AdS/CFT description of D-particle decay

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Unstable D-branes

- Sen's time-dependent boundary state for D-brane decay $|B(t)\rangle \rightarrow$
  Features of D-brane as *source* for closed strings:
  - $$\begin{cases} T_{00} & \text{constant in time,} \\ T_{ij} \xrightarrow{t\,\to\,\infty} 0 & \text{in world volume directions,} \\ T_{ab}, T_{0a}, T_{0i} & \text{identically zero in other directions} \end{cases}$$
  - dilaton charge $\xrightarrow{t\,\to\,\infty}$ 0

D-brane decays without remnants
to pressureless dust

- Couple $|B\rangle$ to closed strings
  → spectrum of radiated closed strings

Lambert
Liu
Maldacena

$$(Q+\bar{Q})\,|\Psi_c\rangle = |B\rangle \longrightarrow |\Psi_c\rangle \xrightarrow{t\,\to\,\infty} \text{outgoing radiation}$$

features of the spectrum:

  - amplitudes exponentially suppressed as level grows,
    but number of available states at each level grows even faster,
    → most energy radiated into very massive, non-relativistic strings
  - total average $\overline{E}$, $\overline{N}$ diverge (naively due to absence of back-reaction; but essentially because $|B\rangle$ is "position" eigenstate)

- Unstable D-branes are sphalerons in string theory configuration space.

Harvey
Hořava
Kraus

D0 in $AdS_5 \times S^5$ ⟷ sphaleron in dual SYM.

Drukker
Gross
Itzhaki

Questions:

- ★ “Can we mimic the analysis of Sen, Lambert et al. in the dual gauge theory?”

- ★ “Which features of the flat space radiation survive?”

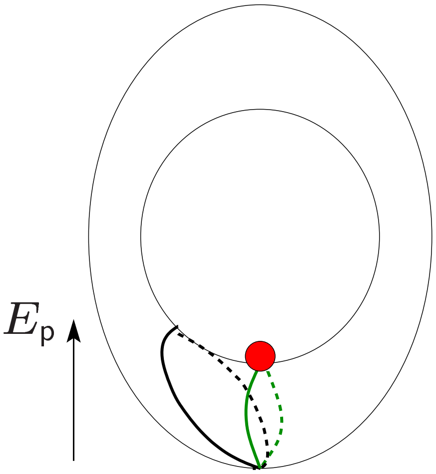
(N.B. Similar questions asked in the context of the 2d string theory ↔ matrix model duality.)

McGreevy
& Verlinde

D0 $\longleftrightarrow$ sphaleron correspondence: STATICS

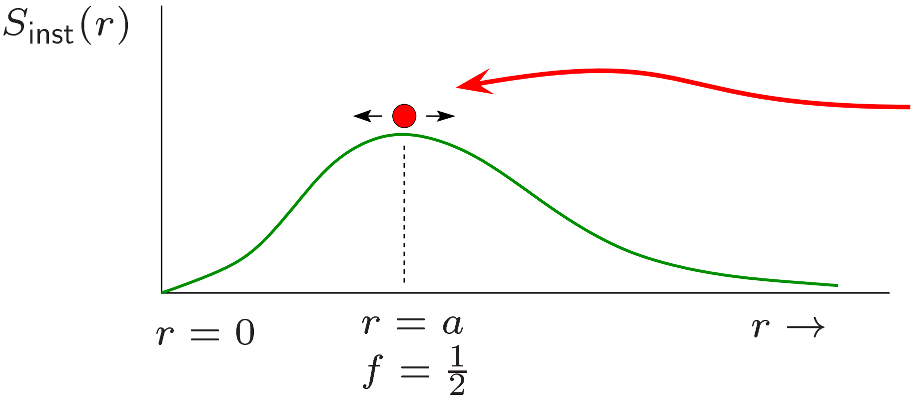
The gauge theory

- Sphalerons: static solutions, associated to saddle points whose existence is guaranteed by existence of *non-contractible* loops in (compact) configuration space. Manton
- SU(2) sphaleron: start from instanton



$$A_\mu = f(r)\partial_\mu U U^\dagger\,, \quad U = \frac{x^\mu \sigma_\mu}{r}$$

$$f(r) = \frac{r^2}{r^2+a^2}\,, \quad r^2 = x_0^2 + x_i^2\,.$$



$\frac{1}{2}\partial_\mu U U^\dagger$ is a solution but *singular*! ($S(r)\sim r^{-1}$)

$$\begin{tikzcd}
R^4 \arrow[r, "\text{conformal trans.}"] \arrow[d] & S^3\times R \arrow[d] & \\
\mathrm{d}s^2 = \mathrm{d}r^2 + r^2\mathrm{d}\Omega_3^2 & \mathrm{d}s^2 = \mathrm{d}t^2 + R^2\mathrm{d}\Omega_3^2 & (r = e^{t/R})
\end{tikzcd}$$

- solution unchanged (due to conformal invariance)
- action density becomes constant $S = \frac{3\pi^2}{g_{YM}^2 R}\int_{-\infty}^{+\infty}\mathrm{d}t$
- can Lorentz rotate
  ($A_t = 0$, A_θ independent of time) → sphaleronic particle (regular)

D0 ⟷ sphaleron correspondence: STATICS

- SU(N) sphalerons: are associated to non-contractible spheres of various dimensions.
  - generic solution: hard
  - special solution: dual to k D-particles on top of each other,
  - ansatz: tensor product of SU(2) sphalerons,

$$\sigma_\mu \to \gamma_\mu = \begin{pmatrix} \sigma_\mu & 0 & \cdots & 0 \\ 0 & \sigma_\mu & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \cdots & \sigma_\mu \end{pmatrix} .$$

  - mass $M_k = kM_{D0}$,
  - number of unstable modes: $k^2 = 1 + 3 + 5 \ldots + (2k - 1)$

D0 ⟷ sphaleron correspondence: STATICS

The AdS side

- AdS/CFT strong-weak in λ → generic state subject to large quantum corrections
- Sphaleron non-SUSY but special ↔ non-contractible loop ↔ instanton which is susy (exists on both sides)
- Conjectured duality

unstable DO at origin of AdS ↔ YM sphaleron on $S^3 \times R$:

static in global time ↔ static YM configuration

at origin of AdS ↔ homogeneous YM configuration

$$\text{sources} \left.\begin{array}{r} g_{\mu\nu} \neq 0 \\ \phi \neq 0 \\ C = 0 \end{array}\right\} \leftrightarrow \left\{\begin{array}{r} \langle T_{\mu\nu} \rangle \neq 0 \\ \langle \mathrm{Tr}(F^2) \rangle \neq 0 \\ \langle \mathrm{Tr}(F\tilde{F}) \rangle = 0 \end{array}\right.$$

number of unstable modes on both sides agrees

mass is renormalised:

$$M_{D0} = \frac{c\lambda^{1/4}}{g_{YM}^2 R} \leftrightarrow \frac{c'}{g_{YM}^2 R}$$

N.B. All string results are *flat space* results "embedded" in an AdS background of large radius.

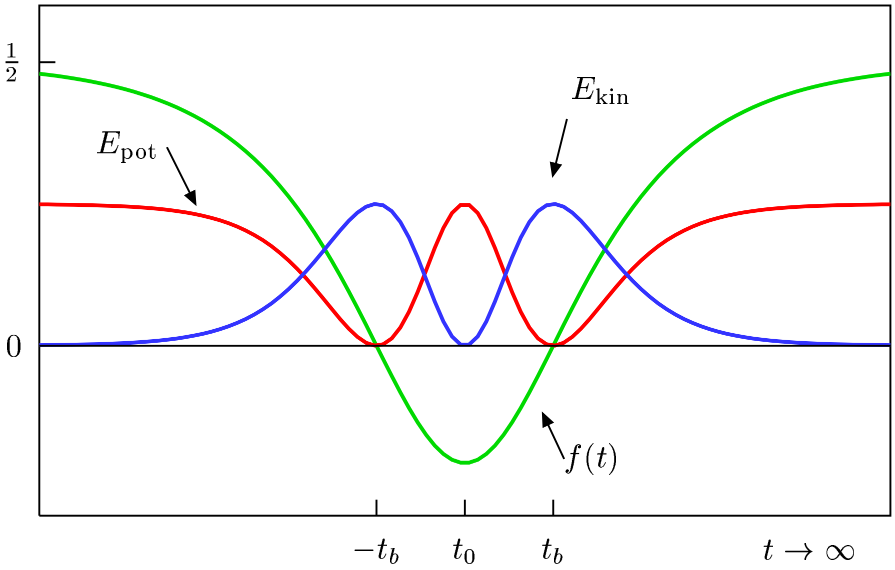
D0–sphaleron correspondence: DYNAMICS

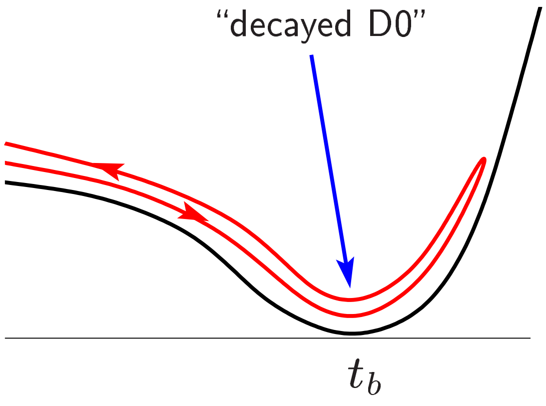
Time-dependent solution

- ansatz which preserves spatial homogeneity of sphaleron

$$A = f(t)\,\sigma^i\Sigma_i \qquad \begin{cases} T_{00} & = -\frac{3}{2}\dot{f}^2 - 6f^2(1-f)^2 \\ T_{ij} & = -\frac{1}{3}T_{00}g_{ij}\,. \end{cases}$$

$$f(t) = \frac{1}{2}\left(-\frac{\sqrt{2}}{\cosh\left(\frac{\sqrt{2}}{R}(t-t_0)\right)} + 1\right)\,.$$





Note: whole process is periodic in AdS:

D0 decays → cloud expands → recollapse to rebuild D0.

- generalisation to k D-particles as before ⟶ look at “synchronised decay”

D0–sphaleron correspondence: DYNAMICS

Constructing the coherent state

- Want a quantum state dual to the decay product.
- Key observation: near the bottom A_μ satisfies free ($g_{\text{YM}} = 0$) e.o.m.

$$A_\mu = f(t)\, U^\dagger \partial_\mu U \,, \qquad \lim_{t \to \text{bottom}} f(t) = 0 \,, \qquad \dot{A} \gg [A, A] \,.$$

  Natural to associate decayed brane with state near the bottom.

- For $A_\mu\Big|_{\text{bottom}}$ use standard machinery of coherent states:
  - Expand A_μ in vector spherical harmonics,
  - Construct

classical values

$$|c\rangle = \mathcal{C} \, \exp \left(g_{\text{YM}}^{-2} \sum_{J,M,y} \text{Tr} \left(A_{JMy} \, \hat{a}^\dagger_{JMy} \right) \right) |0\rangle \,,$$

normalisation

creation operators

- Properties:
  - $\hat{A}_\mu |c\rangle = A_\mu^{+\,\text{classical}} |c\rangle$
  - Contains non-singlets; OK in free theory, but not if $g_{\text{YM}} \neq 0$ (cannot put gluon on sphere) $\longrightarrow$ $|c_{\text{singlet}}\rangle = \mathcal{P}_{\text{singlet}} |c\rangle$
  - $|c\rangle$ built only from vector s-wave on sphere ($J = \frac{1}{2}$; $m, m' = -1, 0, 1$; $y = \frac{1}{2}$).

GOAL: Given $|c\rangle$ $\longrightarrow$ count number of various *bulk* “particles” (states)

Particles and the operator ⟷ state map

- Isomorphism between Hilbert spaces:

  bulk string states ⟷ boundary states ⟷ operators at origin of R^4

- Usually work in position space; heuristically $\hat{O}(\vec{x})$ creates a "particle" ("wave packet") at position $\vec{x}$ at boundary.

- In order to match with string calculation, it's more natural to talk about particles as (angular) momentum eigenstates.

- Sphaleron singular on R^4 ⟶ construct particles (states) on $S^3 \times R$, by mimicking operator–state on R^4: start with $\hat{O}_w^{\mu 1 \cdots \mu_s}(\tau, \phi_i)$,

$$|\hat{O}_w^{(m)}\rangle = \lim_{\tau\to-\infty} \Big\{ e^{-w\tau} \hat{O}_w^{(m)}(\tau) \Big\} |0\rangle \equiv \hat{O}_w^{(m)} |0\rangle \,.$$

$$\hat{O}_w^{(m)}(\tau) = K_w^{(m)} \int_{S^3} \mathrm{d}\Omega \; \hat{O}_w^{\mu 1 \cdots \mu_s}(\tau, \phi_i)\; \underset{\substack{\uparrow \\ \text{lowest spherical harmonics}}}{Y^{(m)}_{\mu_1 \cdots \mu_s}}(\phi_i) \,.$$

  - Number of states agrees with number of states of dimension w, as computed using Polya counting,
  - Can construct orthonormal multi-particle basis.

- Why don't we construct an *alternative* coherent state using $\hat{O}_w^{(m)\dagger}$?

$$|\tilde{c}\rangle = \tilde{\mathcal{C}} e^{\sum_i O_i^{\text{class.}} \hat{O}_i^\dagger} |0\rangle \longrightarrow \text{problem:} \quad \frac{\langle 0|\hat{O}|\tilde{c}\rangle}{\langle 0|\tilde{c}\rangle} \neq O^{(+)}_{\text{classical}}$$

$$\boxed{\text{Why?}}$$

Particle counting

- Naive number operator $\hat{O}_w^\dagger \hat{O}_w$ not good !

can be large

$$[\hat{O},\hat{O}^\dagger] = 1 + \sum_i \frac{\hat{O}_i}{N^{2i}} \quad \Rightarrow \quad \langle n|\hat{O}^\dagger \hat{O}|n\rangle = n + \sum_i \frac{c_i(n)}{N^{2i}}$$

Even when $N \to \infty$, corrections important!

- Calculate probabilities using *orthonormal* basis,

①

$$\mathcal{P} := \frac{\left| \left\langle (\hat{O}_{J_1})^{p_1} \ldots (\hat{O}_{J_M})^{p_M} \middle| c \right\rangle \right|^2}{\langle c|c\rangle \left\langle (\hat{O}_{J_1})^{p_1} \ldots (\hat{O}_{J_M})^{p_M} \middle| (\hat{O}_{J_M})^{p_M} \ldots (\hat{O}_{J_1})^{p_1} \right\rangle}$$

②

$$\bar{N}(J_i) = \sum_{p_1} \cdots \sum_{p_M} p_i \, \mathcal{P}(p_1 \ldots p_i \ldots)$$

$$\bar{E}(J_i) = \sum_{p_1} \cdots \sum_{p_M} w_i \, p_i \, \mathcal{P}(p_1 \ldots p_i \ldots)$$

① → classical expectation values $\langle 0|\hat{O}|c\rangle = \langle 0|c\rangle \, O^{(+)}_{\text{classical}}$.

- Just knowing ① → part of spectrum constrained by symmetry:
  - $\left\langle \hat{F}^2 \middle| c \right\rangle \neq 0$ have dilaton radiation,
    $\left\langle \hat{T}_{\mu\nu} \middle| c \right\rangle = 0$ no gravitational radiation,
    $\left\langle \hat{O}^{\text{NS-NS}}_{\mu\nu} \middle| c \right\rangle = 0$ no NS-NS two-form,
    $\left\langle F_{\nu(\mu_1} \nabla_{\mu_2} \ldots \nabla_{\mu_{s-1}} F^{\nu}_{\mu_s)} - \text{traces} \middle| c \right\rangle = 0$ no twist-two.
  - These agree with flat-space calculations.

What about dynamics of the decay?

Dynamical aspects of the decay

- ① classical expressions, but need tensorial spherical harmonics,

  → Look at simplified, toy version, which keeps the main features of the full $|c\rangle$: scalar coherent state

$$|c\rangle = \mathcal{C}\, e^{\frac{1}{g}\mathrm{Tr}(a_{\mathsf{classical}}\hat{a}^\dagger)}|0\rangle$$

  - Just one oscillator turned on; the one corresponding to a scalar s-wave.
  - Dependence on coupling of the classical amplitude, $a_{\mathsf{classical}}$ non-perturbative (as for the original $|c\rangle$).
  - The states turned on by $|c\rangle$ are

$$\hat{O}_J^\dagger = \frac{1}{\sqrt{J\lambda^J}}\,\mathrm{Tr}\left((a^\dagger)^J\right).$$

- ② hard even in this toy model (even at $g = 0$)

$$\begin{aligned}\mathcal{N}_{J,p} &= \langle \hat{O}_J^p \hat{O}_J^{\dagger\, p}\rangle \\ &= p!\langle \hat{O}_J \hat{O}_J^\dagger\rangle^p + \binom{p}{2}^2 \langle \hat{O}_J^2 \hat{O}_J^{\dagger\, 2}\rangle_{\mathsf{conn.}} \langle \hat{O}_J \hat{O}_J^\dagger\rangle^{(p-2)} + \ldots\end{aligned}$$

Have expansion in $1/N^2$, contributions from planar and non-planar graphs.

Question Which terms in $\mathcal{N}_{J,p}$ does one need to get OK results?

Answer It depends on the numerator, i.e. on the coherent state.

independent of coupling

$$\sim \frac{\eta}{(\lambda^J J)^p (\lambda^K K)^q}$$

grows with J

$$\overline{N}_{O_J} = \sum_{p,K,q} p \frac{\left| \left\langle \hat{O}_J^p \, \hat{O}_K^q \middle| c \right\rangle \right|^2}{\langle c|c \rangle \left\langle \hat{O}_J^p \hat{O}_K^q \,\middle|\, \hat{O}_J^p \hat{O}_K^q \right\rangle}$$

$$\mathcal{N}_{J,K,p,q} = p!q! \left(1 + \frac{b(p,q,J,K)}{N^2} + \ldots \right)$$

Become more and more important as p, q grow.

Larger numerator ⟶ max term in sum attained for larger p_{max}

⟶ corrections more important!

- Remarks:
  - Even with (estimated) total contribution from planar graphs

    ⟶ probability to find particle of type J comes out > 1!
  - Compare to the perturbative coherent state: numerator $\sim \frac{\text{const.}}{N^J J}$

    ⟶ truncation of norms make sense.
- Contribution from all terms in the norms important, even in the $N \to \infty$ limit.
- Compare BMN limit: $J^2 \sim N \to \infty$, contribution from all genera survive!

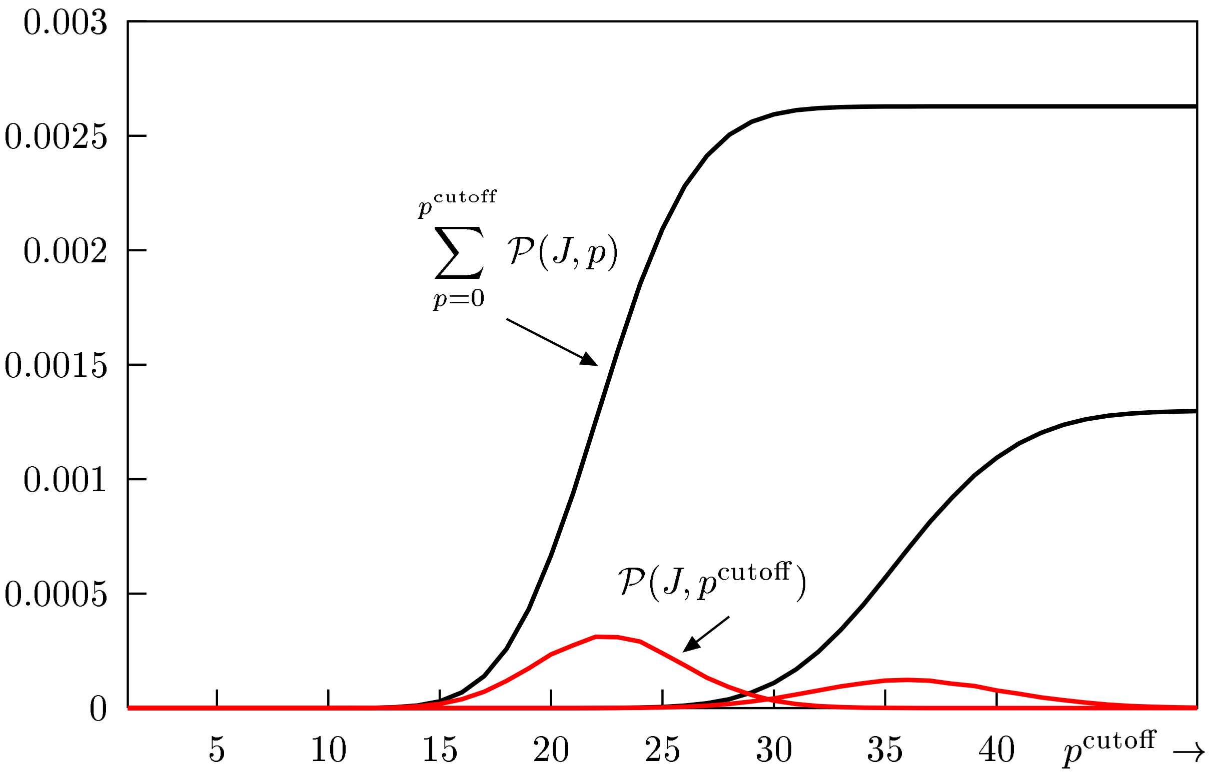
  Here: $p_{\text{max}} \to \infty$ and $N \to \infty$, so that corrections $b(p, J)$ become relevant!

Question Is there simplifying limit one could take?

Numerical results for U(4)

- At the moment, use Monte-Carlo integration to calculate norms $\mathcal{N}_{J,p}$:
  - Get full, planar and non-planar norms.
  - Limitations to low J and N $\longrightarrow$ $U(4)$.
  - Only operators turned on by $|c\rangle$:

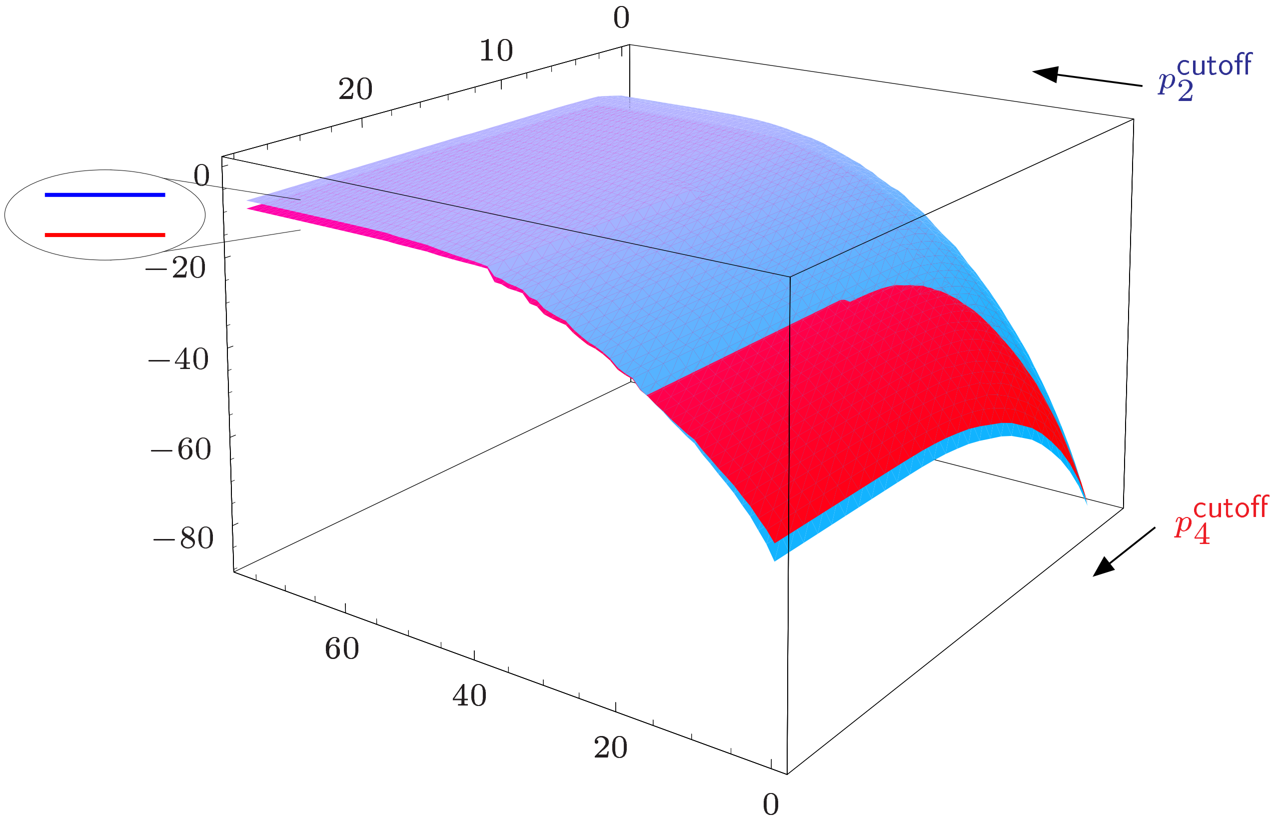
$$\hat{O}_2 = \mathrm{Tr}(a^2)\,, \quad \hat{O}_4 = \mathrm{Tr}(a^4) - \frac{2N^2+1}{N(N^2+1)}\,\mathrm{Tr}(a^2)\,\mathrm{Tr}(a^2)\,.$$



Probability distribution for O_2 particles for different values of λ.

Energy ratio for particles in U(4)

Logarithm of energy in O_2 and O_4 particles:



$E_4 < E_2$ as in gravity.

Summary:

- Analysed D-particle decay in AdS/CFT $\longrightarrow$ get qualitative agreement.
- Dealing with AdS/CFT particles in momentum space.
- Naive number operator $\hat{O}^\dagger \hat{O}$ does not always work (since one deals with composite particles).
- $N \to \infty$ $\longrightarrow$ not just planar subsector. (BMN case: large J; here: large number of particles).
- Setup for analysis of dynamic features, using quantum coherent states.

Outlook:

- Push to higher N $\longrightarrow$ exponential suppression? $\longrightarrow$ finite E_{total}?
- Understand $N \to \infty$, $p \to \infty$ limit systematically. Perhaps using matrix models.
- Quantum corrections, less symmetric cases, . . .