The Black Hole Singularity at Weak Gauge Coupling

Strings 2004

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Understanding spacelike singularities, like the black hole singularity and the Big Bang, may well illuminate basic conceptual aspects of string theory, and may teach us important lessons about its cosmology.

These singularities have proven difficult to study, especially in contrast to timelike (static) singularities, where string theory has had impressive successes.

One approach is to embed examples of these singularities in nonperturbatively well defined string theories.

e.g. "Matrix Cosmology"

2D String Theory

spacetime interpretation subtle

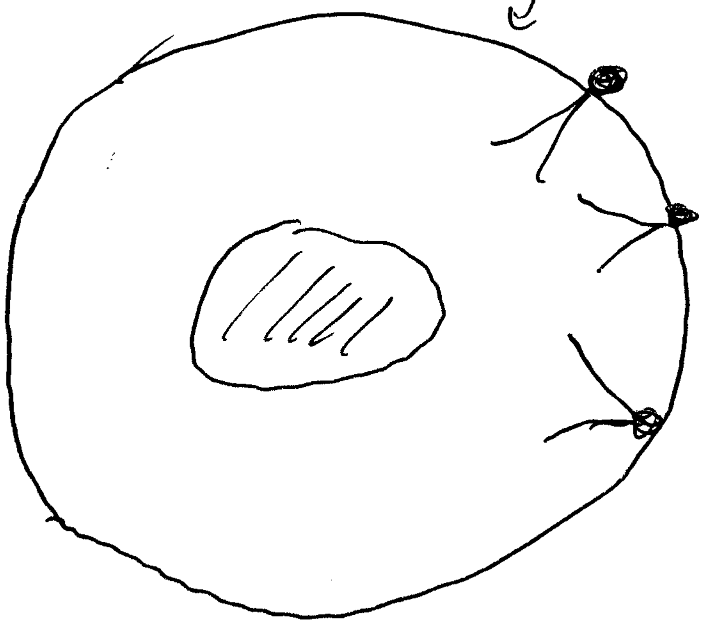
[illegible]

Hertog + Horowitz have realized a "big crunch" in a deformed AdS/CFT context where the boundary theory is unstable. Here the singularity is probably real.

We will focus on a situation where the theory is nonperturbatively well defined, and any singularity is (hopefully) resolved.

Large Black Holes have been realized in a vivid and precise way in AdS/CFT. The boundery description is a hot gauge theory. ~~But~~ The natural obcirvables, thermal gauge theory correlators, correspond to observers at spatial infinity, looking onto the hot black hole horizon.

$(AdS_5 \times S^5)$



A complete (holographic) outside the horizon description

But...

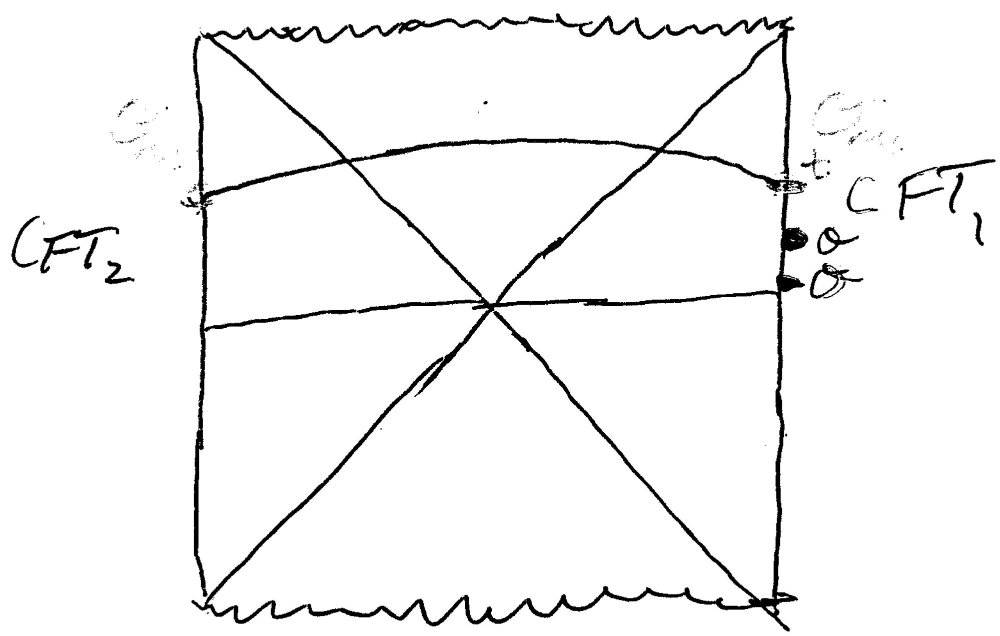
But the black hole singularity lies behind the horizon.

So we must try to extract some insights about behind the horizon physics from CFT correlators. To study the BH singularity.

Some geometrical information can be obtained.

Balasubramanian, Louko, Marolf, Ross, Maldacena, Kraus, Ooguri, S.S. ...)

Consider the eternal AdS-Sch black hole. ($D=3$ for now (BTZ))



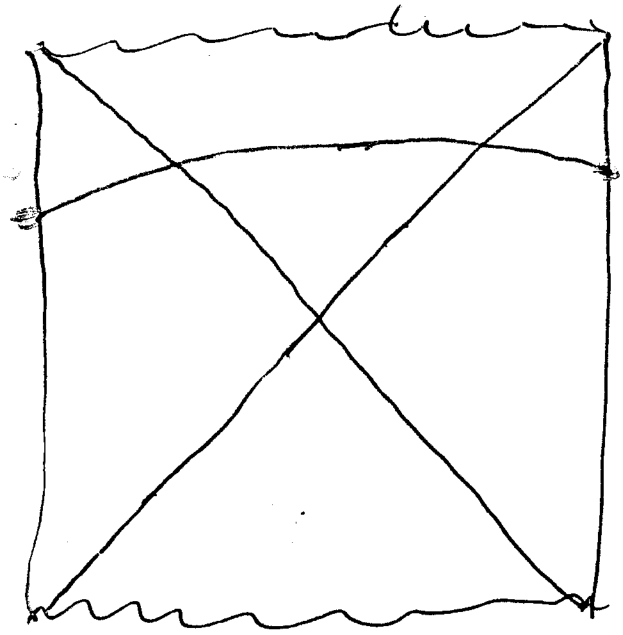
- usual
- new (Maldacena)

$m \to \infty$ geodesic approximation

$$\langle \mathcal{O}_m \mathcal{O}_m \rangle (t) \sim e^{-m \mathcal{L}(t)}$$

$\mathcal{L}(t)$ = proper length of (regulated) geodesic connecting the bdary pts

— goes behind the horizon



$\mathcal{L}(t)$ encodes information about geometry behind the horizon.

No signals are being sent! Pre-existing correlations (and their time development) are being probed.

AdS asymptotics allows spacelike separated probes to yield bulk gauge invariant info.

Really only need one boundary CFT.

shifting $t \to t + \frac{i\beta}{2}$ (β periodic

moves an operator from one bdary to the other.

We want the bulk particle to be heavy ($m \to \infty$) but small ($\equiv$ D-brane)

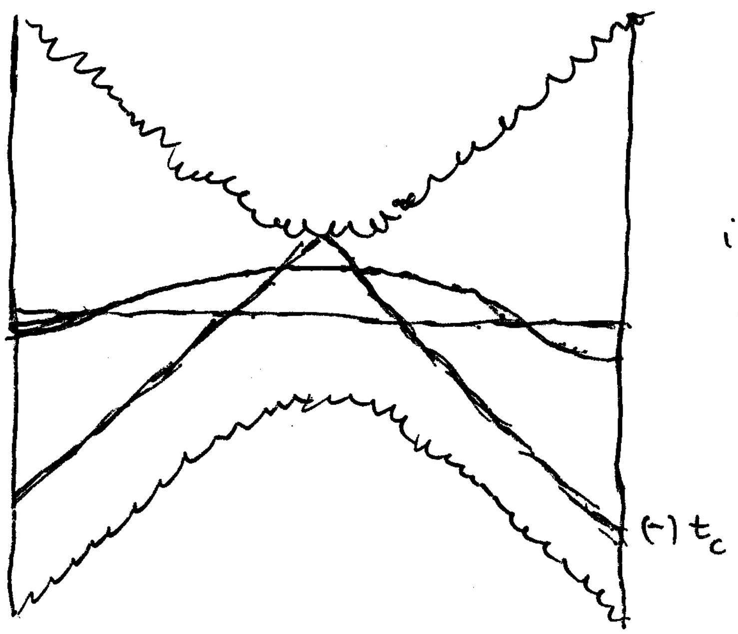
$\mathcal{O}_m \sim \det \phi^i_{ab}$ "giant graviton" wrapped 3-brane $D=5$

$m \sim N$

To find a strong signal of the singularity work in a situation where the curvature diverges there ($D>3$, e.g. $D=5$)

Fidkowski, Hubeny, Kleban, S.S.

AdS-Sch $D=5$ Penrose Diagram



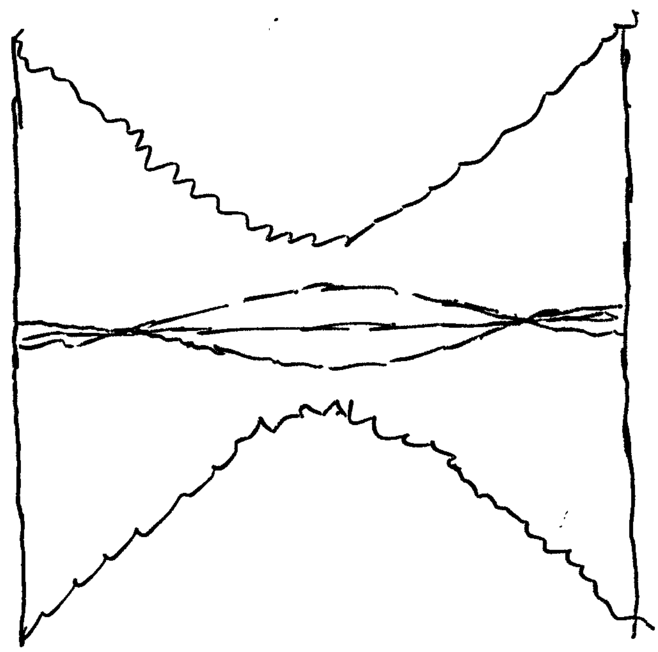
This "bouncing" is a direct consequence of the diverging curvature at the sing.

— null

Naively $\langle \mathcal{O}_m \mathcal{O}_m \rangle (t)$ should have a light-cone singularity $\sim \frac{1}{(t-t_c)^{2m}}$!?

Such a light cone singularity is not possible in the gauge theory.

In fact there are multiple geodesics (some in complex spacetime) linking the two boundary points.



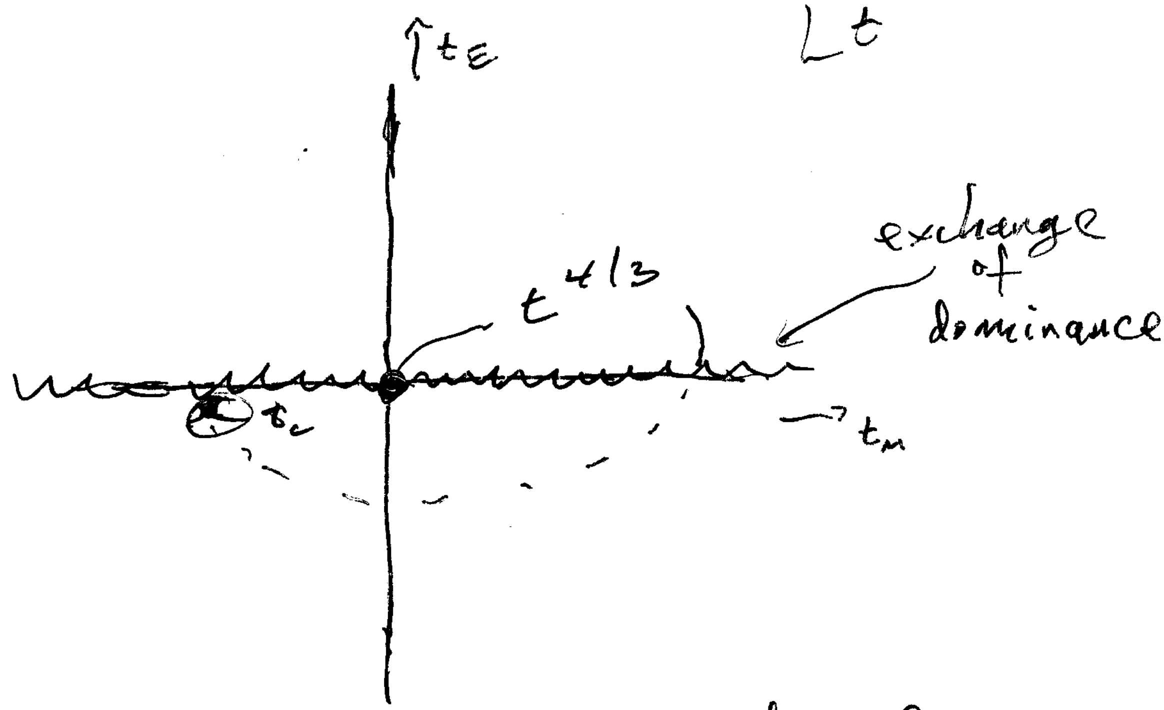
e.g., at $t=0$ there are 3 degenerate geodesics. (caustic)

The null geodesic does not dominate the path integral for $t \approx t_c$. It does not lie on the steepest descent contour.

We analyzed the saddle point (geodesic) dominance problem.

$\langle O_m O_m \rangle (t)$ for $\lambda \to \infty$, $N \to \infty$ ($m \to \infty$)

has the following structure:



Gauge theory results lie on the first sheet. The t_c singularity lies on the second sheet, a bit like a broad resonance.

Properties of the t_c singularity can be effectively determined ~~by~~ from first sheet data.

Slogan:

To go behind the horizon, go onto the second sheet

(analyticity vital)

Good News

There is an unambiguous (albeit subtle) signature of the black hole singularity present in SYM correlators in the limit $\lambda, N \to \infty$, $\alpha', g_s \to 0$. (the t_c singularity)

We can now pose precise questions about the effect of α' and g_s corrections on the singularity.

Bad News

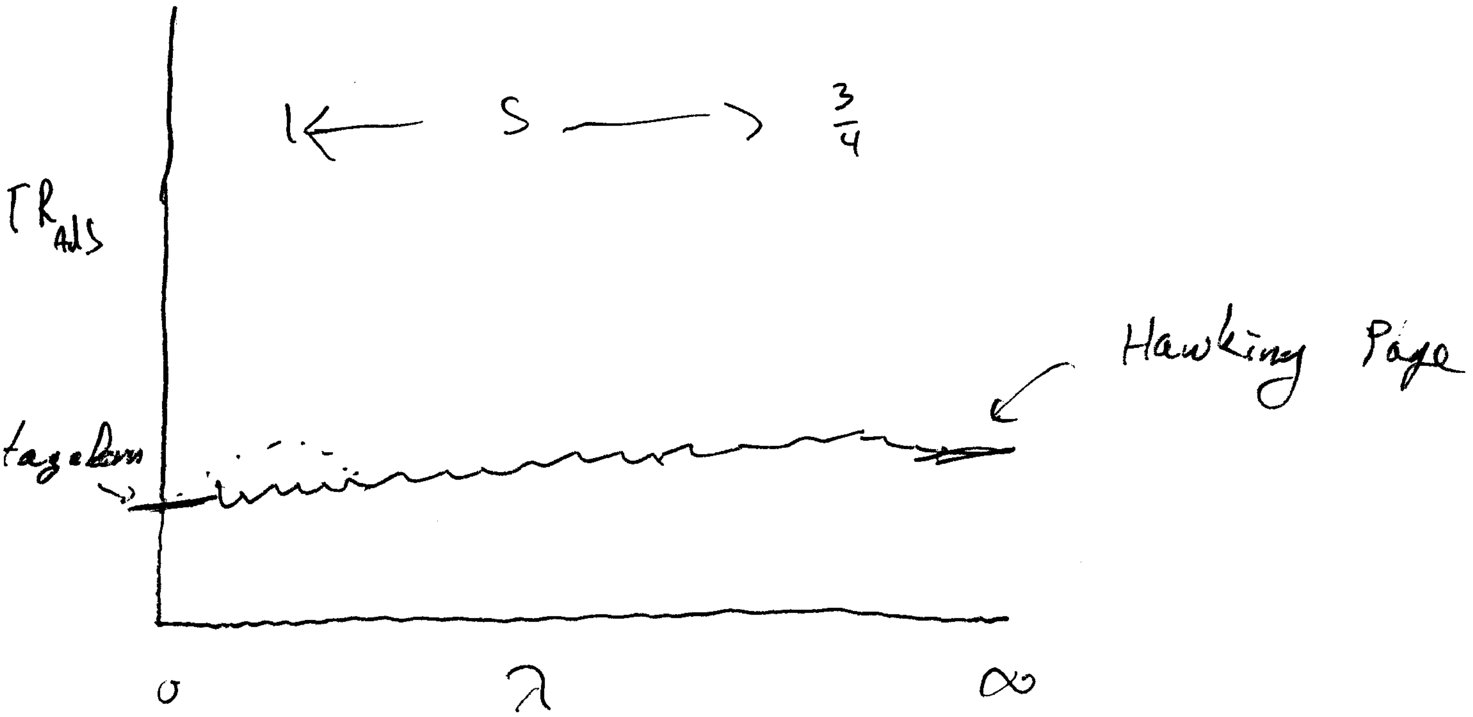
We don't know how to calculate SYM correlators of this type at large λ.

Take advantage of the severity of our problem?

A possible strategy

Finite T phase diagram for $N \to \infty$
$\mathcal{N}=4$ SYM on a sphere

"Conventional Wisdom"



In particular "C.W." is that large TR_{AdS} region is smooth (analytic) except at $\lambda = \infty$

$e^{-1/\lambda} \longrightarrow e^{-\lambda^{1/2}}$

Sharp features (singularities) should move analytically in λ (away from $\lambda = \infty$).
Can follow them to small λ where perturbation theory applies.

e.g.,

Hawking - Page $\longrightarrow$ Hagedorn

Gregory - LaFlamme $\longrightarrow$ Gross-Witten

Try to follow t_c singularity to small λ

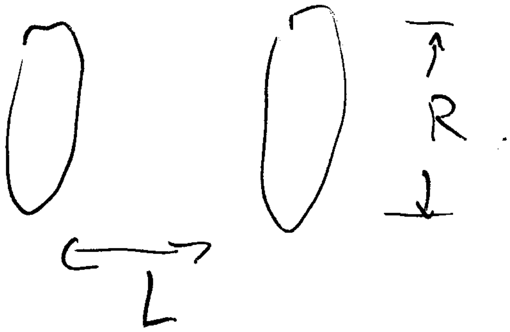
Several subtleties

Hawking - Page — bulk geometry changes
N^2 degrees of freedom rearrange

at t_c sing or $t^{4/3}$ branch pt. bulk geometry stays fixed, D-brane probe changes ($m_D \sim \frac{1}{g_s} \sim N$)
only N out of N^2 d.o.f. affected

Practice problem for D-brane "phase transition"

Two 't Hooft loops in $N=\infty$ AdS/CFT (zero temp)

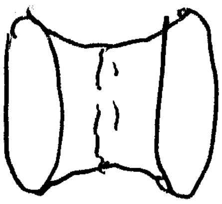


$s = L/R$

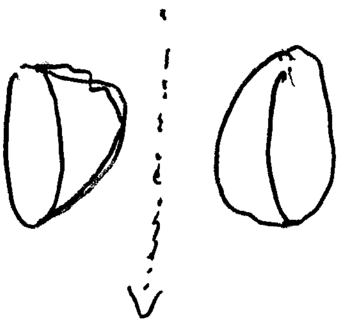
$T_D \sim \frac{1}{g_s} \sim N \to \infty$

saddle surface dominates

$s \ll 1$

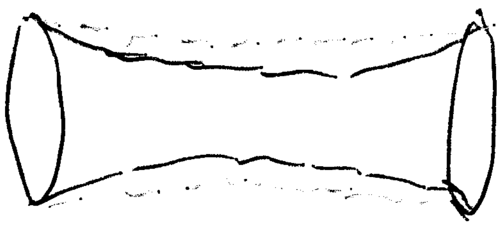


$s = s_0$



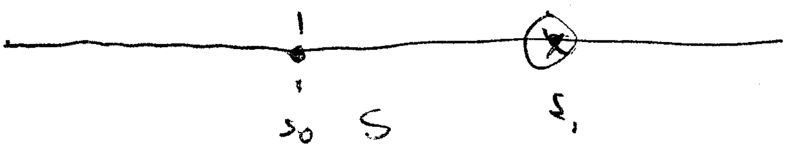
"first order", continue through

$s = s_1$



— min

annihilate



Follow as a function of λ

Study $\langle {}'tH \quad {}'tH \rangle$ at small λ

use semi-classical techniques.

'tHooft loops enforce boundary conditions (holon. $\in \mathbb{Z}_N$) on A_μ: it must be a monopole configuration in some $U(1) \in SU(N)$ conjugate to

$$\begin{pmatrix} N & & & & \\ & -1 & & & \\ & & -1 & & \\ & & & -1 & \\ & & & & \ddots \\ & & & & & -1 \end{pmatrix}$$

Possible conjugates indexed by space

$\mathcal{M} = SU(N) / (SU(N-1) \times U(1)) \sim CP^{N-1}$

N dimensional, N d.o.f.

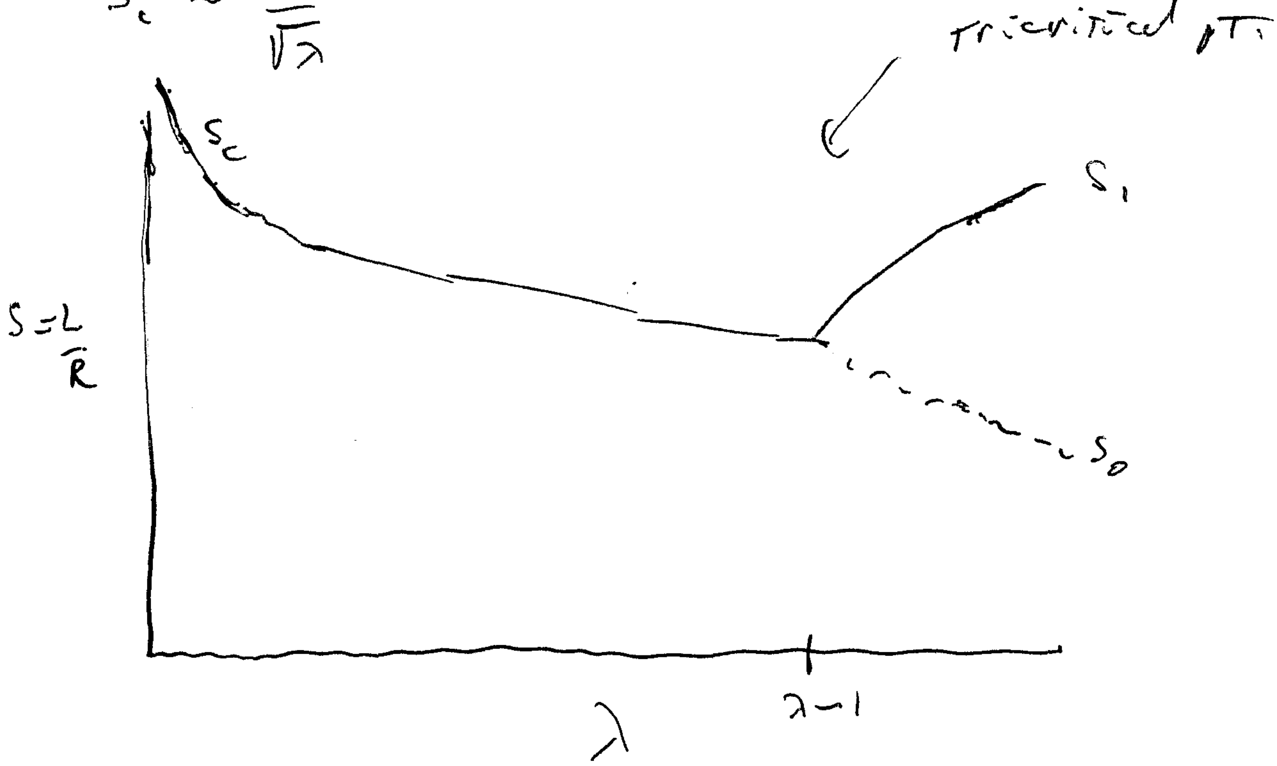
~~Each~~ 'tHooft loops labelled by a relative $U(1)$ orientation $\in \mathcal{M}$

Loops close: attractive energy favors $U(1)$'s almost "parallel"

Loops distant: small energy, "entropy" favors $U(1)$'s almost "perpendicular"

Entropy - Energy balance creates a 2nd order large N transition at

$s_c \sim \frac{1}{\sqrt{\lambda}}$



Back to the black hole singularity.
We are in the middle of this so we will just give a status report.

We want to compute

$$\langle \mathcal{O}_m \mathcal{O}_m \rangle (t) = \langle \det\phi \, \det\phi \rangle (t)$$

for λ small, $(T R_{AdS})$ large. $N \to \infty$

Thermal Feynman diagrams

Combinatorial analysis

Aharony, Antebi, Berkooz, Fishman "Holey Sheets"

c.f. Arnold, Yaffe

Thermal dynamics
Quark-gluon plasma,
Heavy ion collision lit.

Current level of understanding:

1) There are candidates for the continuation of the two geodesic coalescence branch points

2) There does not seems to be a source for the t_c singularity

Possibilities:

1) We are missing something

2) The t_c singularity is smoothed out for all finite λ

$\Rightarrow$ α' effects ($g_s = 0$) resolve (some aspects of) the black hole singularity

α' effects are large $\sim$ α'(curvature)

$\longrightarrow \infty$ at t_c $\sim \dfrac{\alpha'}{(t-t_c)^4}$

Not conventional wisdom

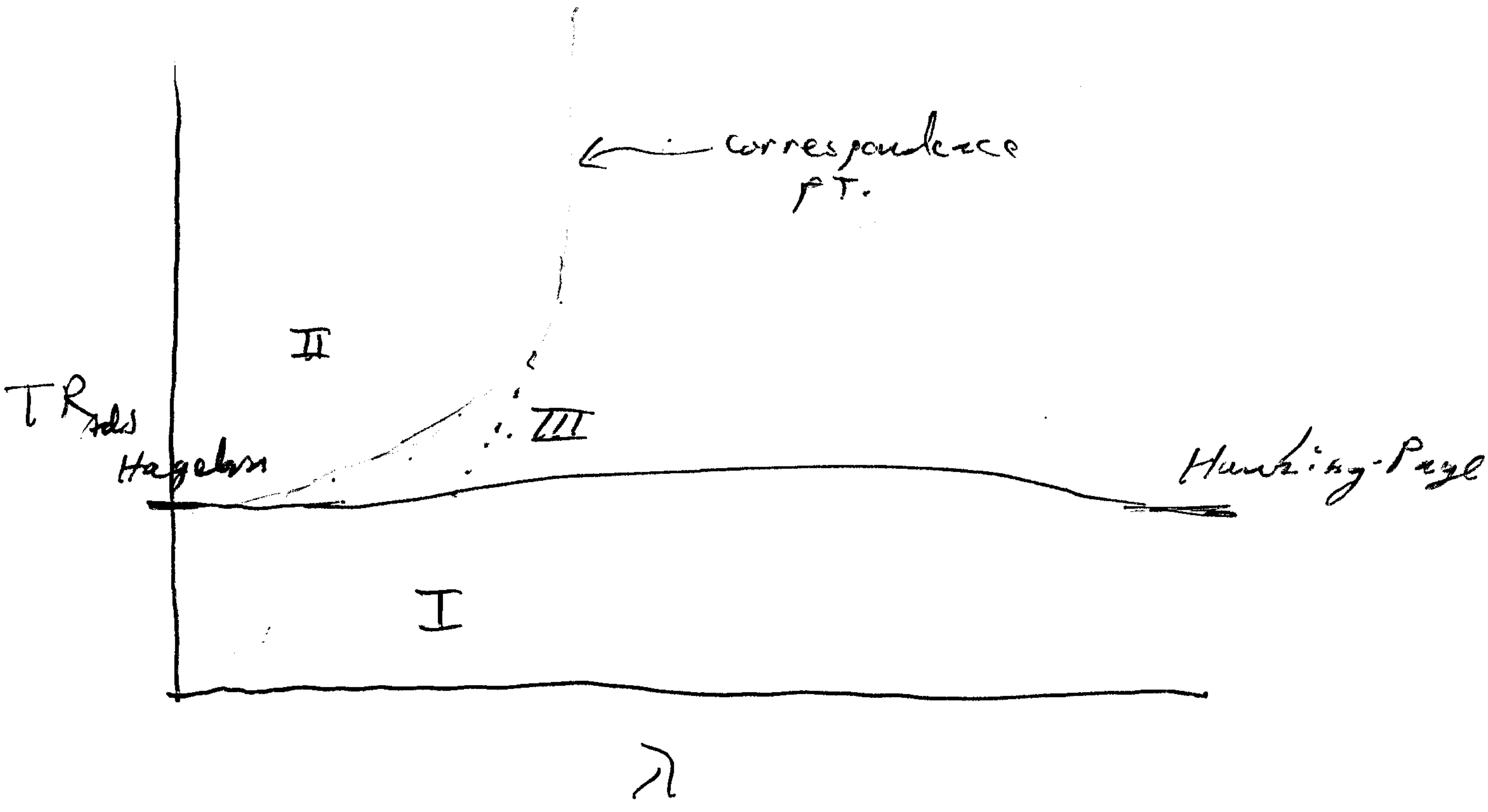
l_p not l_s relevant scale

in null orbifold α' effects insufficient

Could be studied via boundary states in black hole σ-model, if we knew it.

Coset black holes do not have an analog of t_c singularity because effective D-brane metric is flat

3) Conventional Wisdom about phase diagram is wrong.



Maybe a new transition line — dividing the black hole from the perturbative gauge theory, at the correspondence point $R_c \sim l_s$
$\Rightarrow \lambda \sim 1$

At small λ, eigenvalue distributions of Polyakov loop distinguish I, II, III.

Need to extend to large λ

(t_c sing at Hagedorn?)

We hope to narrow down this menu of options in the not too distant future.