Field Theory Supertubes

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STRINGS '04

General Idea: Field theory solitons

- instanton
- monopole/dyon
- vortex
- domain wall
- ⋮

→ $\boxed{\tfrac{1}{2} \text{ susy branes}}$ of

higher-dimensional susy field th.

- What are eqs. for <u>intersections</u> preserving $\tfrac{1}{4}, \tfrac{1}{8} \ldots$ susy?
- Explicit solutions?
- What is M/string interpretation?

Many advances by several groups over last 5 years or so.

This talk concentrates on <u>one</u> aspect:

Supertubes

BIonic perspective

$$\begin{array}{ccc} 11D & & M2 \\ \big\downarrow S^1 & \swarrow & \downarrow \\ 10D & F1 & D2/D0 \end{array}$$

from boost along S^1

$\underbrace{F1 \quad D2/D0}$

all $\frac{1}{2}$ susy. Get $\frac{1}{4}$ susy from $M2 \perp M2$

$$M2 \perp M2$$
$$\downarrow$$
$$\boxed{F1 \perp D2/D0}$$

effective worldvol. description as <u>dyonic BIon</u>

BI. fields
$$\begin{cases} B = \dfrac{v}{\sqrt{1-v^2}} \\[2ex] \underset{\sim}{E} = \dfrac{1}{\sqrt{1-v^2}} \underset{\sim}{\nabla} X \end{cases}$$

transverse displacement

(Tong, Gauntlett, Portugues & PKT)

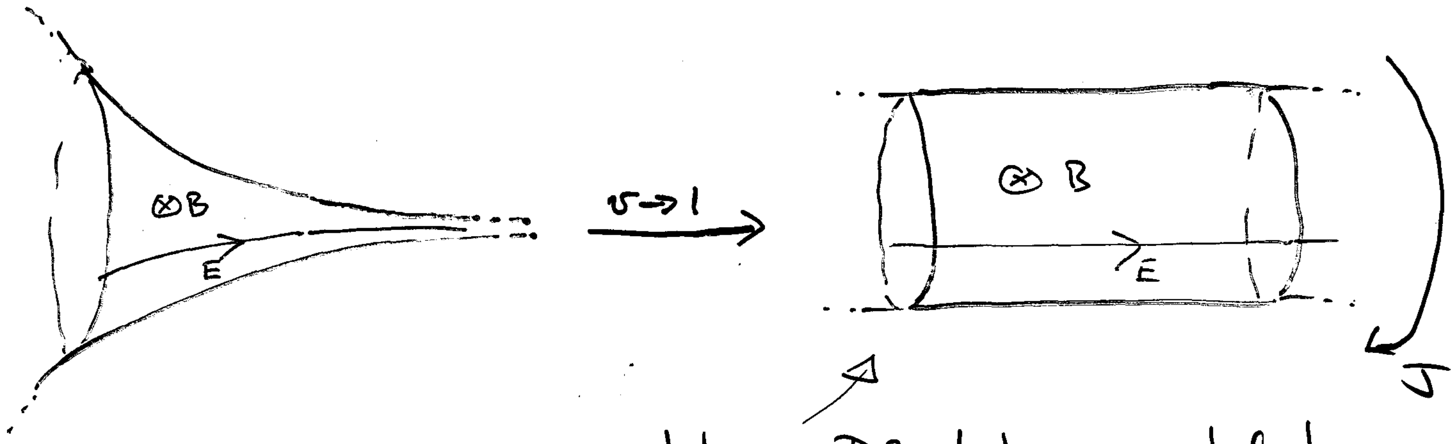
But we could start in <u>any</u> dimension D admitting supermembrane:

$D = 11$: M2
$D = 7$: SYM instanton
$D = 5$: σ-model lump
$D = 4$: WZ model domain wall

} <u>all</u> have effective worldvol. description as supermembrane

$v \to 1$ limit : supertube

Take $v \to 1$ & rescale $B, \underset{\sim}{E}$ so B remains finite



supertube : D2 tube supported by ang. mom. due to B, E fields (Mateos & PKT)

Surprise.

$$\frac{1}{4}\ \text{susy} \Rightarrow \begin{cases} \text{circular x-section} & \text{if } v<1 \\ \text{arbitrary x-section} & \text{if } v=1 \end{cases}$$ (Mateos, Ng, PKT)

For $D = 11/10$ this is effective worldvol. description of supergravity (M/string) supertube (Emparan, Mateos, PKT)

For $D = 7/6$ we expect SYM supertube
For $D = 5/4$ " " σ-model "

These are "field theory supertubes"

Field Theory Supermembranes

- Start with $\boxed{M2 \text{ in } Mink_7 \times K3}$

replace, locally, by KK monopoles

$$\downarrow S^1$$

$$\boxed{D2 \text{ in } D6s}$$

i.e. <u>(2-brane) instanton of 7D SYM</u>

- Start with $\boxed{M2 \text{ in } Mink_5 \times CY}$

replace, locally, by <u>intersecting</u> KK monopoles

$$\downarrow T^3$$

D4 :	1	2	3	4				
D4 :	1	2			5	6		
KK :					×	•	•	•

vacuum <u>soliton</u>

D4 wrapped on S^2

<u>(2-brane) lump of 5D (hyper-Kähler) σ-model</u>

Intersecting Instantons

- 2-brane of 7D SYM is instanton

  = solution of $\frac{1}{2}$-susy eqs $\boxed{F_{ij} = \tilde{F}_{ij}}$

- Intersecting 2-branes (D2⊥D2 in D6s)

  solve unique $\frac{1}{4}$-susy six-dim. eqs

$$\left.\begin{aligned} &F_{12} + F_{34} + F_{56} = 0 \\ &F_{13} + F_{42} = 0 \\ &F_{14} + F_{23} = 0 \\ &\vdots \end{aligned}\right\}$$

(Bak, Lee & Park)

- Compactify on S^1 & boost on S^1 (PKT)

$$\boxed{\begin{aligned} &F_{ij} + \tilde{F}_{ij} + \sqrt{1-v^2}\,\Omega_{ij} D_s\phi \\ &F_{is} = \sqrt{1-v^2}\,\Omega_{ij} D_j\phi \\ &F_{0i} + v D_i\phi = 0 \\ &F_{0s} + v D_s\phi = 0 \\ &D_0\phi = 0 \end{aligned}}$$

$$\Omega = \left(\begin{array}{cc|cc|cc} 0 & 1 & & & & \\ -1 & 0 & & & & \\ \hline & & & & & \\ & & & 0 & 1 & \\ & & -1 & 0 & & \\ \hline & & & & 0 & 1 \\ & & & & -1 & 0 \end{array}\right)$$

← describe $\frac{1}{4}$ susy intersection of instanton string with dyon 2-brane

Higgs field: $\phi = A_6$

$v \to 1$: dyonic instanton tube

- Take $v \to 1$:

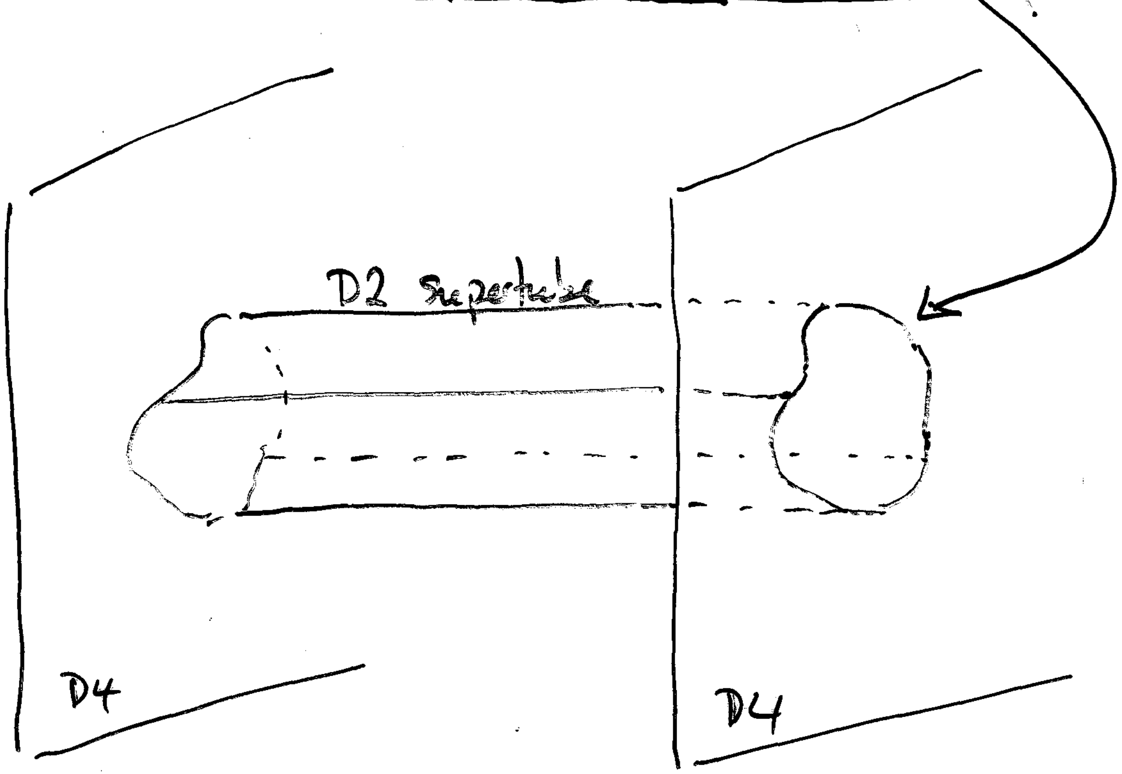
$$\left.\begin{aligned} F_{ij} + \tilde{F}_{ij} &= 0 \\ F_{0i} + D_i\phi &= 0 \\ D_0\phi &= 0 \end{aligned}\right\}$$ dyonic instanton eqs. (Lambert & Tong)

$$\left.\begin{aligned} F_{si} &= 0 \\ F_{0s} + D_s\phi &= 0 \end{aligned}\right\}$$ solve by $\partial_s = 0$ & $A_s = 0$

$\hookrightarrow$ tube with dyonic-instanton x-section

- Dyonic instanton has $\boxed{\text{charge } Q \text{ \& instanton \# } N}$

General $N=2$ solution has Higgs zeros on closed curve $\Rightarrow$ loop of dyon-string (Bak, Kim & Lee)



Intersecting Lumps

Lump = holomorphic map : $\mathbb{C} \longrightarrow S^2 \subset$ target space

e.g. $\boxed{Z(z) = \frac{c}{z}}$ ($|c|$ = size of lump.)

complex field.

= ½ susy 2-brane of 5D susy σ-model

Generic (⅛) susy configuration is quaternionic map

$$\phi\big(\underbrace{x_4 + \mathbb{i}x_1 + \mathbb{j}x_2 + \mathbb{k}x_3}_{q \in \mathbb{H}}\big) \in \mathbb{H}$$

satisfying

$$\boxed{\partial_4 \phi + \underset{\sim}{I} \cdot \underset{\sim}{\nabla} \phi = 0}$$

(Portugues & PKT)

For ALE spaces special ¼-susy solution is holomorphic map : $\mathbb{C}^2 \longrightarrow S^2 \subset$ target

e.g. $\boxed{Z(\xi, z) = \frac{c}{\xi z}}$ ← intersecting 2-branes

Q-kink-lumps

Compactify on S^1 via Scherk-Schwarz:

$$\partial_4 \phi = m\, k(\phi)$$

mass — Killing vector field of target space

S^1

4D σ-model with potential: $V = m^2 |k|^2$

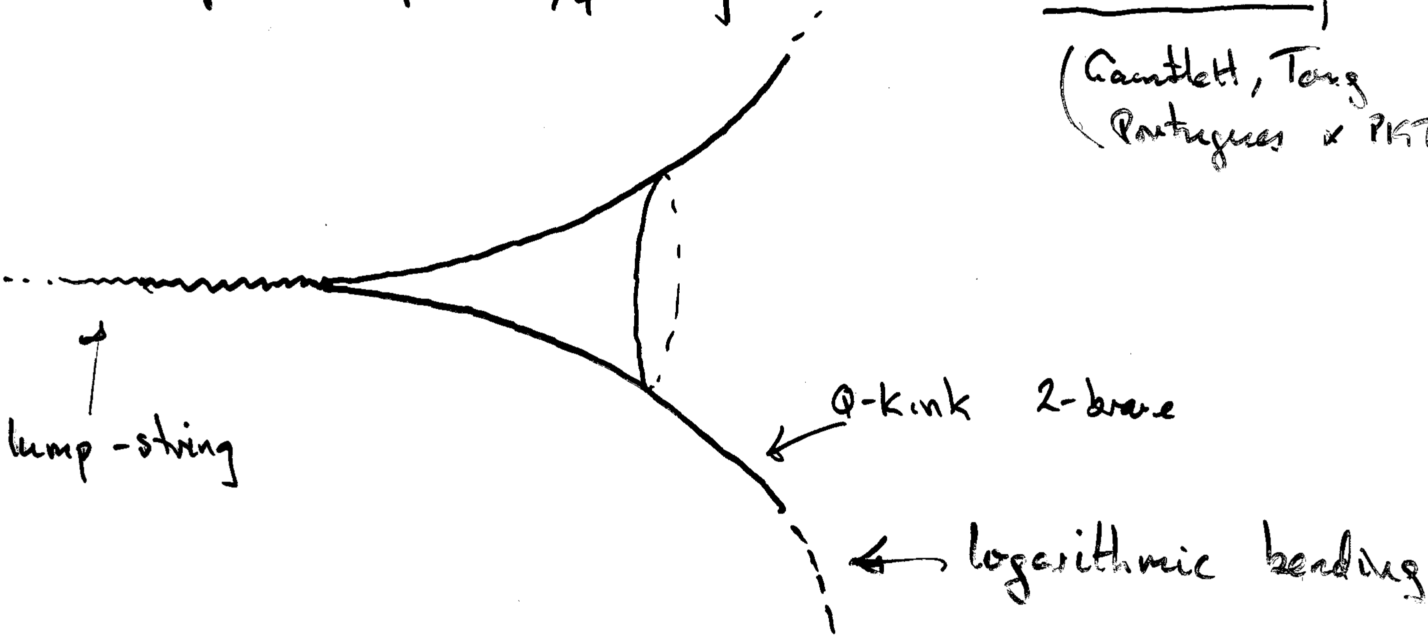
Boost on S^1 to get $\frac{1}{8}$-susy eqs

$$\boxed{\begin{array}{c} I \cdot \nabla \phi + m\sqrt{1-v^2}\, k \\ \dot{\phi} = m v k \end{array}}$$

(Portugues & PKT)

Can find explicit $1/4$-susy solution: Q-kink-lump

(Gauntlett, Tong; Portugues & PKT)



cf BIon, but 2-brane here is part of solution, with non-zero width.

$v \to 1$: Q-lump tube

$v \to 1 \Rightarrow$ $\boxed{\begin{array}{c} \underset{\sim}{I} \cdot \underset{\sim}{\nabla} \phi = 0 \\ \dot{\phi} = m k(\phi) \end{array}}$ ← (Naganuma, Nitta, Sakai)

→ 1/8 susy

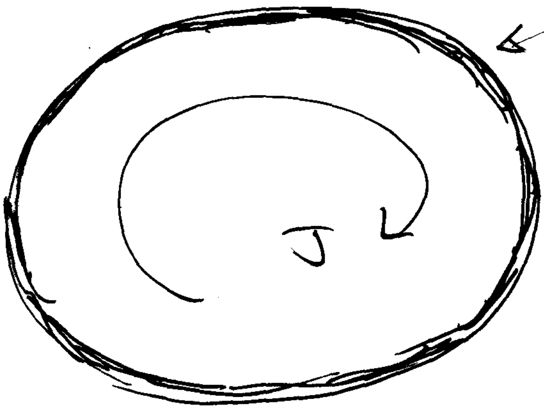
For 1/4 susy we need special case:

$$\left.\begin{array}{l} \nabla_1 \phi + I_3 \nabla_2 \phi \\ \dot{\phi} = m k \end{array}\right\} \text{Q-lump eqs.}$$ (Leese, Abraham)

$\nabla_3 \phi = 0 \Rightarrow$ tube with Q-lump x-section

Explicit Q-lump solutions for ALE target spaces:

- Ang. mom. $J \propto Q$
- For large Q energy density is ring-shaped



loop of Q-kink string

(Abraham & PKT)

(cf dyon string:
Q-kink is analog of dyon)

Summary

- Many aspects of M2/D2 branes in M/string-theory have analogs in SYM-theory & Hyper-Kähler σ-models.

- In particular <u>supertubes</u>

$$F1 \longleftrightarrow \text{instanton or lump} \quad (N)$$

$$D0 \longleftrightarrow \text{electric charge} \quad (Q)$$

In all cases get supersymmetric tubes supported by angular-mom.

- Effective (DBI) description allows <u>arbitrary</u> X-section

Not obvious from field-theory solutions.

$\underbrace{\text{True only in } N \to \infty \text{ limit}}$?

(if so suggests interesting property of ADHM instanton solution: limit of large instanton #.)