

Higher Spin Gauge Theories in Any Dimension

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Symmetric Massless Free Fields



Fronsdal (1978)

$$\varphi_{n_1 \dots n_s}(x)$$

$$\varphi^m{}_{m'}{}^k{}_{k'}{}_{n_5 \dots n_s}(x) = 0$$

Abelian HS gauge symmetry

$$\delta \varphi_{n_1 \dots n_s}(x) = \partial_{\{n_1} \varepsilon_{n_2 \dots n_s\}}(x)$$

$$\varepsilon^r{}_{rn_3 \dots n_{s-1}} = 0$$

Invariant Free Action

$$\begin{aligned} S^s = & \frac{1}{2}(-1)^s \int d^d x \{ \partial_n \varphi_{m_1 \dots m_s} \partial^n \varphi^{m_1 \dots m_s} \\ & - \frac{s(s-1)}{2} \partial_n \varphi^r{}_{rm_1 \dots m_{s-2}} \partial^n \varphi^k{}_{k'}{}^{m_1 \dots m_{s-2}} + s(s-1) \partial_n \varphi^r{}_{rm_1 \dots m_{s-2}} \partial_k \varphi^{nkm_1 \dots m_{s-2}} \\ & - s \partial_n \varphi^n{}_{m_1 \dots m_{s-1}} \partial_r \varphi^{rm_1 \dots m_{s-1}} - \frac{s(s-1)(s-2)}{4} \partial_n \varphi^r{}_{r'}{}^n{}_{m_1 \dots m_{s-3}} \partial_k \varphi^t{}_{t'}{}^{km_1 \dots m_{s-3}} \} \end{aligned}$$

$s = 1$	Maxwell
$s = 2$	Einstein
$s > 2$	Unifying gauge principle ?

Geometric formulations

Metric-like

de Wit, Freedman (1980)

Siegel, Zwiebach (1987)

Francia, Sagnotti (2002)

Bekaert, Boulanger (2003)

de Medeiros, Hull (2003)

Frame-like

M.V.(1980)

Alkalaev, Shaynkman, M.V.(2003)

Higher Spin Problem

Find a nonlinear HS gauge theory

- Correct free field limit
- Unbroken HS gauge symmetries
- Non-Abelian global HS symmetry of a vacuum solution

Motivations

- Massive HS modes in Superstring from breaking of HS gauge symmetries?!

Gross, Mende (1988)

- *AdS/CFT*: HS symmetries are unbroken in the Sundborg–Witten limit

$$\lambda = g^2 N \rightarrow 0$$

$$l_{str}^2 \Lambda_{AdS} \rightarrow \infty$$

Sundborg hep-th/0103247

Witten (2001)

Sezgin, Sundell hep-th/0105001

Klebanov, Polyakov hep-th/0204051

Difficulties

- S – matrix argument

Coleman, Mandula (1967)

- HS-gravity interaction problem

Aragone, Deser (1979)

$$\partial_n \rightarrow D_n = \partial_n - \Gamma_n \quad [D_n, D_m] = \mathcal{R}_{nm} \dots$$

$$\delta\varphi_{nm\dots} \rightarrow D_n \varepsilon_{m\dots}$$

$$\delta S_s^{cov} = \int \mathcal{R} \dots (\varepsilon \dots D\varphi \dots) \neq 0 \quad ?!$$

↑
Weyl tensor for $s > 2$

Higher Derivatives in HS Interactions

A.Bengtsson, I.Bengtsson, L.Brink (1983)

Berends, Burgers, van Dam (1984)

$$S = S^2 + S^3 + \dots$$

$$S^3 = \sum_{p,q,r} (D^p \varphi)(D^q \varphi)(D^r \varphi) \ell^{p+q+r+\frac{1}{2}d-3}$$

Role of AdS Background

HS theories:

Fradkin, M.V. (1987)

$$\ell = \Lambda^{-\frac{1}{2}} = R_{AdS}$$

$$[D_n, D_m] \sim \Lambda \sim O(1)$$

String:

$$\ell \sim \sqrt{\alpha'}$$

SUGRA by Gauging SUSY Algebra

$$o(d-1, 2)$$

$$o(N)$$

$$T^{AB}$$

$$Q_\alpha^p$$

$$t^{pq}$$

$$A, B, \dots = 0, \dots d$$

$$\omega_n^{AB}$$

$$\psi_{n\alpha}^p$$

$$A_n^{pq}$$

$$p, q, \dots = 1, \dots N$$

$$a, b, \dots = 0, \dots d-1$$

$$s = 2$$

$$g_{nm} \longrightarrow e_n^a \longrightarrow \{e_n^a, \omega_n^{ab}\} \longrightarrow \omega_n^{AB}$$



MacDowell, Mansouri (1978)

Stelle, West (1980)

Higher Spin Gauge Fields

$s \geq 2$

$$\varphi_{n_1 \dots n_s} \rightarrow e_n^{a_1 \dots a_{s-1}} \rightarrow \{e_n^{a_1 \dots a_{s-1}}, \omega_n^{a_1 \dots a_{s-1}, b_1 \dots b_{s-1}}\} \rightarrow$$

$$\rightarrow \omega_n^{A_1 \dots A_{s-1}, B_1 \dots B_{s-1}}$$

Lopatin, M.V. (1988)

M.V. hep-th/0106200

$$\left. \begin{aligned} \omega_n^{\{A_1 \dots A_{s-1}, A_s\} B_2 \dots B_{s-1}} &= 0 \\ \omega_n^{A_1 \dots A_{s-3} C} C_{C, B_1 \dots B_{s-1}} &= 0 \end{aligned} \right\}$$

$$\begin{array}{c} s-1 \\ \begin{array}{|c|c|c|c|c|c|c|c|} \hline & & & & & & & \\ \hline \end{array} \\ o(d-1, 2) \end{array}$$

Dynamical frame-like field

$$e_n^{a_1 \dots a_{s-1}} = \omega_n^{a_1 \dots a_{s-1}, B_1 \dots B_{s-1}} V_{B_1} \dots V_{B_{s-1}}$$

$$V_B = \delta_B^{\hat{d}} \quad \hat{d} : (d+1)^{th} - \text{direction}$$

Fronsdal field $\varphi_{n_1 n_2 \dots n_s} = e_{\{n_1, n_2 \dots n_s\}}$

Auxiliary Lorentz–connection type fields

$$\omega_n^{a_1 \dots a_{s-1}, b_1 \dots b_t} = \omega_n^{a_1 \dots a_{s-1}, b_1 \dots b_t B_{t+1} \dots B_{s-1}} V_{B_{t+1}} \dots V_{B_{s-1}} : \left(\frac{1}{\sqrt{\Lambda}} \frac{\partial}{\partial x} \right)^t (e)$$

$$t > 0$$

HS Algebra $hu(1|2:[d-1, 2])$

As CFT_{d-1} HS algebra: Eastwood [hep-th/0206233](#)

Generators

$T_{A_1 \dots A_n, B_1 \dots B_n}$

n

 $o(d-1, 2)$

$T_{\{A_1 \dots A_n, A_{n+1}\} B_2 \dots B_n} = 0$

$T^C_{C A_3 \dots A_n, B_1 \dots B_n} = 0$

Oscillator Realization

M.V. hep-th/0304049

$$[Y_i^A, Y_j^B]_* = \epsilon_{ij} \eta^{AB}$$

canonical pair $Y_1^A = Y^A, Y_2^A = P^A$

$$\eta_{AB} = \eta_{BA} \quad A, B = 0, \dots, d$$

$$\epsilon_{ij} = -\epsilon_{ji} \quad i, j = 1, 2$$

$$T^{A,B} = -T^{B,A} = \frac{1}{2} \{Y_i^A, Y_j^B\}_* \epsilon^{ji}$$

$o(d-1, 2)$

$$t_{ij} = t_{ji} = \frac{1}{2} \{Y_i^A, Y_{jA}\}_*$$

$sp(2)$

$$[t_{ij}, T^{AB}]_* = 0$$

Howe duality

$$hu(1|2:[d-1, 2])$$

Lie bracket: $[f(Y), g(Y)]_*$

- $[f(Y), t_{ij}]_* = 0$
- $f(Y) = t^{ij} * f_{lij} = f_{rij} * t^{ij} \sim 0$

Gauging HS Algebra

$$\omega(Y|x) = \sum_{n=0}^{\infty} dx^m \omega_{m A_1 \dots A_n, B_1 \dots B_n}(x) Y_1^{A_1} \dots Y_1^{A_n} Y_2^{B_1} \dots Y_2^{B_n}$$

$$\begin{array}{c} 0 \\ \parallel \\ 0 = Dt_{ij} = dt_{ij} + [\omega, t_{ij}]_* = 0 \end{array}$$

imposes Young properties

$$\omega_m \{A_1 \dots A_n, A_{n+1}\} B_2 \dots B_n = 0 \quad \begin{array}{c} n \\ \boxed{\begin{array}{cccccc} & & & & & \end{array}} \end{array}$$

Field strength

$$R = d\omega(Y|x) + \omega(Y|x) \wedge * \omega(Y|x)$$

Different Spins: Homogeneous Polynomials

$$\omega(\mu Y|x) = \mu^{2(s-1)} \omega(Y|x)$$

maximal finite dimensional subalgebra

$$o(d-1, 2) \quad \oplus \quad u(1)$$

bilinears

$$s=2$$

constants

$$s=0$$

Matrix valued fields:

$$\omega \rightarrow \omega_q^p(Y|x) \quad p, q = 1, \dots, n \quad u(1) \rightarrow u(n)$$

Lower Spin Examples

$s = 2 : R^a = 0$

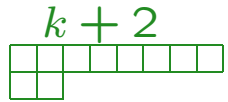
zero torsion

$\downarrow$ **Weyl tensor**
 $R^{ab} = e_c \wedge e_d C^{ac, bd}$

Ricci is zero

Bianchi identities imply

$$\partial_{n_1} \dots \partial_{n_k} C_{a_1 a_2, b_1 b_2} \sim C_{c_1 \dots c_{k+2}, d_1 d_2}$$

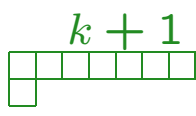


Einstein equations: $C_{a_1 \dots a_{k+2}, b_1 b_2}$

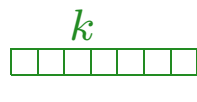
traceless

Linearized $s = 2$ equations in flat space

$$dC_{a_1 \dots a_l, b_1 b_2} = e^c \left(k C_{a_1 \dots a_l c, b_1 b_2} + 2 C_{a_1 \dots a_l \{b_1, b_2\} c} \right)$$

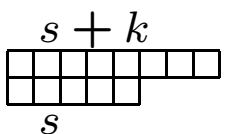
$S=1$: **Maxwell equations**  **traceless**

$$F = e_c \wedge e_d C^{c,d}, \quad dC_{a_1 \dots a_l, b} = e^c \left((l+1) C_{a_1 \dots a_l c, b} + C_{a_1 \dots a_l b, c} \right)$$

$S=0$: **Klein-Gordon equations**  **traceless**

$$dC_{a_1 \dots a_k} = e^c (k+2) C_{a_1 \dots a_k c}$$

$$C_{a_1 \dots a_k b}{}^b = 0$$

Any S :  **traceless**

$$C_{a_1 \dots a_{s+k}, b_1 \dots b_s}$$

AdS_d Vacuum

$\omega_0 \in o(d-1, 2)$:

$$\omega_0^{A B} \neq 0, \quad \omega_0^{A_1 \dots A_{s-1}, B_1 \dots B_{s-1}} = 0 \quad s > 2$$

AdS_d vacuum:

$$R^{A B}(\omega_0) = 0$$

Linearization:

$$\omega(Y|X) = \omega_0(Y|X) + \omega_1(Y|X)$$

Central On-Mass-Shell Theorem

All spins: free equations + constraints on auxiliary fields

M.V. hep-th/0106200

$$\begin{aligned} R_1(Y|x) &= \frac{1}{2} e_0^a \wedge e_0^b \frac{\partial^2}{\partial Y_i^a \partial Y_j^b} \varepsilon_{ij} C(Y|x) \Big|_{V_A Y_i^A = 0} \\ \widetilde{D}_0(C) &= 0 \end{aligned}$$

$$R_1 = d\omega + \omega_0 * \omega_1 + \omega_1 * \omega_0, \quad \widetilde{D}_0(C) = dC + \omega_0 * C - C * \widetilde{\omega}_0$$

$$\widetilde{f}(Y) = f(\widetilde{Y}), \quad \widetilde{Y}_i^A = Y_i^A - \frac{2}{V^2} V^A V_B Y_i^B$$

$$t_{ij} * C = C * t_{ij}$$

Unfolded Dynamics

$\omega^\Phi(x)$: a set of differential forms

$$d\omega^\Phi = F^\Phi(\omega) \qquad d = dx^n \frac{\partial}{\partial x^n}$$

Consistency condition

$$F^\Phi \wedge \frac{\delta F^\Omega}{\delta \omega^\Phi} = 0$$

FDA's

- (HS) gauge invariance

$$\delta\omega^\Phi = d\varepsilon^\Phi - \varepsilon^\Omega \frac{\delta F^\Phi}{\delta\omega^\Omega} = 0$$

$$\deg \varepsilon^\Phi = \deg \omega^\Phi - 1$$

- Invariance under diffeomorphisms
- Interactions: nonlinear deformation of F^Φ
- Degrees of freedom: 0-forms

Infinite dimensional module dual to

the space of one-particle states ($\forall x$)

- Universality

Nonlinear Construction

- Find a larger algebra g' such that

in $\star \quad \omega \rightarrow W = \omega + \omega C + \omega C^2 + \dots$

$$dW + W \wedge W = 0$$

reconstructs nonlinear equations

- Find restrictions on W that reconstruct $\star$ in all orders

Guiding principle: unbroken $sp(2)$

Result: interaction is unique up to field redefinitions.

YM constant $g^2 = |\Lambda|^{\frac{d-2}{2}} \kappa^2$ can be rescaled away in the classical HS model

Doubling of Oscillators

$$g \longrightarrow g' : \quad Y_i^A \longrightarrow (Z_i^A, Y_i^A) \quad \text{M.V. (1992)}$$

$$\omega(Y|x) \longrightarrow W(Z, Y|x), \quad C(Y|x) \longrightarrow B(Z, Y|x)$$

$$S(Z, Y|x) = dZ_i^A S_A^i \quad : \text{ connection along } Z_i^A$$

Noncommutative connection

$$\mathcal{W} = d + W + S$$

Star Product

$$(f * g)(Z, Y) = \int dS dT f(Z + S, Y + S) g(Z - T, Y + T) \exp 2S_A^i T_i^A$$

$$[Y_i^A, Y_j^B]_* = -[Z_i^A, Z_j^B]_* = \varepsilon_{ij} \eta^{AB}$$

$$Z - Y : Z + Y \quad \text{normal ordering}$$

Klein operator

$$\mathcal{K} = \exp \frac{2}{V^2} V_A Z^{Ai} V_B Y^B_i, \quad \mathcal{K} * f = \tilde{f} * \mathcal{K}, \quad \mathcal{K} * \mathcal{K} = 1$$

Nonlinear HS Equations

M.V. hep-th/0304049

$$\mathcal{W} * \mathcal{W} = \frac{1}{2}(dZ_A^i dZ_i^A + 4\Lambda^{-1} dz^i dz_i B * \mathcal{K}) \quad \mathcal{W} * B = B * \widetilde{\mathcal{W}}$$

$$\widetilde{\mathcal{W}}(dZ, Z, Y) = \mathcal{W}(\widetilde{dZ}, \widetilde{Z}, \widetilde{Y}) \quad dz_i = \frac{1}{\sqrt{V^2}} V_B dZ_i^B$$

Manifest gauge invariance

$$\delta \mathcal{W} = [\varepsilon, \mathcal{W}]_*, \quad \delta B = \varepsilon * B - B * \widetilde{\varepsilon}, \quad \varepsilon = \varepsilon(Z, Y|x)$$

Nonlinear $sp(2)$: two dimensional fuzzy hyperboloid
in noncommutative space of Y_i^A and Z_j^A .

Radius depends on HS curvature $B(x)$.

Perturbative Analysis:

$$W = W_0 + W_1, \quad S = dZ_i^A Z_A^i + S_1, \quad B = B_1$$

$$W_0 = \frac{1}{2} \omega_0^{AB}(x) Y_A^i Y_{iB} \quad \omega_0^{AB}(x) : \quad AdS_d$$

Central On-Mass-Shell Theorem

reproduced to lowest order

Singletons in any Dimension

M.V. hep-th/0404124

$hu(1|(1, 2):[d-1, 2]):$ **fermionic extension**

$$sp(2) \rightarrow osp(1, 2) \quad Y_i^A \rightarrow (Y_i^A, \phi^A), \quad \{\phi^A, \phi^B\} = -2\eta^{AB}$$

Fock modules of $hu(1|2:[M, 2])$ and $hu(1|(1, 2): [M, 2])$:

Bars, Deliduman,

massless scalar S_M and spinor F_M in M dimensions

Andreev hep-th/9803188

Generalized Flato-Fronsdal theorem

$$S_{d-1} \otimes S_{d-1} = \sum_{s=0}^{\infty} \oplus \overbrace{\square \square \square \square \square \square}^s \quad m=0 \quad \text{bosons in } AdS_d$$

Mikhailov (2002); Das, Jevicki (2003); Gopakumar (2003)

$$F_{d-1} \otimes S_{d-1} = \sum_{s=\frac{1}{2}}^{\infty} \oplus \overbrace{\square \square \square \square \square \square}^{s-1/2} \quad m=0 \quad \text{fermions in } AdS_d$$

$$F_{d-1} \otimes F_{d-1} = \sum_{p,q} \oplus \overbrace{\begin{array}{c} \square \square \square \square \square \square \\ \square \\ \square \\ \square \end{array}}^{p+1} \quad m=0 \quad \text{bosons in } AdS_d$$

$\oplus m > 0$ **antisymmetric tensors**

Matching: gauge fields –UIRs

HS superalgebras in any d

Finite dimensional subsuperalgebras: $d = 3, 4, 5$.

Conclusions

- Nonlinear HS theories exist

any d

- HS geometry

fuzzy hyperboloid

Other Topics

- Twistor realization:

$$Y_i^A \rightarrow \text{spinor oscillators } y_\alpha$$

M.V. (1988)

$$AdS_4, AdS_5, N=4 \quad SYM$$

Sezgin, Sundell
Alkalaev, M.V.

- HS fields: superfields in space-time with matrix

(central charge) coordinates

Fronsdal (1985)

Bandos, Lukierski, Sorokin hep-th/9904109

M.V. hep-th/0106149

Nonlinear nonlocal HS currents

Gelfond, M.V. hep-th/0304020

- Applications of Unfolded Dynamics

Classification of all (free) conformal equations

Tipunin, Shaynkman, M.V. hep-th/0401086

To Do

- Free and interacting mixed symmetry HS gauge fields
in AdS_d
- Nonlinear HS dynamics at the action level
- $sp(2M)$ covariant formulation of nonlinear HS theories

- Solutions of nonlinear HS field equations:

to break down HS gauge symmetries =

to introduce a massive parameter

$$m^2 \neq \Lambda \qquad \Lambda^{-\frac{1}{2}} D \sim 1$$

Low energy expansion in $\left(\frac{1}{m} \frac{\partial}{\partial x}\right)^p$

- To understand HS symmetries in String Theory?!