i F^{\bar{j}} R_{i\bar{j} a\bar{b}},$$

$$A_{abc} = F^i D_j (e^{\hat{K}/2} Y_{abc}).$$

Gaugino masses:

$$m_{gI} = F^i \partial_i \log(\text{Im} f_I).$$

IV) 3-form fluxes and soft SUSY breaking

E.g. Soft masses for (0,3)-flux $G_{0,3}$:

Scalar masses:

$$\begin{aligned}
 (m_{i\bar{i}}^{33})^2 &= |m_{3/2}|^2 K_{C_i^3 \bar{C}_i^3} - |F^{\mathcal{T}^i}|^2 R_{\mathcal{T}^i \bar{\mathcal{T}}^i i\bar{i}}^3 = 0, \\
 (m_{i\bar{i}}^{7,j})^2 &= |m_{3/2}|^2 K_{C_i^{7,j} \bar{C}_i^{7,j}} - \sum_{k,l} F^{\mathcal{T}^k} \bar{F}^{\bar{\mathcal{T}}^l} R_{\mathcal{T}^k \bar{\mathcal{T}}^l i\bar{i}}^{7,j}, \\
 (m^{37_a})^2 &= |m_{3/2}|^2 K_{C^{37_a} \bar{C}^{37_a}} - \sum_{k,l} F^{\mathcal{T}^k} \bar{F}^{\bar{\mathcal{T}}^l} R_{\mathcal{T}^k \bar{\mathcal{T}}^l}^{37_a}, \\
 (m^{7a7b})^2 &= |m_{3/2}|^2 K_{C^{7a7b} \bar{C}^{7a7b}} - \sum_{k,l} F^{\mathcal{T}^k} \bar{F}^{\bar{\mathcal{T}}^l} R_{\mathcal{T}^k \bar{\mathcal{T}}^l}^{7a7b}.
 \end{aligned}$$

Gaugino masses:

$$\begin{aligned}
 m_{g,D3,j} &= 0, \\
 m_{g,D7,j} &= \frac{F^{\mathcal{T}^j}}{(\mathcal{T}^j + \bar{\mathcal{T}}^j) - \alpha'^2 f^k f^l (S + \bar{S})}.
 \end{aligned}$$

V) $G_{3,0}$ -flux and dynamical generation of gaugino condensation

Question: Do certain 3-form fluxes correspond to the dynamical formation of a gaugino condensate in the gauge sector of the theory?

Motivation:

- Large N Transition: Flux $\longleftrightarrow \langle \lambda\lambda \rangle$ *Vafa (2001)*
- Heterotic string compactifications with gaugino condensate: need (3,0)-flux for stabilization.
- Type IIB orientifolds: $G_{3,0} \longleftrightarrow F^S \neq 0$, corresponds to dilaton dominated spontaneous SUSY breaking, very similar to gaugino condensation.

Using the AdS/CFT correspondence between $\mathcal{N} = 1^* \text{ SU}(N)$ gauge theory $\longleftrightarrow$ **Polchinski-Strassler solution** of type IIB supergravity



Generation of $G_{3,0}$ is associated with the dynamical formation of $\langle \lambda\lambda \rangle \neq 0$.

G.L. Cardoso, G. Curio, G. Dall'Agata, D.L., hep-th/0406118

V) $G_{3,0}$ -flux and dynamical generation of gaugino condensation

$\mathcal{N} = 1^*$: mass deformation of $\mathcal{N} = 4$ $SU(N)$

$$W = m_{ij} \phi_{(i} \phi_{j)}, \quad m_{ij} = m \quad , \quad i = 1, 2, 3$$

Confining phase: $\langle \bar{\lambda} \lambda \rangle = m^3$ at large $g_{\text{YM}}^2 N$.

Dual supergravity solution describing RG-flow: warped solution with 3-form flux

$$ds^2 = Z^{-1/2} \eta_{\mu\nu} dx^{\mu\nu} + Z^{1/2} g_{mn} dx^m dx^n \quad ,$$

$$G_3 = F_3 - \tau H_3 \quad , \quad F_3 = dC_2 \quad , \quad H_3 = dB_2$$

$$G_3 = G_3(x^m) \quad , \quad \tau = \tau(x^m)$$

Exact supergravity solution is not known. Iterative expansion in powers of m :

Polchinski-Strassler (2000): order m , $\tau = \tau_0$

Freedman/Minahan (2000): order m^2 , $d\tau \neq 0$

Here:

- Supergravity solution at order m^3 : G_3 has $(3, 0)$ -piece!

V) $G_{3,0}$ -flux and dynamical generation of gaugino condensation

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$$G^{(3,0)} = \partial A_2^{(2,0)}, \quad A_2^{(2,0)} = m^3 r^{-\Delta} \varepsilon_{ijk} \left(\frac{\bar{z}^2}{r^2} \right)^2 \frac{z^i}{r} d \left(\frac{z^j}{r} \right) \wedge d \left(\frac{z^j}{r} \right),$$

AdS/CFT: $A_2^{(2,0)}$ corresponds to $\langle \lambda \lambda \rangle$!

- $G^{(3,0)}$ preserves $\mathcal{N} = 1$ supersymmetry! How can that be?

3 kinds of supersymmetric Killing spinor solutions:

Type A(ndy): Heterotic/type II string with H-flux.

Type B(ecker): supersymmetric ISD $G_{2,1}$ -flux in type IIB.

Graña, Polchinski (2000); Giddings, Kachru, Polchinski (2001)

Type D(all'Agata):

G. Dall'Agata, hep-th/0403220

Polchinski/Strassler background at order m^3 with IASD flux $G^{(3,0)}$ is
supersymmetric with $SU(2)$ group structure.

Type D ansatz: manifestation of dielectric nature of the solution, see also

C. Pilch, N. Warner, hep-th/0403005