GAUGE THEORY WITH SU(N) GAUGE GROUP

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STRING THEORY WITH 1/N AS THE STRING COUPLING CONSTANT?

AN OLD GOAL, AND APPARENTLY TRUE...

FROM THE AdS/CFT CORRESPONDENCE

WE KNOW THAT GAUGE/STRING

DUALITY HOLDS FOR

$\mathcal{N}=4$ SUPER YANG-MILLS

ALL KINDS OF ORBIFOLDS THEREOF

AND MULTITUDES OF THEORIES

OBTAINED BY RELEVANT

PERTURBATIONS AND R.G. FLOWS

MORALLY SPEAKING, FOR ALL

THEORIES OF MATRIX-VALUED FIELDS

IN $D \leq 4$

BUT AT PRESENT, IT IS ONLY A USEFUL COMPUTATIONAL TOOL FOR LARGE g^2N, WHERE WE CAN USE SUPERGRAVITY.

THAT ISN'T ENOUGH IF WE WISH TO UNDERSTAND THE MYSTERIES OF QCD, SUCH AS QUARK CONFINEMENT.

QCD IS ASYMPTOTICALLY FREE SO $g^2N \ll 1$ AT SHORT DISTANCES.

A STRING THEORY DUAL TO QCD MUST LOOK AT SHORT DISTANCES LIKE THE STRING DUAL OF A WEAKLY COUPLED GAUGE THEORY.

BUT WHAT IS THAT?

... I'LL DESCRIBE A WEAK COUPLING GAUGE/STRING DUAL, BUT I DON'T KNOW IF IT IS THE ONE WE NEED.

FOR A STRING DUAL OF $\mathcal{N}=4$ SUPER YANG-MILLS, WE WANT A TARGET SPACE WITH PSU(4|4) SYMMETRY. THE SYMMETRY OF THE $\mathcal{N}=4$ THEORY.

THE AdS/CFT STORY IS BASED ON

$$PSU(4/4)/SO(5)\times SO(5)$$

WHOSE BOSONIC PART IS

$$AdS_5 \times S^5$$

I'LL CONSIDER INSTEAD A STRING THEORY WITH TARGET $CP^{3/4}$, A HOMOGENEOUS SPACE WITH FOUR BOSE AND FOUR FERMI COORDINATES

$$Z^I = (X^\alpha, \psi^A)$$

$$\alpha, A = 1 \cdots 4$$

X^α NOT ALL ZERO AND

$$Z^I \simeq \lambda Z^I, \quad \lambda \neq 0$$
$$\lambda \in \mathbb{C}^*$$

PRECISELY BECAUSE THE NUMBER OF FERMIONS EQUALS THE NUMBER OF BOSONS, $\mathbb{CP}^{3|4}$ IS A CALABI-YAU SUPERMANIFOLD

VOLUME FORM

$$\Omega_0 = \epsilon_{\alpha_1 \ldots \alpha_4} dX^{\alpha_1} \ldots dX^{\alpha_4} \; \epsilon_{A_1 \ldots A_4} d\psi^{A_1} \ldots d\psi^{A_4}$$

ON $\mathbb{C}^{4|4}$ IS $\mathbb{C}^*$-INVARIANT

AND SO DESCENDS TO CALABI-YAU VOLUME FORM ON $\mathbb{CP}^{3|4}$

WE KNOW VARIOUS REASONS

AROUND THAT $\mathcal{N}=4$ SUPER-YANG-MILLS

IS SPECIAL.

THE β FUNCTION VANISHES

PRECISELY FOR $\mathcal{N}=4$ ...

AND $\mathbb{CP}^{3|\mathcal{N}}$ IS CALABI-YAU

PRECISELY FOR $\mathcal{N}=4$ ...

BEING A CALABI-YAU MANIFOLD, $\mathbb{CP}^{3/4}$ HAS A "TOPOLOGICAL B-MODEL."

ONE STARTS WITH THE 2-DIM'L $N=(2,2)$ SUPERSYMMETIC σ MODEL WITH TARGET $\mathbb{CP}^{3/4}$

$$\mathcal{L} = \int d^2x \left[\partial_\alpha Z^I \partial^\alpha \bar{Z}_I + \text{fermions} \right]$$

AND MAKES A "TOPOLOGICAL TWIST"

THE TWIST MEANS WE REINTERPRET ONE OF THE SUPERCHARGES, Q, WHICH HAS $Q^2 \equiv 0$, AS A "BRST OPERATOR"....

SPACE OF PHYSICAL STATES

= COHOMOLOGY OF Q

I WANT TO DO THE

B-MODEL FOR OPEN

STRINGS ... ASSOCIATED WITH

$\mathbb{CP}^{3|4}$-FILLING D-BRANES.

LET US FIRST REMEMBER

THE TOPOLOGICAL B-MODEL

OF AN ORDINARY (BOSONIC)

CALABI-YAU^3-MANIFOLD W

THE BASIC OPEN STRING EXCITATION IS A "GAUGE FIELD" $A_{\bar{i}}(X, \bar{X})$ {WITH VALUES IN $U(N)$ FOR N D-BRANES... AND ACTION

$$\int dX^{\bar{i}} dX^{\bar{j}} dX^{\bar{k}} \, \mathrm{Tr}\left(A_{\bar{i}} \partial_{\bar{j}} A_{\bar{k}} + \tfrac{2}{3} A_{\bar{i}} A_{\bar{j}} A_{\bar{k}}\right) \wedge \Omega$$

FOR $\mathbb{CP}^{3|4}$, IT IS MUCH THE SAME, EXCEPT THE GAUGE FIELD $a_{\bar{z}}(x, \bar{x}, \psi)$ DEPENDS ON FERMIONIC COORDINATES ψ^A AS WELL AS BOSONIC COORDINATES $x, \bar{x}$

$$a_{\bar{z}}(x, \bar{x}, \psi) = A_{\bar{z}}(x, \bar{x}) + \psi^A \chi_{\bar{z}A}(x, \bar{x}) + \psi^A \psi^B \phi_{\bar{z}AB}(x, \bar{x}) + \dots$$

THE ACTION IS STILL

A CHERN-SIMONS (0,3)-FORM

$$I = \int d\bar{X}^{\bar{\imath}}\, d\bar{X}^{\bar{\jmath}}\, d\bar{X}^{\bar{k}}$$

$$\mathrm{Tr}\left(a_{\bar{\imath}}\, \partial_{\bar{\jmath}} a_{\bar{k}} + \tfrac{2}{3} a_{\bar{\imath}} a_{\bar{\jmath}} a_{\bar{k}}\right)$$

$$\wedge\ \Omega$$

WHERE NOW, ROUGHLY,

$$\Omega = d^3X\, d^4\psi$$

THIS ACTION HAS THE AMAZING PROPERTY THAT - ALTHOUGH DEFINED ON THE SPACE $\mathbb{CP}^{3|4}$ OF THE WRONG DIMENSION - IT HAS THE SAME SPECTRUM AS $\mathcal{N}=4$ SUPER YANG-MILLS ON MINKOWSKI SPACE

... AND SOME OF THE SAME INTERACTIONS

HELICITIES:

$$\mathcal{A} = A + \psi\chi + \psi\psi\phi + \psi^3\tilde{\chi} + \psi^4 C$$

$$\begin{array}{ccccc} | & | & | & | & | \\ 1 & \tfrac{1}{2} & 0 & -\tfrac{1}{2} & -1 \end{array}$$

THIS STATEMENT IS EQUIVALENT TO THE "PENROSE TRANSFORM" FROM WAVE EQUATIONS IN MINKOWSKI SPACE TO $\bar{\partial}$ OR SHEAF COHOMOLOGY IN $\mathbb{CP}^{3|4}$

HOWEVER, WE DON'T GET THE INTERACTIONS OF $\mathcal{N}=4$ SYM ...

ROUGHLY SPEAKING, WE CAN'T, SINCE WE HAVE AN EXTRA $U(1)_R$ SYMMETRY...

$\mathcal{N}=4$ SYM HAS $SO(4)_R$

BUT $\mathbb{CP}^{3|4}$ HAS $U(4)_R$ ACTING ON ψ^A, $A=1\ldots 4$

THE EXTRA U(1)

$$Z^I = (x^\alpha, \psi^A)$$
$$S=0 \quad S=1$$

DOES NOT LEAVE INVARIANT

THE VOLUME-FORM

$$\Omega = d^3x\, d^4\psi$$

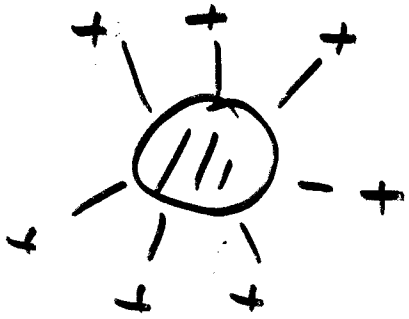
SO IT IS <u>ANOMALOUS</u>

IN THE B-MODEL

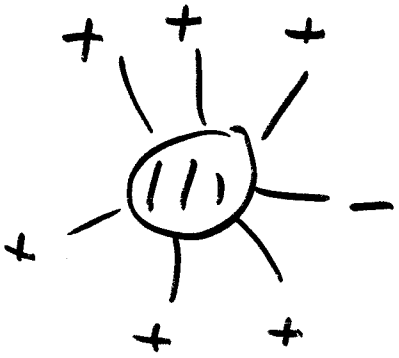
THE $\mathcal{N}=4$ SYM ACTION IS A SUM OF TERMS WITH DIFFERENT VALUES OF THE S-CHARGE.

THE TERMS OF $S=-4$ (IN SOME CONVENTION I WON'T EXPLAIN) COME FROM THE CHERN-SIMONS (0,3)-FORM, AND THE REST FROM A D-INSTANTON CORRECTION

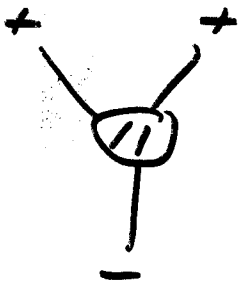
TREE-LEVEL YANG-MILLS SCATTERING AMPLITUDES ARE REPRODUCED AS FOLLOWS:



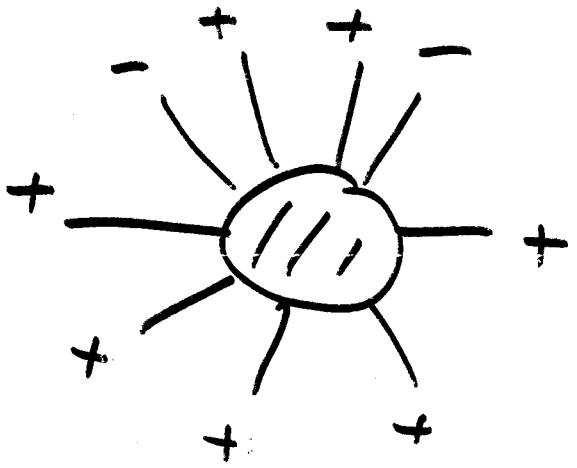
$= 0$, A STANDARD RESULT de Witt, ...



$= 0$ EXCEPT



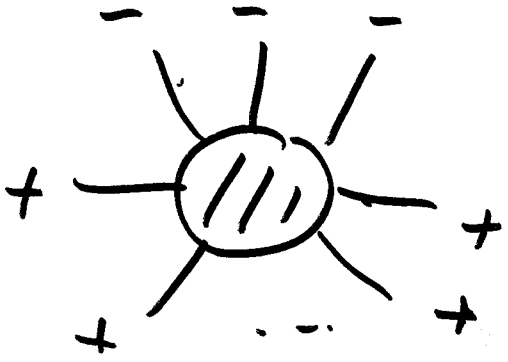
FROM THE CHERN-SIMONS $(0,3)$-FORM



= Parke-Taylor MHV amplitude

arises as a

one-instanton effect



two instantons

$(g^+ + + \ldots + \underbrace{- - \ldots -}_{q})$

p

FROM $g-1$ INSTANTONS

THE APPROACH VIA INSTANTONS IN $\mathbb{CP}^{3|4}$ HAS REPRODUCED AND ILLUMINATED STANDARD YM TREE AMPLITUDES

Roiban, Spradlin, Volovich

Cachazo, Svrcek, EW

HOWEVER NO CALCULATIONS YET FOR LOOPS

BEAUTIFUL RESULTS TO AIM FOR

-DIXON'S TALK- BERN, DIXON, KOSOWER ....

THE LOOPS, IN ANY EVENT, WILL NOT DESCRIBE PURE SYM BECAUSE CLOSED STRING MODES WILL COME IN...

THE CLOSED STRING MODES DESCRIBE $\mathcal{N}=4$ CONFORMAL SUPERGRAVITY (BERKOVITS & EW)

I SEE NUMEROUS PROBLEMS,

AMONG THEM

* LEARN TO CALCULATE THE LOOPS

** FIND A VERSION OF TWISTOR STRING THEORY THAT DESCRIBES PURE SUPER YANG-MILLS WITHOUT THE CONFORMAL SUPERGRAVITY

WE NEED TO SOLVE THE SECOND PROBLEM IF WE REALLY WANT TO ACHIEVE THE ORIGINAL GOAL:

FIND A STRING THEORY DUAL TO WEAKLY COUPLED STRING THEORY GAUGE