Free fermions and BPS geometries

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H. Lin and J. M. , to appear

Emergent space

- Matrix quantum mechanics $\longrightarrow$ free fermions (Brezin, Itzykson, Parisi, Zuber '78)

- Low energy 1+0 $\rightarrow$ 1+1 dimensions: ripples on the fermi sea

- Geometry of fermi sea $\sim$ space-time geometry

- Large N $\sim$ classical limit

Outline

- Example $\subset$ AdS/CFT as 1/2 BPS configurations

- Other theories with $R \times SO(4) \times SO(4)$ symmetry

- Solutions with $\widetilde{SU}(2|4)$ supergroup and associated theories

Free fermions from SYM

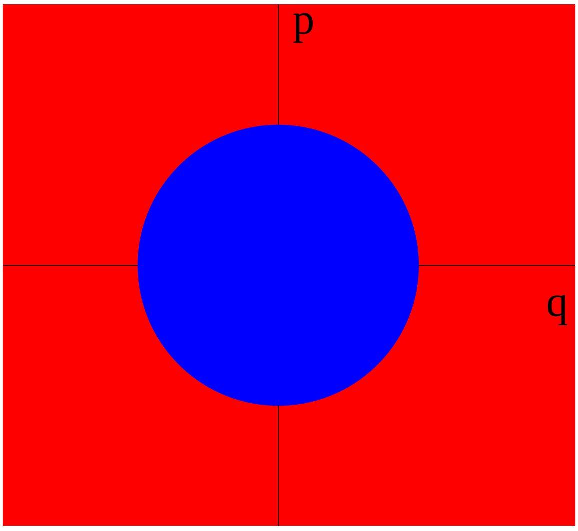
- Pick J in SO(6), rotates

$$Z = \phi_1 + i\phi_2$$

- Select states with $E = J$ or $\Delta = J$.

- Only the zero mode of the field Z survives. Theory on $R \times S^3 \to$ harmonic oscillator for Z.

Corley, Jevicki, Ramgoolam; D. Berenstein



Phase space

Gravity Solutions

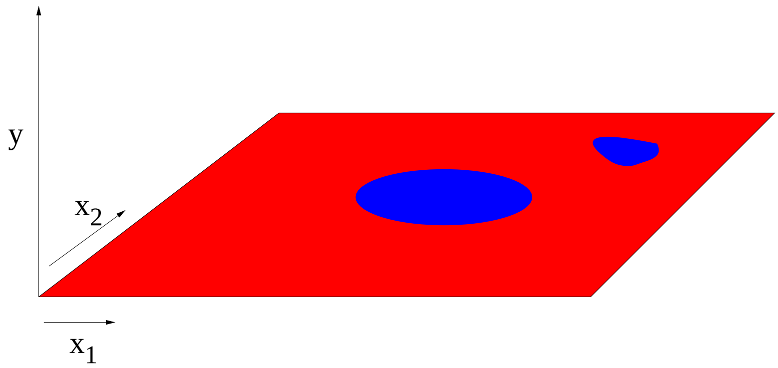
Symmetries:

$(E-J) + SO(4)_{S^3} + SO(4)_{SO(6)} + 16$ SUSY

$$ds^2 = -*(dt+V)^2 + *(dy^2 + dx_1^2 + dx_2^2) + *d\Omega_3{}^2 + *d\Omega_3{}^2$$

The solution is specified by z

$$\partial_1^2 z + \partial_2^2 z + y\partial_y \left(\frac{\partial_y z}{y} \right) = 0$$

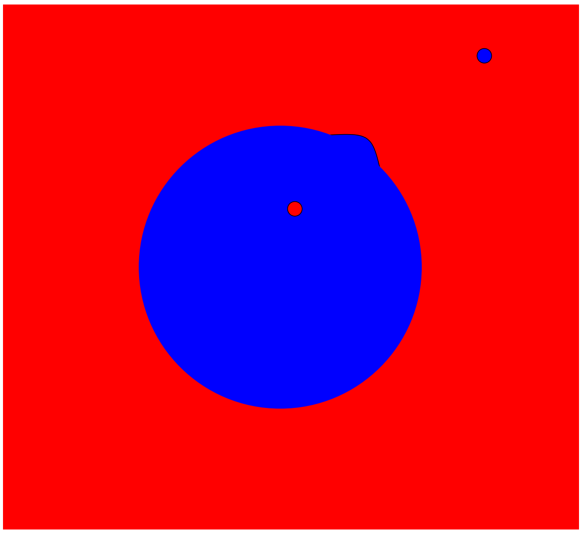


- $y \geq 0$

- 2 types of boundary conditions at $y = 0$

$$z = \frac{1}{2}, \qquad S^3 \to 0$$
$$z = -\frac{1}{2}, \qquad S^3 \to 0$$

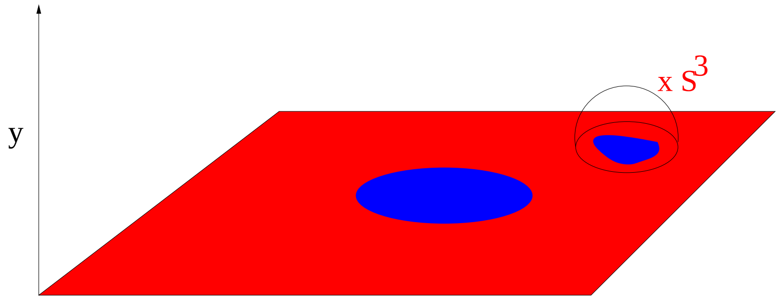
- $y = 0$ plane $\leftrightarrow$ Fermion phase space



- ripples $\longrightarrow$ gravitons

- single fermions $\longrightarrow$ giant gravitons $S^3 \subset AdS_5$

- single holes $\longrightarrow$ giant gravitons $S^3 \subset S^5$



- Geometric transitions S^3 that branes wrap $\longrightarrow$ contractible

- Fermion/hole number $\longrightarrow F_5$ flux on 5-sphere

- Topology of solution $\longrightarrow$ topology of droplets

Quantum Foam

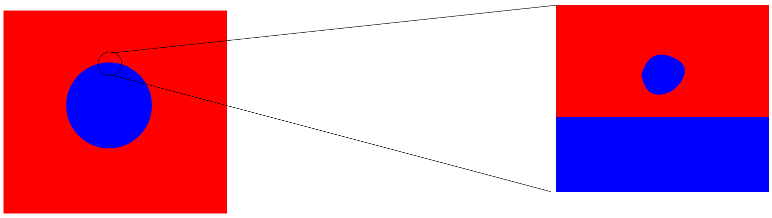
Small topological fluctuations $\longrightarrow$ already included as gravitons

(Boson = fermion) $\leftrightarrow$ (Graviton = Foam)

If density $0 < \rho < 1 \rightarrow$ null singularity

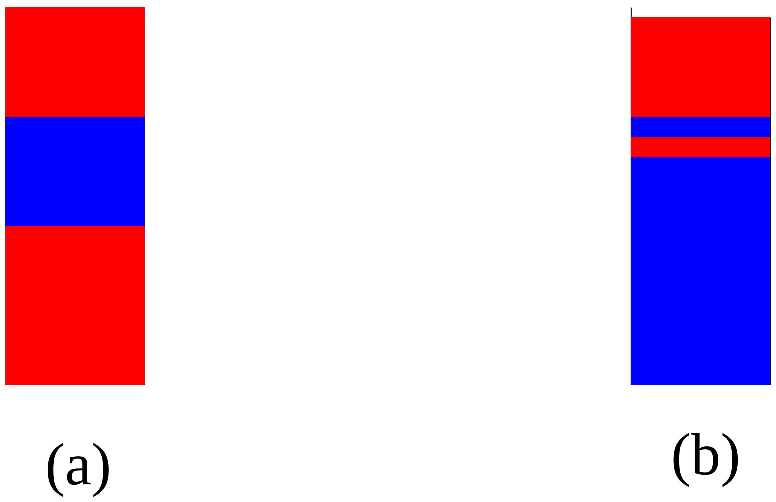
If density $\rho < 0$ or $1 < \rho \rightarrow$ closed time-like curves

Milanesi, O'Laughlin



- 1/2 BPS excitations on the IIB plane wave $\longrightarrow$ relativistic fermion.

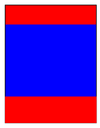
Compact x_1



- (a) D4 brane on $R \times S^1 \times S^3$. SUSY vacua = states of 2d QCD on a cylinder.

- (b) M2 brane theory with a mass deformation. Pope, Warner ; Bena, Warner

Compact $x_1,\ x_2$



- Little string theory with $R \times T^2 \times S^3{}_K \times S^3{}_N$ boundary.

  Itzhaki, Kutasov, Seiberg

- Low energies $SU(N)_K$ or $SU(K)_N$ Chern Simons theory on $R \times T^2$.

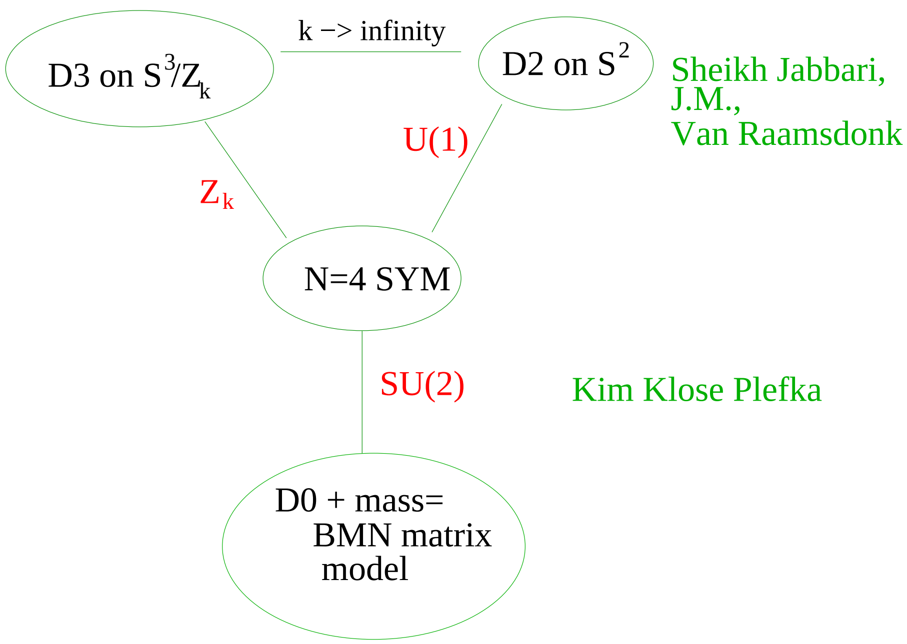
- Droplet configurations $\rightarrow$ different vacua

Poincare susy algebra in 1+1 or $2+1$

$$\{Q, Q\} = p_\mu + J_{SU(2)} + J_{SU(2)}$$

non-central charges. Possible in $d \leq 3$.

Theories with $\widetilde{SU}(2|4)$ symmetry



$Z_k \subset U(1) \subset SU(2)_L \subset SO(4) \subset SO(2,4)$

All have SO(6) symmetry

Gravity solutions

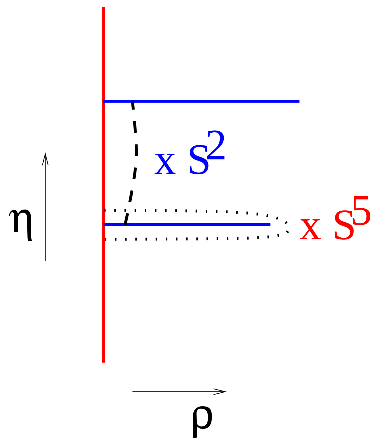
View the system in IIA.

$$ds^2 = -*(dt+\omega)^2 + *(d\rho^2 + d\eta^2) + + * d\Omega_5^2 + * d\Omega_2^2 \quad (1)$$

Single function

$$\frac{1}{\rho}\partial_\rho(\rho\partial_\rho V) + \partial_\eta^2 V = 0$$

Electrostatic problem



- Charge on disks: D2 charge

- Separation: NS-5 charge

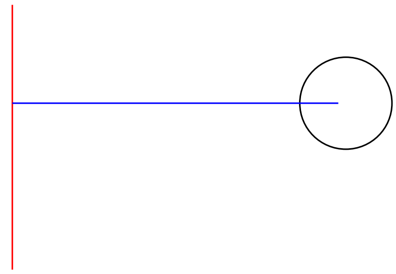
- Different configurations of disks $\rightarrow$ different vacua

- Study 1/2 BPS states on these geometries with $E = J$, $J \subset SO(6)$

- We can study near BPS states (pp-wave limit)

- These states live near the tip of the disks

Near BPS states for D2 on S^2



Single disk $\longrightarrow$ IIA pp wave

$$ds^2 = -2dx^+dx^- - (\vec{r}^{\,2} + 4\vec{y}^{\,2})(dx^+)^2 + d\vec{r}^{\,2} + d\vec{y}^{\,2}$$

Hyun, Shin

Bosons: masses 1 and 2. Universal result.

Near BPS:

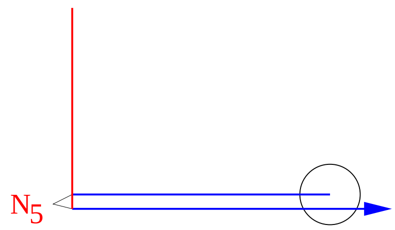
$$(E - J)_n = \sqrt{1 + (g_{YM}^2 N)^{\frac{2}{3}} \frac{n^2}{J^2}}$$

Coefficient is not universal.

Weak coupling:

$$(E - J)_n = 1 + (g_{YM}^2 N)\frac{n^2}{J^2} + \cdots$$

Solutions dual to the BMN matrix model



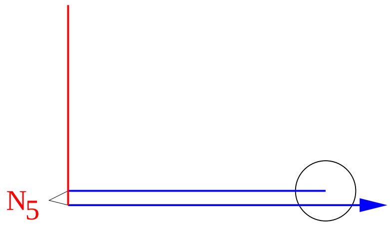
- Conducting plane at $\eta = 0$

- Vacua of BMN matrix model $\longrightarrow$

  dimension N SU(2) representations $\longrightarrow$

  configurations of disks with fixed total dipole moment.

Near BPS states in the BMN matrix model



- IIA pp wave + N_5 NS-5-branes.

- Lighcone worldsheet $\rightarrow$ (4,4) super-symmetric theory.

- (linear dilaton) x $SU(2)_{N_5}$ WZW + potential

Near BPS, large N_5

$$(E-J)_n = \sqrt{1+(g_{YM}^2 N_2)^{\frac{1}{2}}\frac{n^2}{J^2}}$$

$N = N_2 N_5$

Weak coupling $N_5 = 1$:

$$\begin{aligned}(E-J)_n &= 1 + f(x)\frac{n^2}{J^2} + \cdots \\ f(x) &= x + c_2 x^2 + c_3 x^3 + c_4 x^4 + \cdots \\ x &= g_{YM}^2 N\end{aligned}$$

Klose, Plefka ; Fischbacher, Klose, Plefka

We cannot do $N_5 = 1$

Conclusions

- Precise matching between field theories with large number of vacua, $e^{\sqrt{N}}$ and supergravity

- Unified gravity description for all theories with $\widetilde{SU}(2|4)$ supergroup.

- Interesting strong coupling results for many spin chains. Close relation to $\mathcal{N} = 4$ SYM.