Brane Tilings, Dimers and Quiver Gauge Theories

Amihay Hanany

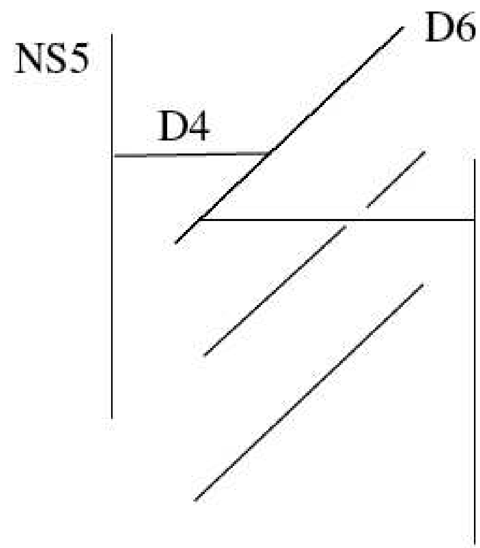
Strings 2005, Toronto

Thanks to: Benvenuti, Franco, Kazakopoulos, Kennaway, Martelli, Sparks, Uranga, Vegh, Wecht

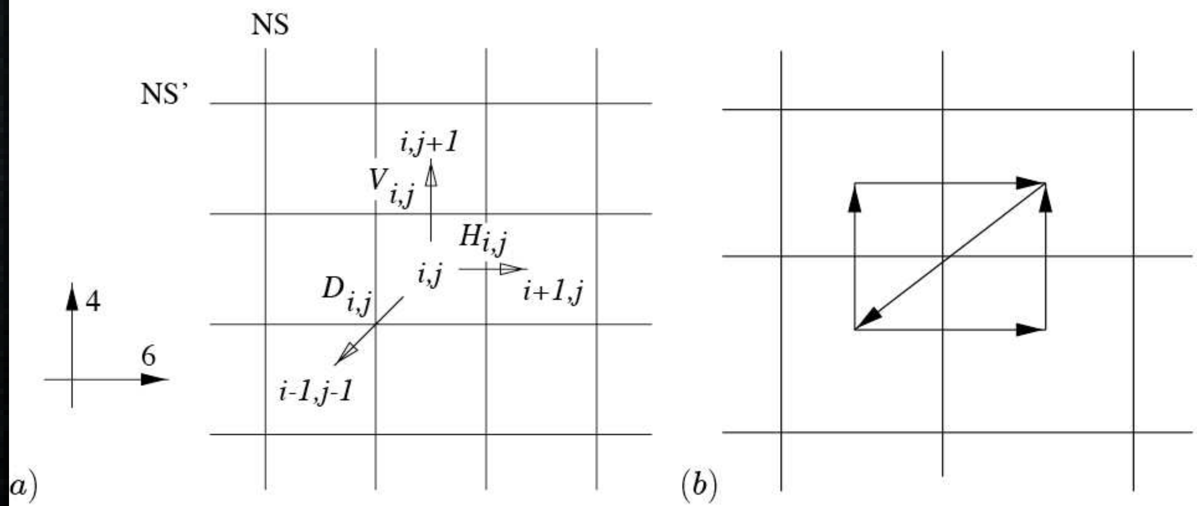
Gauge Theories in String Theory

- Gauge theories arise in String Theory in many varieties
- Pure geometry - Geometric Engineering
- D branes probing a singularity
- Intersecting D branes

Today: intersecting branes and their relation to branes on singular manifolds

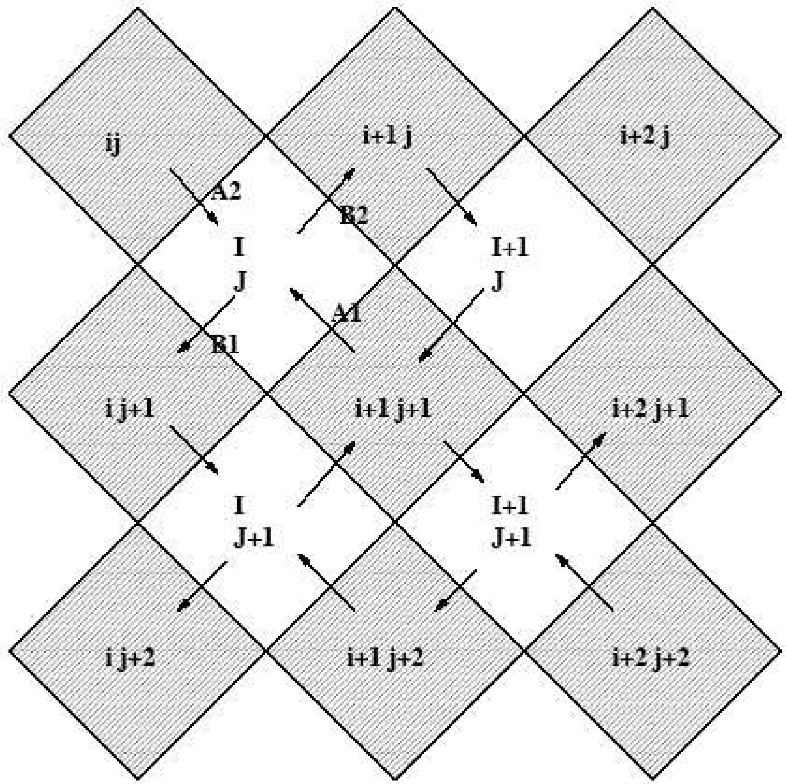


Recall: Brane Intervals



Brane Boxes

Hanany Zaffaroni

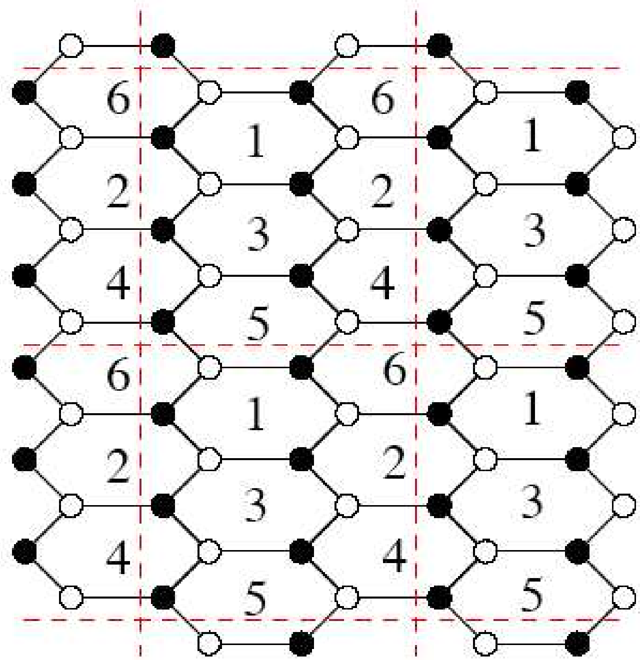


Brane Diamonds

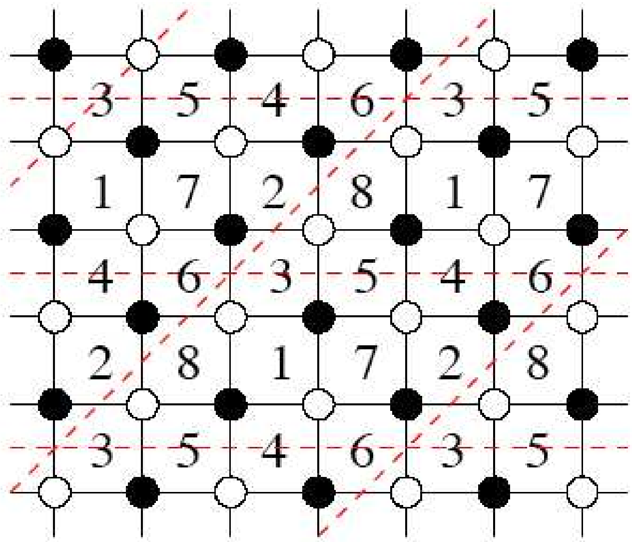
Aganagic, Karch, Lüst, Miemiec

A new concept: Brane Tilings

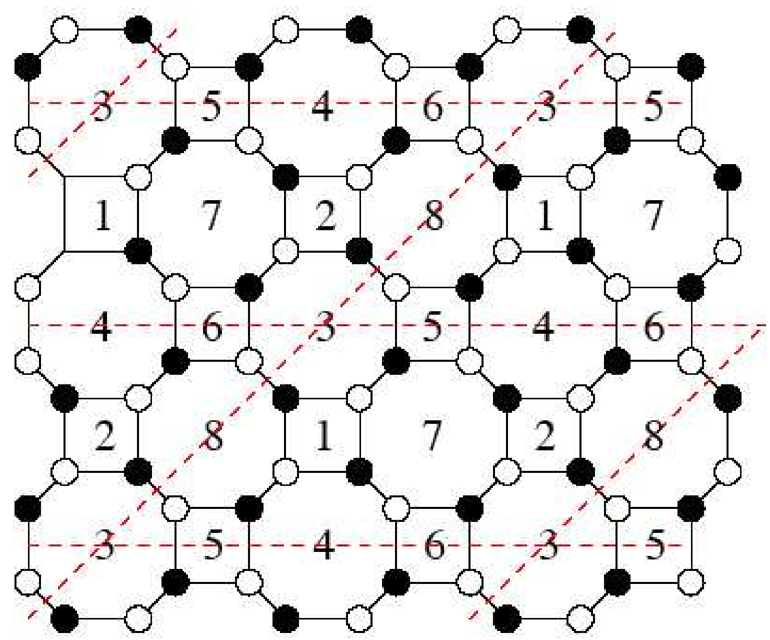
Periodic tilings of the two dimensional plane



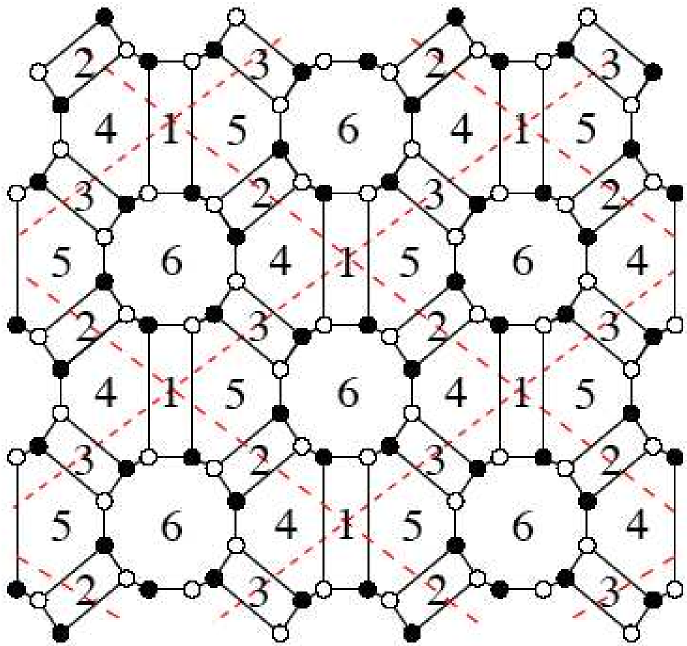
Hexagons



Squares



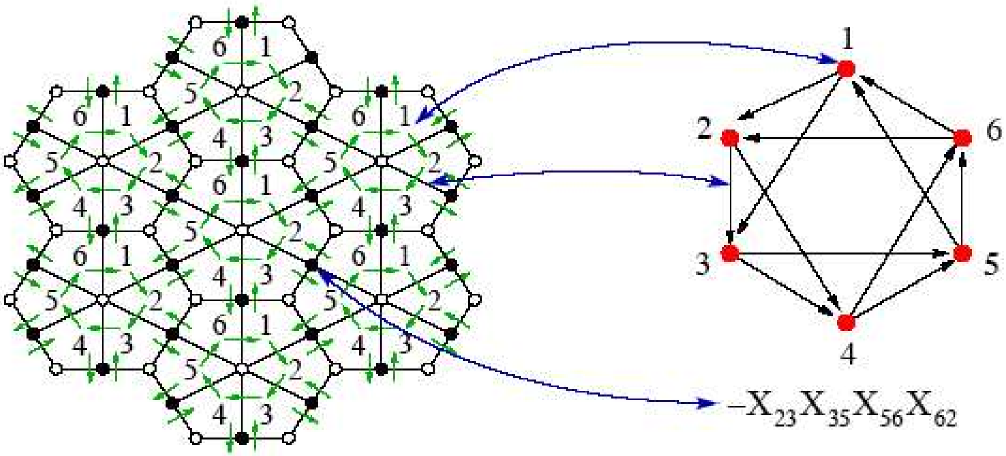
Octagons & Squares



Dodecagons, Hexagons & Squares

Tiling - Quiver dictionary

- 2n sided face - Gauge group with n flavors
- Edge - A bi-fundamental chiral multiplet charged under the two gauge groups corresponding to the faces it separates.
- k valent node - A k-th order interaction term in the superpotential



Example: Del Pezzo 3, Model I

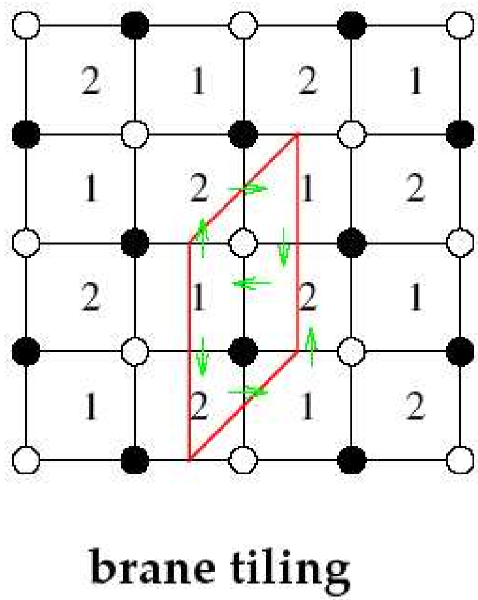
Comments

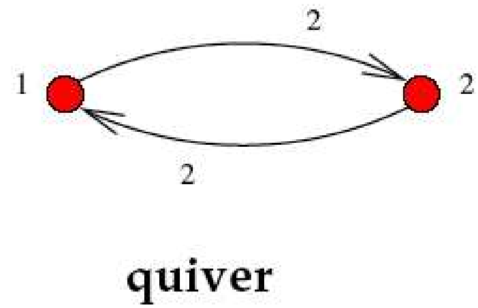
- Arrows are oriented in an alternating fashion
- Graph is bi-partite: Nodes alternate between clockwise (white) and counterclockwise (black) orientations of arrows
- black (white) nodes connected to white (black) only

More comments

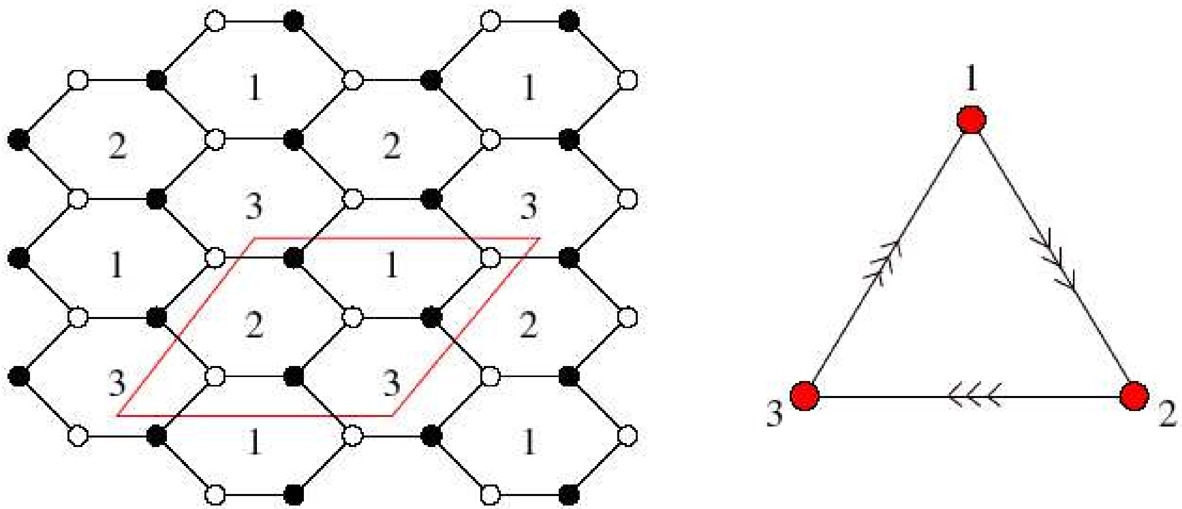
- odd sided faces are forbidden by anomaly cancellation condition
- white nodes with + sign in the superpotential
- black nodes with - sign in the superpotential
- These rules define a unique Lagrangian

$CY_6 = conifold$





Example: Conifold



Z_3 orbifold of C^3

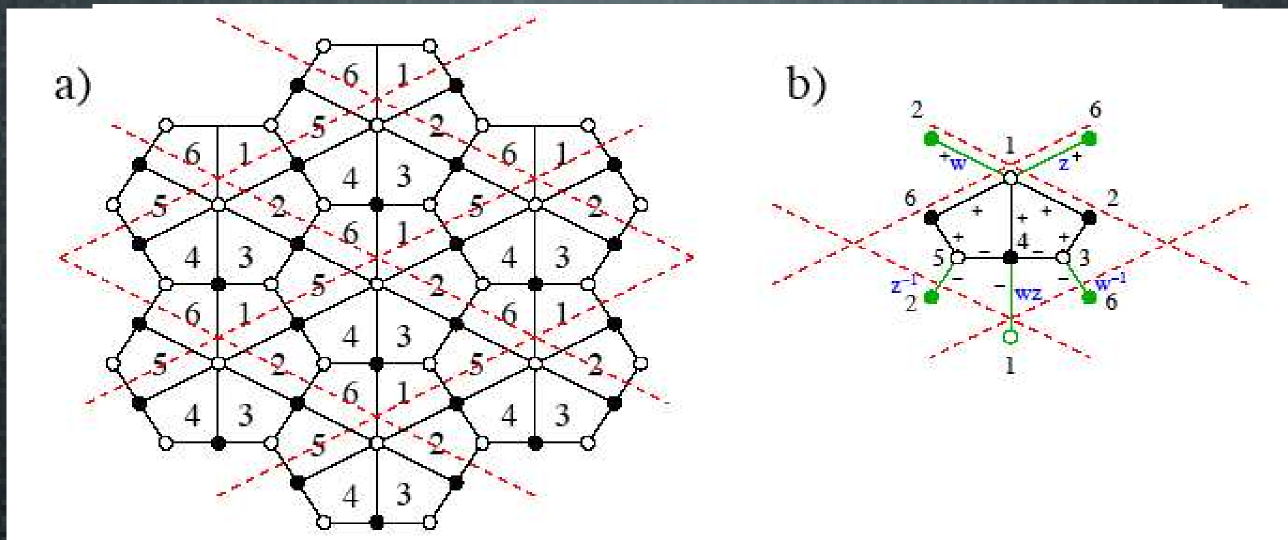
Brane tiling	**Gauge theory**
F: number of faces	N_g: number of gauge groups
E: number of edges	N_f: number of fields
N: number of nodes	N_W: number of superpotential terms

$$N_g + N_W - N_f = 0.$$

Euler's formula

Moduli Space of Vacua

- All quiver theories arising from periodic tilings have CY as their moduli space of vacua
- Computed using the Kasteleyn matrix:
- Adjacency matrix between white and black nodes



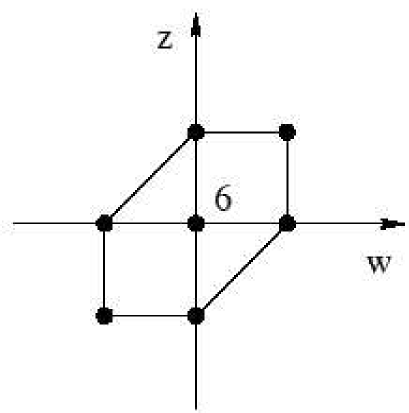
$$K = \left(\begin{array}{c|ccc} & 2 & 4 & 6 \\ \hline 1 & 1+w & 1-zw & 1+z \\ 3 & 1 & -1 & -w^{-1} \\ 5 & -z^{-1} & -1 & 1 \end{array}\right)$$

Kasteleyn matrix

R. Kenyon

$$P(z,w) = \det K$$

$$P(z,w) = w^{-1}z^{-1} - z^{-1} - w^{-1} - 6 - w - z + wz.$$



Toric diagram of dP3I

Bonus: multiplicities

- The coefficients of $P(z,w)$ are integers and are the multiplicities of the linear sigma model fields used to define the CY as a toric variety

Resolves a long standing problem: What is the gauge theory on a D3 brane at the tip of a CY cone

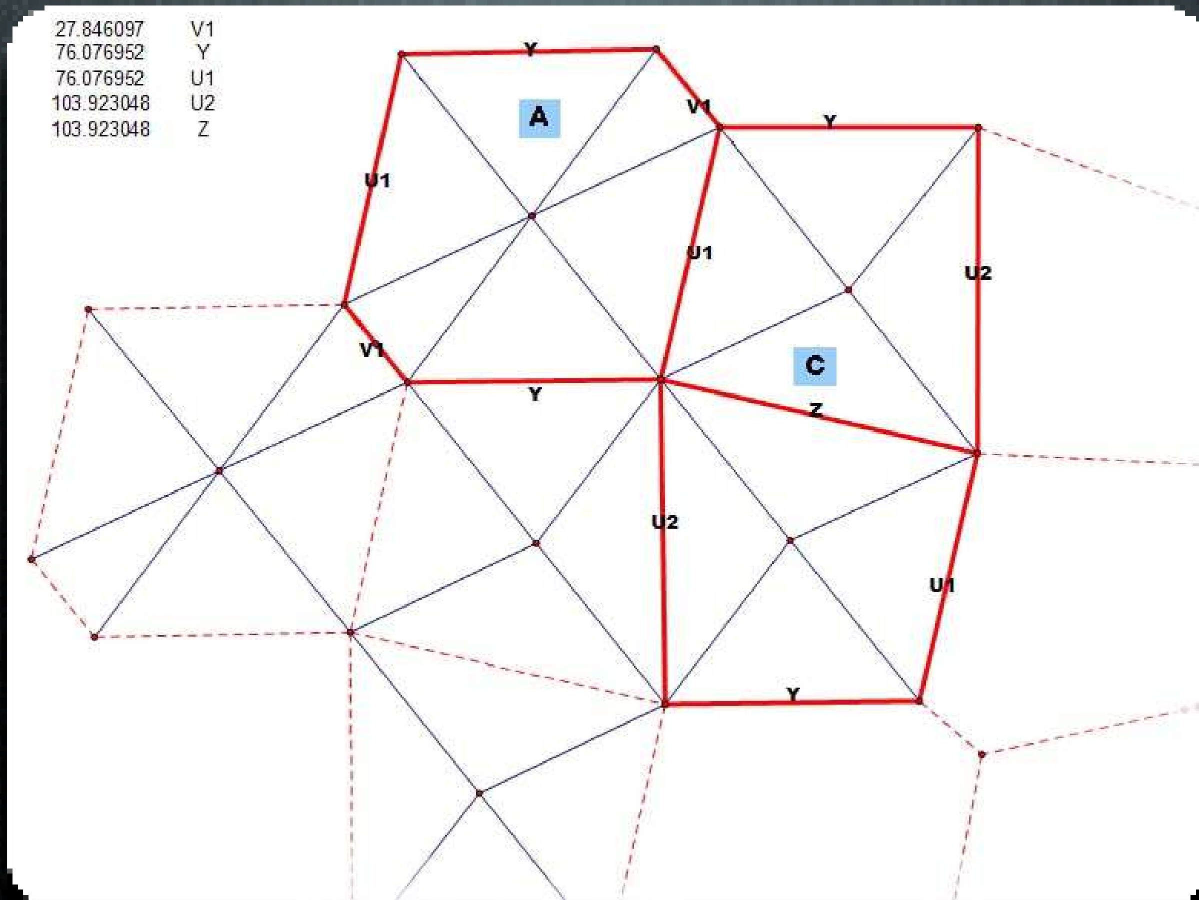
$$\sum_{i \in edges\ around\ node} R_i = 2 \qquad \text{for each node}$$

$$\sum_{i \in edges\ around\ face} (1 - R_i) = 2 \qquad \text{for each face}$$

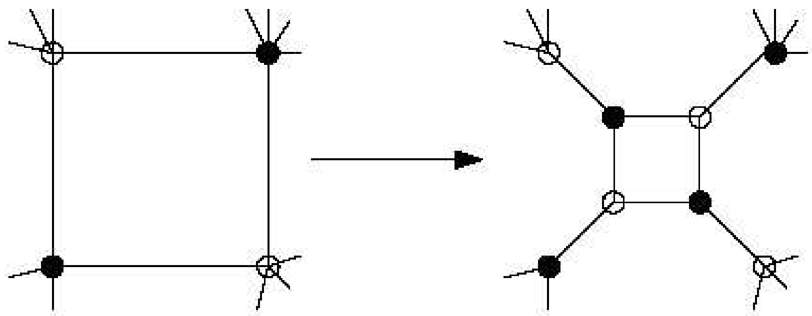
$$\sum_{i \in edges\ around\ node} (\pi R_i) = 2\pi \qquad \text{for each node}$$

$$\sum_{i \in edges\ around\ face} (\pi R_i) = (\#edges - 2)\pi \qquad \text{for each face}$$

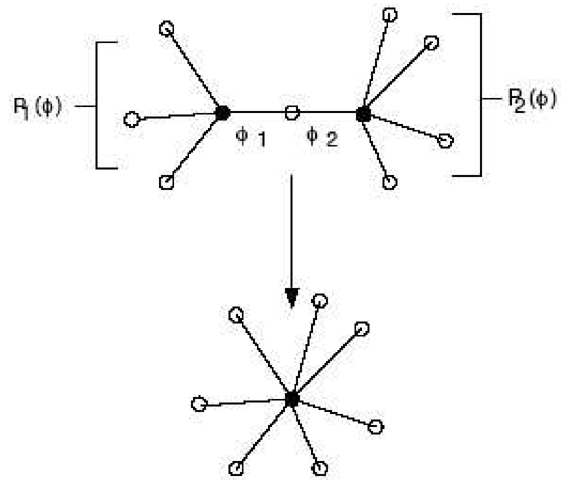
IR fixed point


27.846097 V1
76.076952 Y
76.076952 U1
103.923048 U2
103.923048 Z
A
C
Y
V1
U1
U2
Z

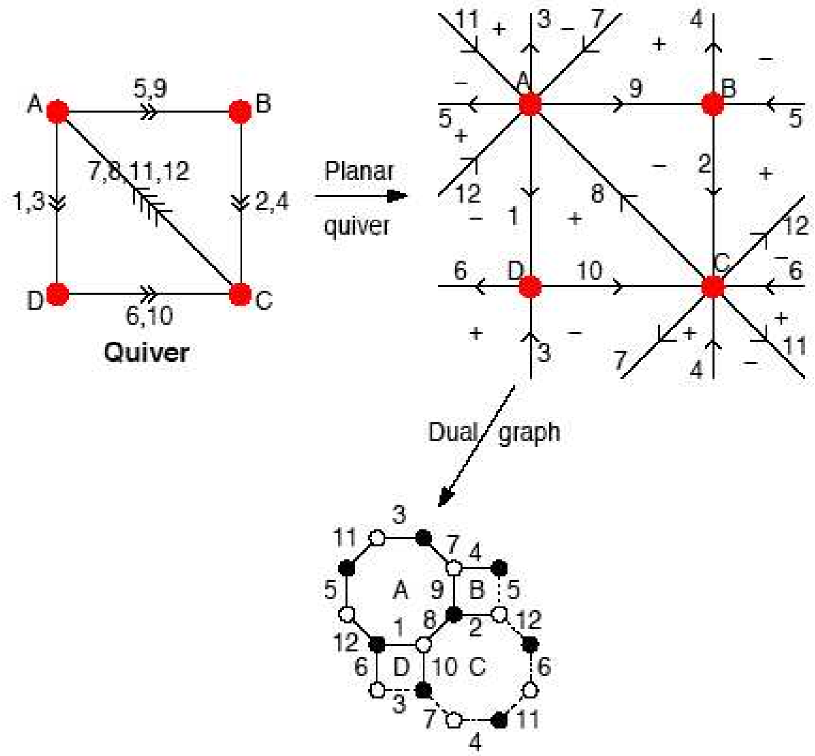
SPP tiling



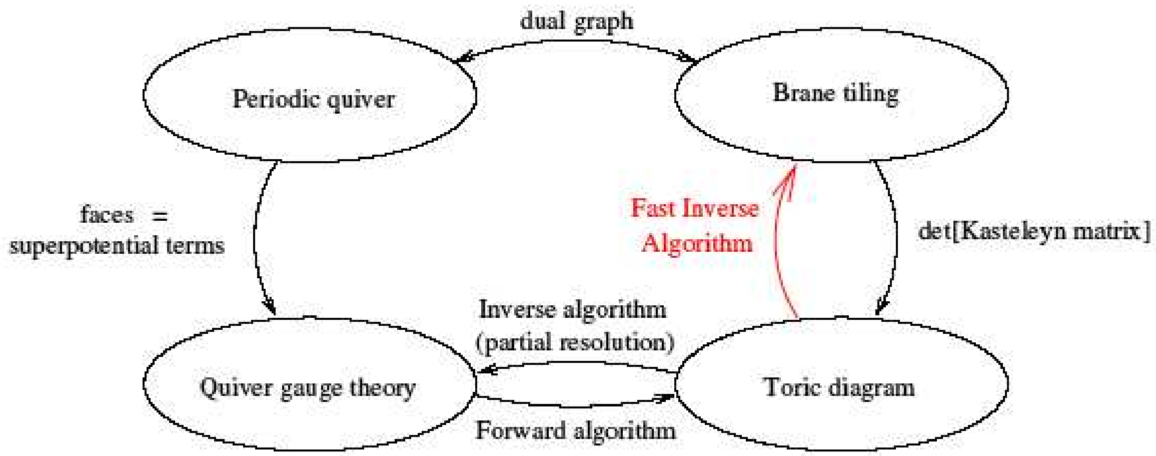
Seiberg Duality



Integrating out massive fields



Periodic Quiver



Logical flow chart

Dimers change the way we think of Quiver Gauge Theories

Conclusions

- Periodic tilings of 2d plane - N=1 SCFT's
- compute properties of quiver gauge theories using dimer techniques
- Solved a long standing problem - computing superpotetials for D3 brans probing singular CY's
- Construct infinite families of quiver gauge theories ($Y^{p,q}$ $L^{a,b,c}$ Martelli's talk)

Dimers

- New connections with combinatorics (domino tilings), condensed matter systems, derived categories, etc..