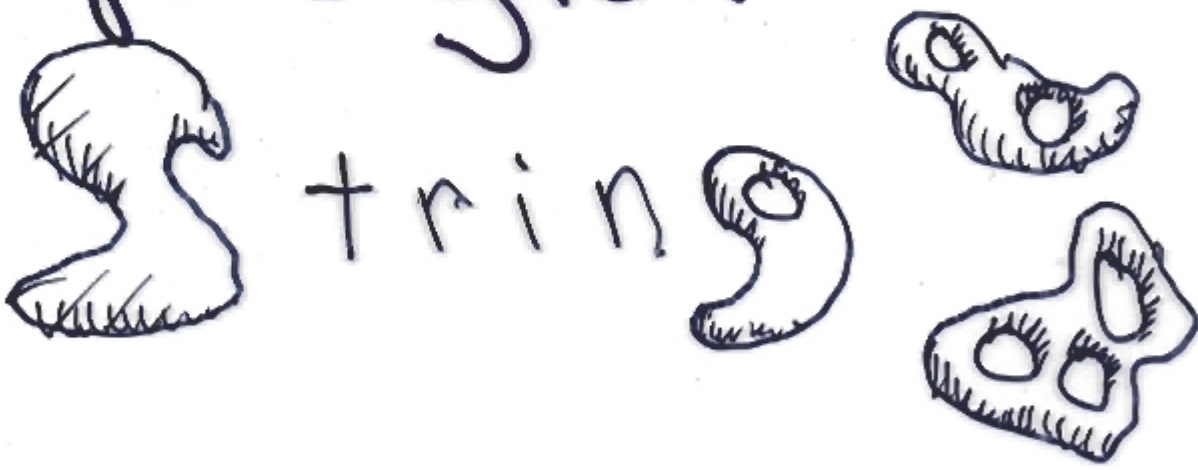
Black Hole Attractors and the Topological String



In collaboration with

[redacted]

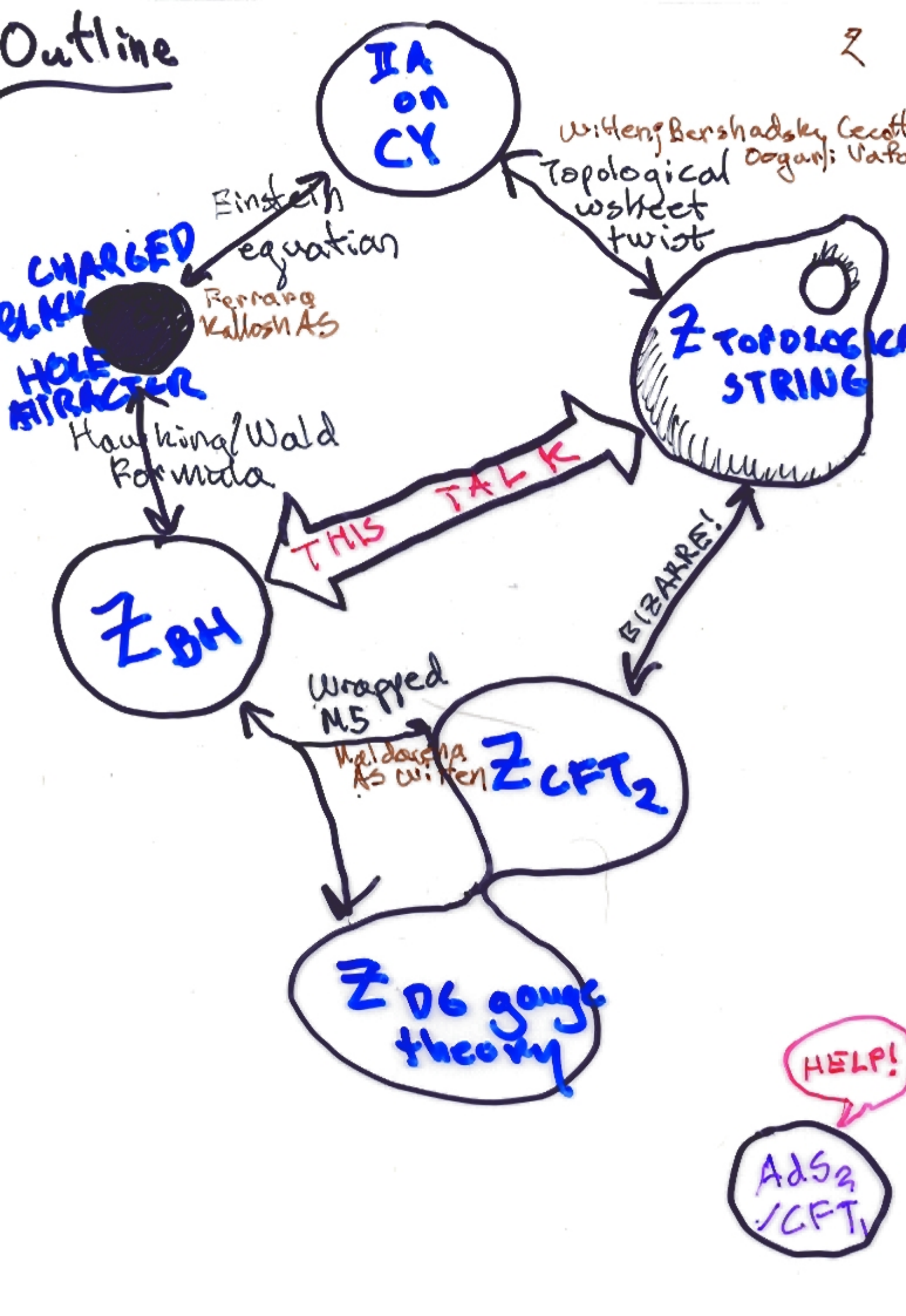
Hirosi Ooguri

and

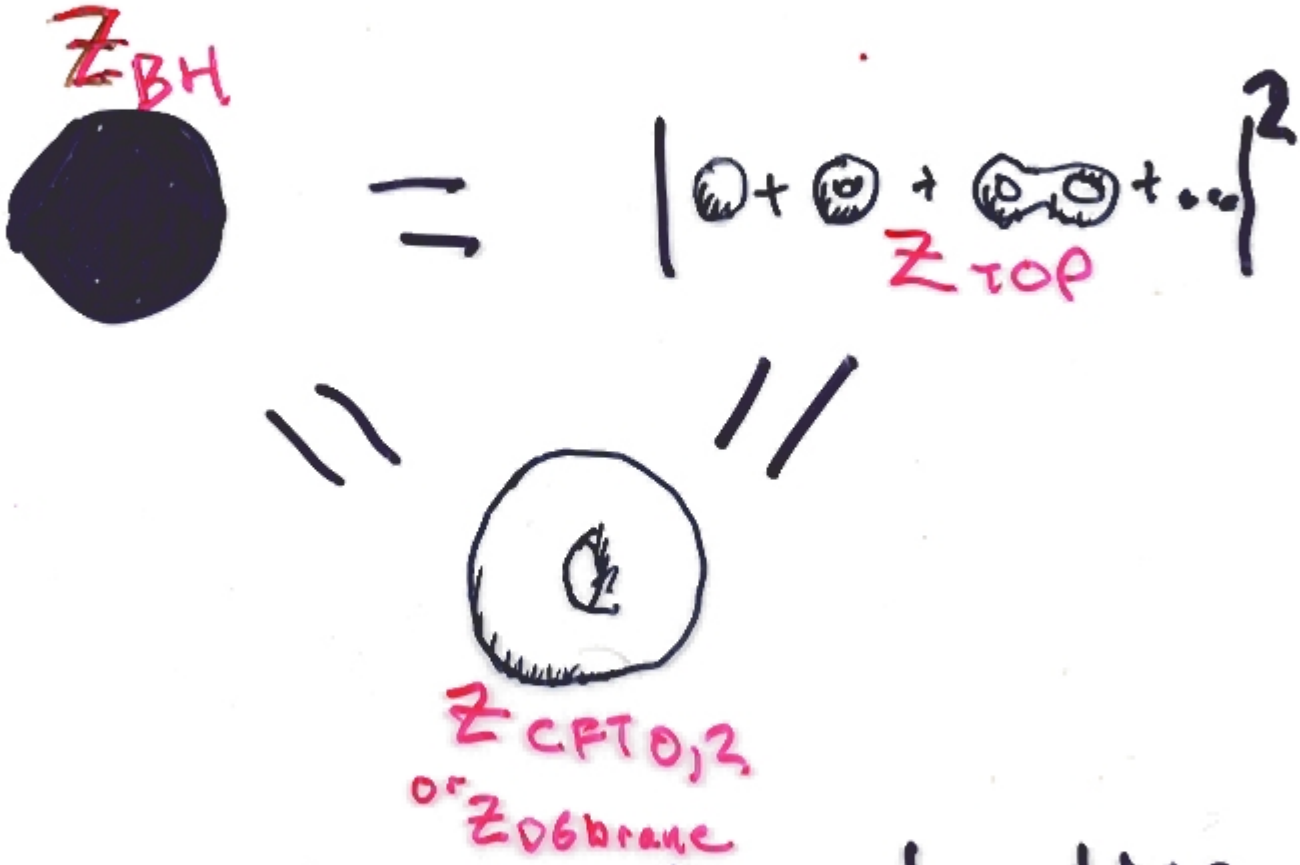
Cumrun Vafa

hep-th 0405146

Outline


IIA on CY
Witten; Bershadsky Cecotti Ooguri; Vafa
Topological worldsheet twist
Einstein equation
CHARGED BLACK HOLE ATTRACTOR
Ferrara Kallosh AS
Hawking/Wald Formula
THIS TALK
BIZARRE!
Z_{BH}
Z_{CFT_2}
Wrapped M5
Maldacena AS Witten
Z_{D6} gauge theory
HELP!
AdS_2/CFT_1

Summary of Conjecture



Why do these objects live on the same space?

Why is Z_{TOP} squared?

Lightning review of the topological string.

$$\log Z_{TOP} = \sum_h F_h(X)\, g_{TOP}^{2h-2}$$

- Z_{TOP}: 2nd quantized partition function.
- F_h: Gromov witten invariants
- X: kahler moduli
- witten BCOV
- g_{TOP}: genus counting parameter

(i) Twisted ws sees only kahler moduli (A model)

(ii) F_h counts genus h holomorphic maps

(iii) Computes F-terms

graviphoton

$$\int d^4x \sqrt{-g} F_h(X) (R_{\mu\nu\rho\sigma})^2 (T_{\mu\nu})^{2h}$$

BCOV
Antoniadis Gava Narain Taylor

Why is this beautiful mathematical structure underlying such an uninteresting amplitude?

Lightning review of BH attractors

Ferrara Kallosh AS
AS

Moduli space

Kahler moduli

BH CHARGES

magnetic $p^\Lambda = \mathrm{Re}\, C X^\Lambda$

electric $q_\Lambda = \mathrm{Re}\, C F_\Lambda$

C: graviphoton charge

$F_\Lambda = \partial_\Lambda F_0(X)$

= periods

This had to happen so that the entropy

Area $= S_{BH} = S_{BH}(p^\Lambda, q_\Lambda, \cancel{\text{moduli}})$

$= \frac{\pi i}{2} \left[\bar{C} \bar{X}^\Lambda q_\Lambda - \bar{C} \bar{F}_\Lambda p^\Lambda \right]_{\text{Attractor point}}$

Notice that both Z_{TOP} and $Z_{BH} \sim e^{S_{BH}}$ depend on half the moduli (Kahler ~~complex~~).

Further since the F-term

$$\left[\sum_h \int d^4x \sqrt{-g}\, F_h(X) (R_{\mu\nu\rho\sigma})^2 (T_{\mu\nu})^{2h}\right]_{\text{Black Hole}} \neq 0$$

computed by Z_{TOP}, we expect corrections to Z_{BH}, with

$$g_{TOP}^2 \sim (T_{\mu\nu})^2_{\text{horizon}} \sim \frac{1}{Q^2_{\text{graviphoton}}}$$

or

$$\boxed{g_{TOP} \sim \frac{1}{Q}}$$

Cardoso de Wit Mohaupt

So what is the relation between Z_{TOP} and Z_{BH}?

Important subtleties

Z_{TOP} Holomorphic Anomalies (Bershadsky Cecotti Ooguri Vafa)

!!?

Z_{BH} AdS_2 fragmentation (M. Jackson, Maldacena, S) Multiple attractor basins (Moore, Denef)

!!?

Z_{CFT} Non-compact Coulomb branches

These arise at higher orders and have been ignored. They lead to anomalous background moduli dependence in all three pictures. Hopefully they enrich, not destroy the picture.

$\frac{1}{Q}$ corrected S_{BH}

Wald; Cardoso, deWit & Mohaupt

Corrected attractor eqns.

$$p^\Lambda = \mathrm{Re}\, C X^\Lambda$$

$$q_\Lambda = \mathrm{Re}\, C F_\Lambda$$

same as before except full F

$$c^2 T^2 = 256 \quad F_\Lambda = \partial_\Lambda \sum_h F_h(X, T^2)$$

$$= \frac{64}{Q^2}$$

Corrected entropy

$$S_{BH} = \frac{\pi i}{2}\left[\bar{C}\bar{X}^\Lambda q_\Lambda - p^\Lambda \bar{C}\bar{F}_\Lambda\right]_{attr}$$

$$+ 128\pi i \left[\frac{\partial \bar{F}}{\partial \bar{T}^2} - \frac{\partial F}{\partial T^2}\right]_{attractor}$$

What does this complicated-looking formula mean?

Reinterpretation

Path integrals compute mixed canonical/microcanonical ensemble:

magnetic charge

$$Z_{BH}(p^\Lambda, \phi^\Lambda) = \sum_{q_\Lambda} \Omega_{BH}(p^\Lambda, q_\Lambda) e^{-\phi^\Lambda q_\Lambda}$$

electric potential

$$\boxed{\phi^\Lambda = \pi \, \mathrm{Im}\, \mathcal{C} X^\Lambda}$$

$$\Rightarrow S_{BH} = \ln Z_{BH} - \phi^\Lambda \frac{\partial}{\partial \phi^\Lambda} \ln Z_{BH}$$

Complicated formula for S_{BH} arises from

$$Z_{BH} = e^{\pi \, \mathrm{Im}\, F(\mathcal{C}X^\Lambda)}$$

with $\mathcal{C}X^\Lambda = p^\Lambda + i \frac{\phi^\Lambda}{\pi}$

or

$$\boxed{Z_{BH} = e^{F_{TOP} + \bar{F}_{TOP}} = |Z_{TOP}|^2}$$

Concluding remark

Why?