SEARCHING FOR A 2-D BLACK HOLE

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STRINGS 04 - PARIS -

GOAL / MOTIVATION

- FIND A SIMPLE SYSTEM CONTAINING BLACK HOLES WHICH IS COMPLETELY SOLVABLE

→ UNDERSTAND NATURE OF THE SPACETIME BEHIND THE HORIZON

PLAN:

- DISCUSS ASPECTS OF THE 2-d BLACK HOLE -

IN PROGRESS

2-D STRING THEORY = MATRIX QUANTUM MECHANICS AS $N \to \infty$

$$S_{W.S.} = \int \underbrace{(\partial X^0)^2}_{\text{TIME}} + \underbrace{(\partial\phi)^2 + \mu e^{\phi}}_{\text{LIOUVILLE}}$$

$$\mathcal{L} = \int dt\, \mathrm{Tr}\left[(D_0 M)^2 + M^2\right]$$

$\downarrow$

DIAGONALIZE M

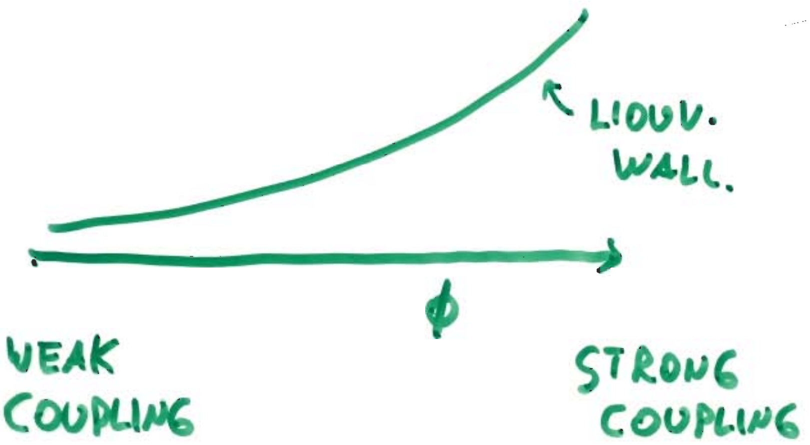
$\downarrow$

EIGENVALUES

$\downarrow$

FERMIONS

SPACE-TIME $\to$ 2-D. (X^0, ϕ).



MATRIX QUANTUM MECHANICS

$\updownarrow$

STRINGS $\rightarrow$ 2D TARGET SPACE

$\downarrow$

CONTAINS GRAVITY

$\downarrow$

BLACK HOLES

• BLACK HOLE $\longleftrightarrow$?? IN MATRIX MODEL

• **BLACK HOLE**:

1. NON ZERO ENTROPY
2. NON ZERO ABSORPTION

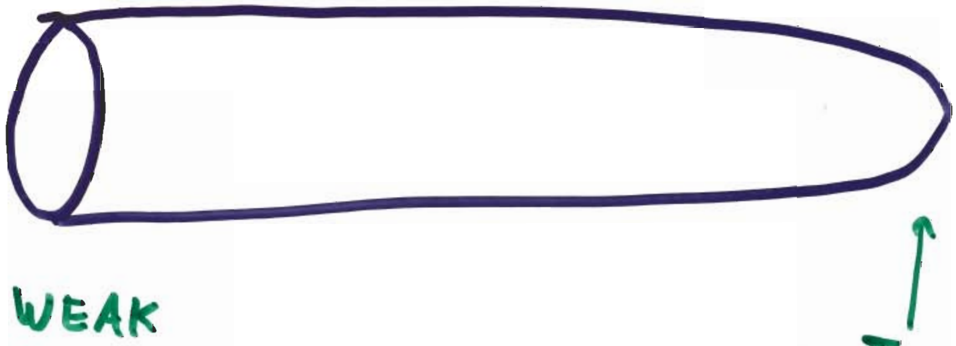
} SEMICLASSICALLY

- ON THE GRAVITY SIDE $\rightarrow$ NEED TO SOLVE EXACTLY IN α'

- BEST CANDIDATE $\rightarrow$ $SL(2)/U(1)$ WITTEN

• 2-D BLACK HOLE

$SL(2)_k/U(1)$ → COSET CFT
↓
EXACTLY SOLVABLE



• VALUE OF COUPLING:

- NORMALIZABLE DEFORMATION FOR $k>3$
- NOT NORMALIZABLE FOR $k \leq 3$

$$\delta\Phi_0 \sim e^{\frac{2}{\sqrt{k-2}}\phi} + e^{\frac{2(\frac{k}{2}-1)}{\sqrt{k-2}}\phi}\, e^{i\sqrt{k}(x_L-x_R)} + c.c.$$

↑ WAVE FUNCTION → CONTAINS PIECES WITH DIFFERENT WINDING

→ WINDING PIECE BECOMES NOT NORMALIZABLE

LESSON:

- NAIVE ANALYSIS USING GRAVITY LEADS TO INCORRECT QUALITATIVE ANSWERS

QUESTION	GRAVITY	STRING
PARAMETERS WE NEED TO FIX AT $\phi \to -\infty$? → PARAMETERS THAT THE FREE ENERGY DEPENDS ON ?	R	R, λ
$\exists$ ZERO MODE AT TREE LEVEL ? $\exists$ ONE LOOP DIVERGENCE ?	YES	NO
CAN WE VARY R ? $\exists$ A NON-SINGULAR SOLUTION FOR A RANGE OF R VALUES ?	NO	YES

WINDING MODES $\xleftrightarrow{\text{T-DUAL}}$ MOMENTUM MODES

$$e^{\alpha\phi}\cos\left(\frac{X}{R}\right)$$

↑

SINE-LIOUVILLE

$\mathcal{N}=2$ $SL(2)_k/U(1)$ $\xleftrightarrow[\text{MIRROR}]{\text{T-DUAL}}$ $\mathcal{N}=2$ SINE LIOUVILLE

HORI - KAPUSTIN

BOTH → $U(1)_R$ SYMMETRY

→ QUOTIENT BOTH SIDES

BOSONIC $SL(2)_k/U(1)$ ⟷ BOSONIC SINE-LIOUVILLE

FATEEV, (ZAMOLODCHIKOV)^2

KAZAKOV - KOSTOV - KUTASOV

EUCLIDEAN B.H. IN THE MATRIX MODEL

WINDING → WILSON LINES

$$\mathcal{L}_{M.M.} = \int dt\, \mathrm{Tr}\left[(D_0 M)^2 + M^2 + \dots \right] + \lambda \left[\mathrm{Tr}_F\, e^{\oint A} + c.c. \right]$$

↑ NON-LOCAL ACTION.

T-DUALITY → MOMENTUM → SINE GORDON → TODA EQN.

DEFINE : $\chi \equiv \partial_\mu^2 F \equiv \partial_\mu^2 \log(Z(\lambda,\mu,R))$

EQN FOR χ :

$$1 = \mu\, e^{\frac{\chi}{R}} + (R-1)\lambda^2\, e^{\frac{2-R}{R}\chi}$$

MOORE
KKK

. ENTROPY ?

. . NAIVE → $S = -\partial_T F \sim \lambda^{\frac{4}{2-R}} \log \lambda \sim e^{-2\Phi_0}\, \Phi_0$

→ FROM MICROSTATES ?

- 2 PARAMETERS λ, R

$\approx$ FUGACITY OR CHEMICAL POTENTIAL FOR QUARKS

- ENTROPY AT FIXED m :

$$S \sim -m \log m \sim -\log m!$$

KAZAKOV TSEYTLIN

$$Z = \int DM\,DA\; e^{\int \mathrm{Tr}(\dot{M}^2 + M^2)} \sum_m \frac{\lambda^{2m}}{m!\,m!} \left(\mathrm{Tr}\, e^{\oint A}\right)^m \left(c.c.\right)^m$$

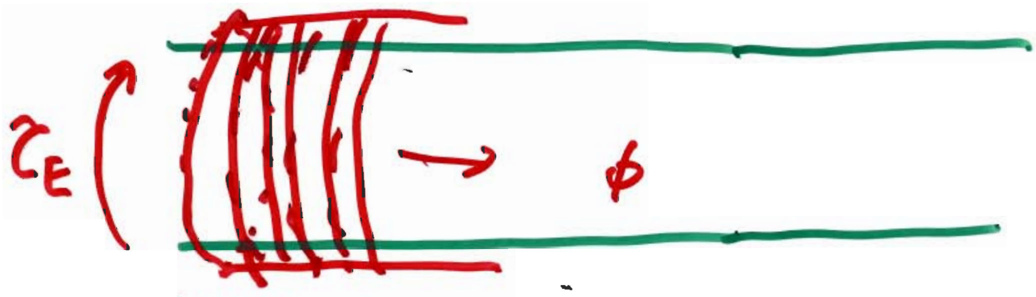
$T \log \lambda^2$ $\approx$ FUGACITY OR CHEMICAL POTENTIAL

- CONFIGURATIONS WITH DIFFERENT VALUES OF m DIFFER IN ENERGY BY AN INFINITE AMOUNT AS $N \to \infty$

MARCHESINI, ONOFRI
GROSS – KLEBANOV

→ ABSORBED IN RENORMALIZING λ

TARGET SPACE PICTURE:



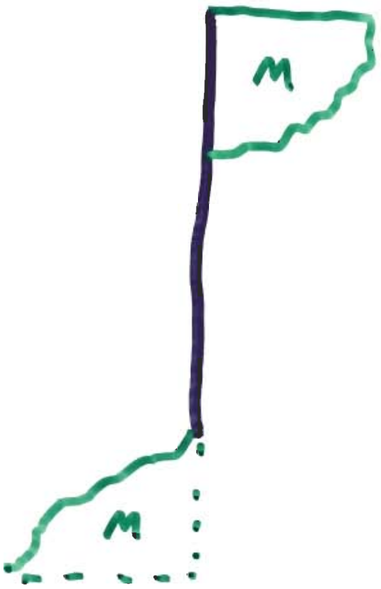
WEAK COUPLING

m STRINGS & m ANTI-STRINGS WRAPPED ALONG EUCLIDEAN TIME

- QUARKS & ANTIQUARKS

$\downarrow$

REPRESENTATIONS OF $U(N)$



- $$Z = \sum_{m} \frac{\lambda^{2m}}{(m!)^2} \sum_{R_m} C_{R_m} \; Z_{R_m}$$

Z_{R_m}: PARTITION FUNCTION IN A FIXED REPRESENTATION

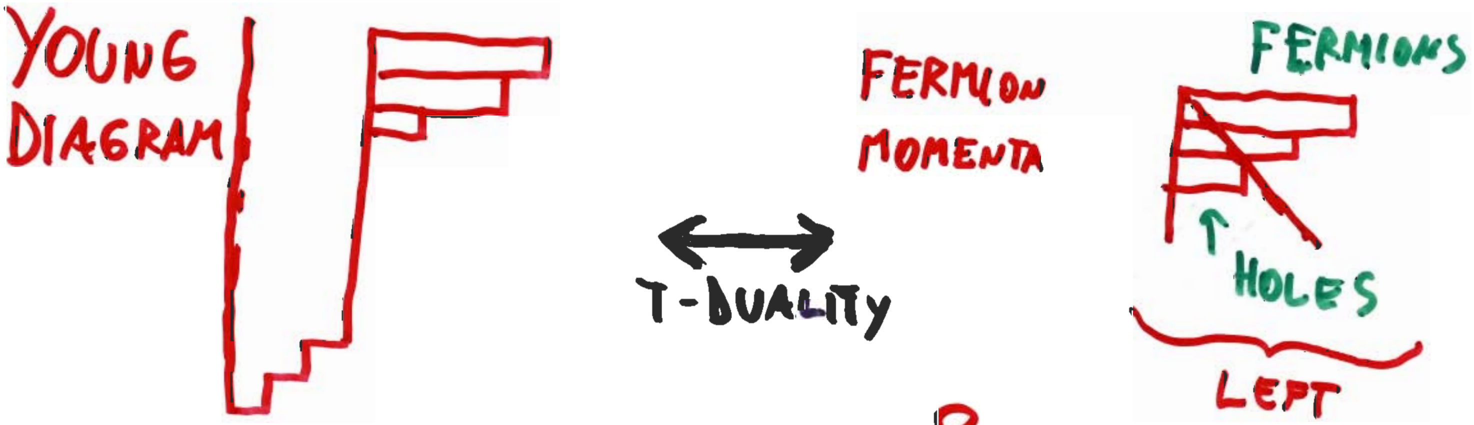
C_{R_m}: NUMBER OF TIMES THAT THE REPRESENTATION R APPEARS IN $(\square)^m \times (\bar{\square})^m$

- THE MATRIX MODEL IN EACH FIXED REPRESENTATION IS A DIFFERENT SUPER SELECTION SECTOR

QUESTION:

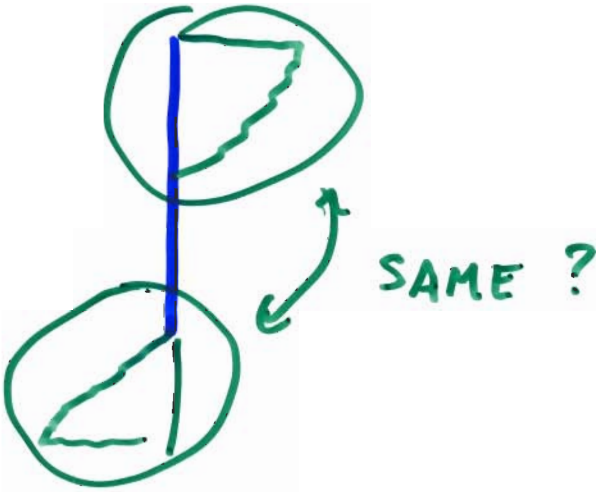
- WHAT IS Z_R FOR A FIXED REPRESENTATION?

- ASSUME T-DUALITY



& SIMILAR FOR RIGHT MOVING ONES

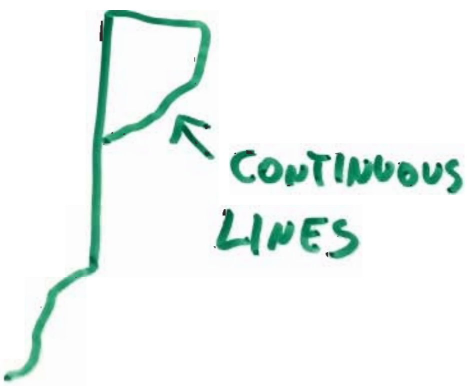
- REDUCES TO THE S-MATRIX AMPLITUDE OF LEFT & RIGHT MOVING (EUCLIDEAN) FERMIONS



- $$Z = Z_0 \prod_j R(m_j)$$

REFLECTION AMPLITUDE FOR EACH FERMION.

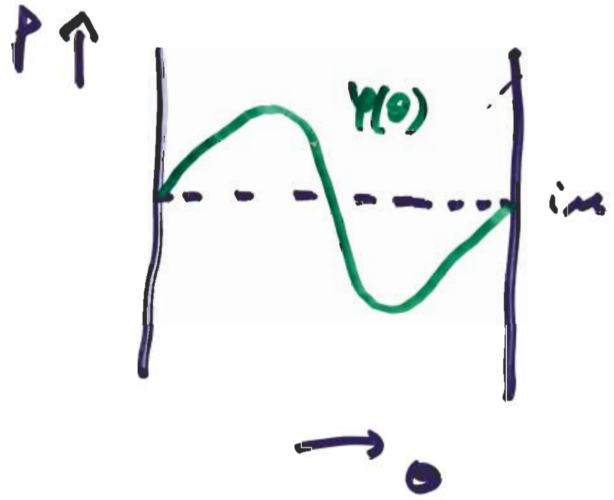
- LARGE m



$$\mathcal{F} = \mathcal{F}_0 + \int dp\, m(p) \log R(p)$$

FERMION DENSITY

- $$\log Z = \beta \int_0^{2\pi} d\theta \left[\left(i\varphi(\theta)+\mu\right)^2 \log\left(\mu + i\varphi(\theta)\right) + c.c. \right]$$

- 

- T-DUAL $$S \sim \left(\mu' + \frac{im}{R'}\right) \log\left(\mu' + i m/R'\right)$$

$$\mu' = \mu R$$
$$R' = 1/R$$
$\rightarrow$ GET OVERALL FACTOR OF R.

GET A SIMPLE ANSWER

$$\log Z = \beta f(\text{CONTOUR} = \text{REPRESENTATION})$$

↑ INDEPENDENT OF β

- NO ENTROPY FOR FIXED REPRESENTATION
  → IN THE CLASSICAL LIMIT WHERE $\mu, m \to \infty$ μ^2/m ARBITRARY

- EACH REPRESENTATION → ≠ SUPERSELECTION SECTOR

- IN EACH REPRESENTATION FERMIONS ARE INTERACTING

$$H = \sum_i p_i^2 - \lambda_i^2 + \frac{1}{N} \frac{T^R_{ij} T^R_{ji}}{(\lambda_i - \lambda_j)^2}$$

BOULATOV
KAZAKOV

MINAHAN
POLYKRONAKOS

- INTEGRABLE ?

- COEFFICIENTS $C_R(m)$.

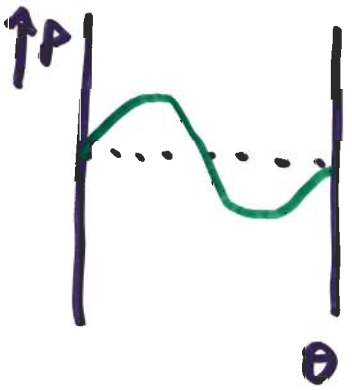
$$C_R = \int d\Omega \; \chi_R(\Omega) \, (\mathrm{Tr}\,\Omega)^m \, (\mathrm{Tr}\,\Omega^\dagger)^m$$

$$\downarrow$$

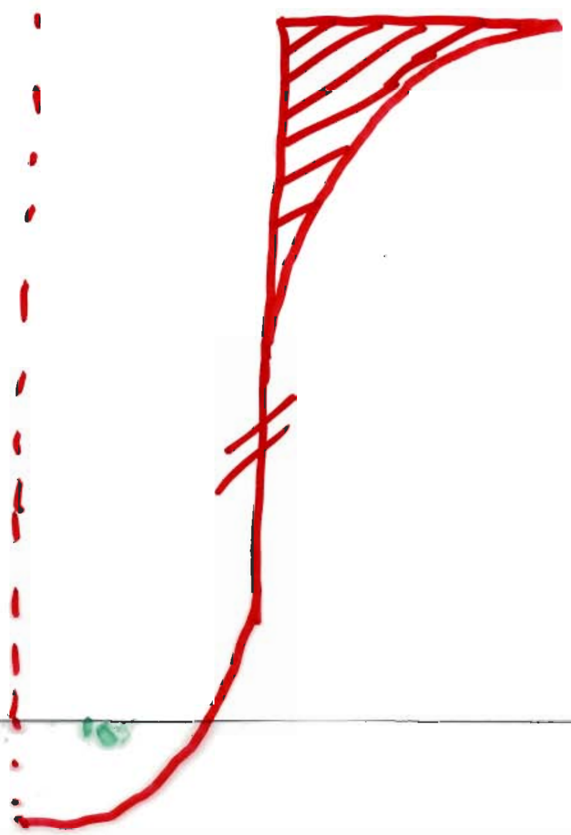
$$\langle \Psi_{\text{FERMIONS},R} \,|\, \alpha_{-1}^m \, \bar{\alpha}_{-1}^m \,|\, 0 \rangle$$

→ PEAKS ON A REPRESENTATION

$$\sim \alpha_{-1}^m |0\rangle \sim e^{\sqrt{m}\,\alpha_{-1}} |0\rangle$$



YOUNG TABLEAU →



NUMBER OF TIMES IT APPEARS:

$$C_R \sim m!$$

- WE REPRODUCE THE LEADING TERM IN THE ENTROPY

$$Z = \sum_m \frac{\lambda^{2m}}{(m!)^2} \cdot \underset{\uparrow \, m!}{C_R} \quad \underset{\uparrow}{Z_R}$$

NO ENTROPY

$\rightarrow$ GET $e^S = \frac{1}{m!}$

- SUBLEADING TERM IN THE ENTROPY

$$S_0 \sim -\log m! + A(\beta)\, m$$

$\uparrow$

REQUIRES MINIMIZING

$$\sum_R C_R Z_R$$

....

$\downarrow$

EXPECT ONLY ONE CONTRIBUTING REPRESENTATION

ABSORPTION

- EXACT CFT → NON VANISHING ANSWER

GIVEON et al.

- FIXED REPRESENTATION ?

→ HARD ⇝ NEED CORRELATORS OF MOMENTUM AND WINDING

- CONJECTURE

⇝ MIGHT COME FROM AVERAGING PHASES IN DIFFERENT SUPERSELECTION SECTORS

CONCLUSIONS

- ENTROPY IS COUNTING THE MULTIPLICITY OF THE REPRESENTATION THAT MAKES THE BIGGEST CONTRIBUTION
- EACH REPRESENTATION → NO CLASSICAL ENTROPY

FUTURE

- ABSORPTION ?
- WINDING & MOMENTUM CORRELATORS
- LEG - POLES ?
- DIRECT COMPUTATION, WITHOUT USING T-DUALITY
  → LORENTZIAN PICTURE