Holography and hydrodynamics

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Collaboration and some Refs.

- Dam Son
- Giuseppe Policastro
- Chris Herzog
- Alvaro Nunez
- Pavel Kovtun
- Alex Buchel
- Jim Liu
- Andrei Parnachev
- Paolo Benincasa

- hep-th/0507026
- hep-th/0506184
- hep-th/0506144
- hep-th/0406124
- hep-th/0405231
- hep-th/0309213
- hep-th/0302026
- hep-th/0205052
- hep-th/0205051

Prologue

- Our goal is to understand thermal gauge theories, e.g. thermal QCD
- Of particular interest is the regime described by fluid dynamics, e.g. quark-gluon plasma
- This near-equilibrium regime is completely characterized by values of transport coefficients, e.g. shear and bulk viscosity
- Transport coefficients are hard to compute from “first principles”, even in perturbation theory. For example, no perturbative calculation of bulk viscosity in gauge theory is available.

Prologue (continued)

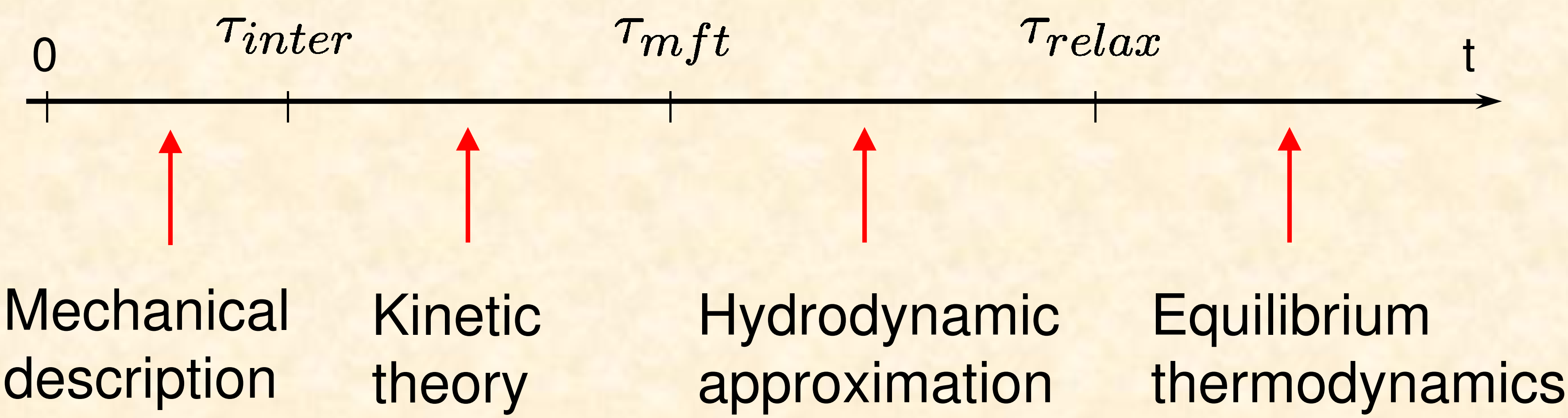
- Transport coefficients of some gauge theories can be computed in the regime described by string (gravity) duals – usually at large N and large ‘t Hooft coupling
- Corrections can in principle be computed
- Shear viscosity result is *universal*. Model-independent results may be of relevance for RHIC physics
- Certain results are *predicted* by hydrodynamics. Finding them in gravity provides a check of the AdS/CFT conjecture

Outline

- Shear and bulk viscosities
- Computing viscosity in AdS/CFT
- Universality of shear viscosity in the regime described by gravity duals
- Bulk viscosity and the speed of sound from gravity
- Relation to RHIC (if any)
- Conclusions

What is hydrodynamics?

Hierarchy of times (example)



Hierarchy of scales

$$l_{mfp} \ll l \ll L$$

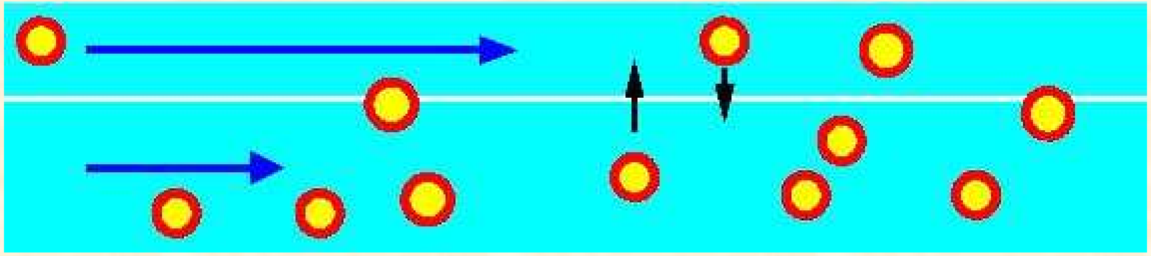
(L is a macroscopic size of a system)

What is viscosity?

Friction in Newton's equation: $$\frac{d(mv_i)}{dt} + \gamma v_i = F_i$$

Friction in Euler's equations

$$\frac{\partial(\rho v_i)}{\partial t} = -\frac{\partial}{\partial x^k}\left(P\delta_{ik} + \rho v_i v_k\right) + \frac{\partial}{\partial x^k}\sigma_{ik}^{fric}$$



$$\sigma_{ik}^{fric} \sim \partial v_i/\partial x^k$$

$$\sigma_{ik}^{fric} \sim \partial v_i/\partial x^k + \partial v_k/\partial x^i$$

$$\sigma_{ik}^{fric} = \eta\left(\frac{\partial v_i}{\partial x^k} + \frac{\partial v_k}{\partial x^i} - \frac{2}{d}\delta_{ik}\frac{\partial v_l}{\partial x^l}\right) + \zeta\delta_{ik}\frac{\partial v_l}{\partial x^l} + \cdots$$

Viscosity of gases and liquids

Gases (Maxwell, 1867): $\eta \sim \rho \, \bar{v} \, l_{mfp} \sim \frac{m_o \bar{v}}{\sigma} \sim \frac{m_o^{1/2}}{\sigma} \sqrt{T}$

Viscosity of a gas is

- § independent of pressure
- § scales as square of temperature
- § inversely proportional to cross-section

Liquids (Frenkel, 1926): $\eta \sim A(P,T) \; \mathrm{exp} \, \frac{W}{T}$

- § W is the "activation energy"
- § In practice, A and W are chosen to fit data

Computing transport coefficients from "first principles"

Fluctuation-dissipation theory
(Callen, Welton, Green, Kubo)

Kubo formulae allows one to calculate transport coefficients from microscopic models

$$\eta = \lim_{\omega \to 0} \frac{1}{2\omega} \int dt\, d^3x e^{i\omega t} \langle \left[T_{xy}(t,x), T_{xy}(0,0) \right] \rangle$$

In the regime described by a gravity dual the correlator can be computed using AdS/CFT

Universality of shear viscosity in the regime described by gravity duals

$$ds^2 = f(w)\left(dx^2 + dy^2\right) + g_{\mu\nu}(w) dw^\mu dw^\nu$$

$$\left.\begin{aligned}
\eta &= \lim_{\omega \to 0} \frac{1}{2\omega} \int dt\, dx e^{i\omega t} \langle \left[T_{xy}(t,x), T_{xy}(0,0) \right] \rangle \\
\sigma_{abs} &= -\frac{16\pi G}{\omega}\, \mathrm{Im}\, G^R(\omega) \\
&= \frac{8\pi G}{\omega} \int dt\, dx e^{i\omega t} \langle \left[T_{xy}(t,x), T_{xy}(0,0) \right] \rangle
\end{aligned}\right\} \quad \eta = \frac{\sigma_{abs}(0)}{16\pi G}$$

Graviton's component h_y^x obeys equation for a minimally coupled massless scalar. But then $\sigma_{abs}(0) = A_H$.

Since the entropy (density) is $s = A_H / 4G$ we get

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

Three roads to universality of η/s

- **The absorption argument**

  D. Son, P. Kovtun, A.S., hep-th/0405231

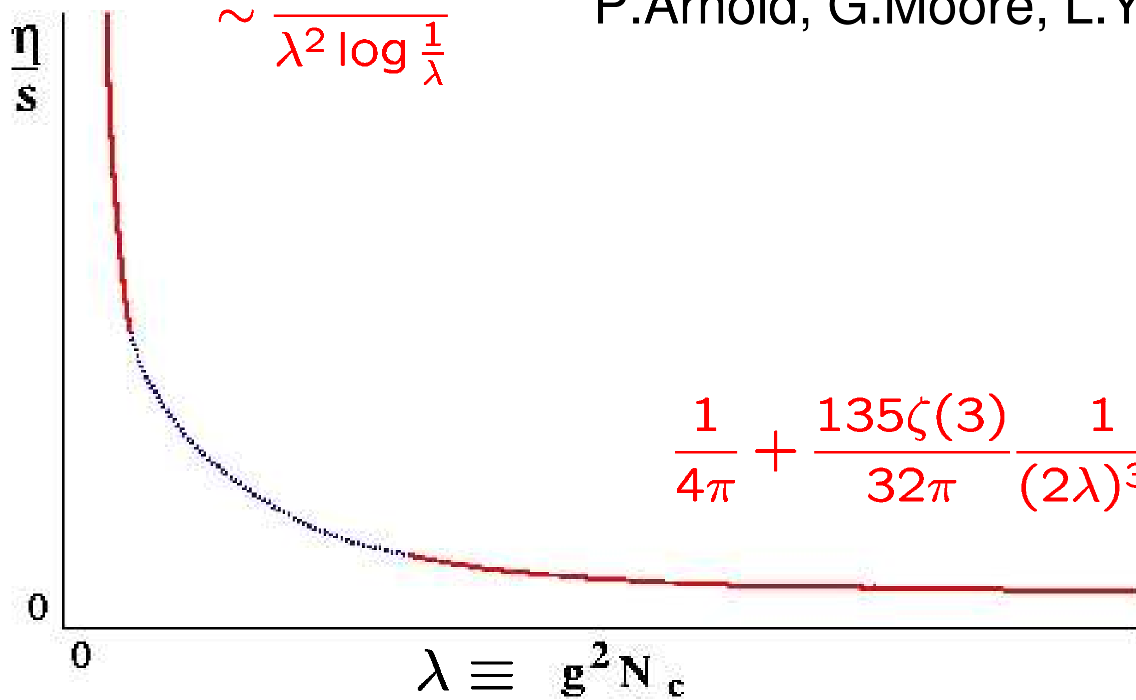
- **Direct computation of the correlator in Kubo formula from AdS/CFT** A.Buchel, hep-th/0408095

- **"Membrane paradigm" general formula for diffusion coefficient + interpretation as lowest quasinormal frequency = pole of the shear mode correlator + Buchel-Liu theorem**

  P. Kovtun, D.Son, A.S., hep-th/0309213, A.S., to appear,
  P.Kovtun, A.S., hep-th/0506184, A.Buchel, J.Liu, hep-th/0311175

Shear viscosity in $\mathcal{N} = 4$ SYM

P.Arnold, G.Moore, L.Yaffe, 2001



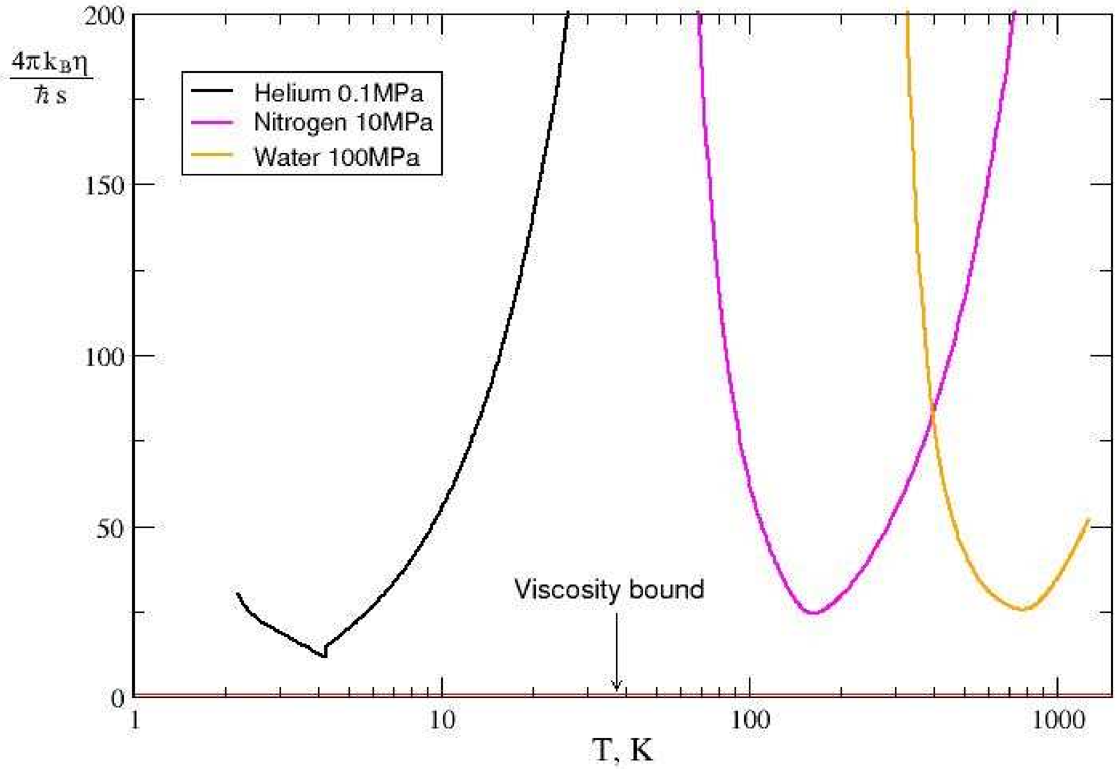
$$\sim \frac{1}{\lambda^2 \log \frac{1}{\lambda}}$$

$$\frac{1}{4\pi} + \frac{135\zeta(3)}{32\pi} \frac{1}{(2\lambda)^{3/2}} + \cdots$$

Correction to $1/4\pi$: A.Buchel, J.Liu, A.S., hep-th/0406264

A viscosity bound conjecture

$$\frac{\eta}{s} \geq \frac{\hbar}{4\pi k_B} \approx 6.08 \cdot 10^{-13} K \cdot s$$



P.Kovtun, D.Son, A.S., hep-th/0309213, hep-th/0405231

Hydrodynamics as an effective theory

Thermodynamic equilibrium: $\langle T^{00}\rangle = \epsilon\,,\ \langle T^{0i}\rangle = 0$

$$T^{ij} = P(\epsilon)\delta^{ij}$$

Near-equilibrium: $T^{00} = \epsilon + \tilde{T}^{00}$

$$T^{ij} = P\delta^{ij} + \frac{\partial P}{\partial \epsilon}\tilde{T}^{00} + \tilde{T}^{ij}$$

(with $\frac{\partial P}{\partial \epsilon}$ circled and labelled v_s^2)

$$\tilde{T}^{ij} = -\frac{1}{\epsilon+P}\left[\eta\left(\partial_i\tilde{T}^{0j} + \partial_j\tilde{T}^{0i} - \frac{2}{3}\delta^{ij}\partial_k\tilde{T}^{0k}\right) + \zeta\delta^{ij}\partial_k\tilde{T}^{0k}\right] + \cdots$$

Eigenmodes of the system of equations $\partial_\mu T^{\mu\nu} = 0$

Shear mode (transverse fluctuations of $\tilde{T}^{0i}$): $\omega = -\frac{i\eta}{\epsilon+P}q^2$

Sound mode: $\omega = v_s q - \frac{i}{2}\frac{1}{\epsilon+P}\left(\zeta + \frac{4}{3}\eta\right)q^2$

For CFT we have $\zeta = 0$ and $\epsilon = 3P$ ⇒ $v_s = 1/\sqrt{3}$

Two-point correlation function of stress-energy tensor

Field theory

Zero temperature: $\langle T_{\mu\nu} T_{\alpha\beta} \rangle_{T=0} = \Pi_{\mu\nu,\alpha\beta} F(k^2) + Q_{\mu\nu,\alpha\beta} G(k^2)$

Finite temperature: $\langle T_{\mu\nu} T_{\alpha\beta} \rangle_{T} = S^{(1)}_{\mu\nu,\alpha\beta} G_1(\omega,q) + S^{(2)}_{\mu\nu,\alpha\beta} G_2(\omega,q)$

$+ \; S^{(3)}_{\mu\nu,\alpha\beta} G_3(\omega,q) + S^{(4)}_{\mu\nu,\alpha\beta} G_4 + S^{(5)}_{\mu\nu,\alpha\beta} G_5$

Dual gravity

- Five gauge-invariant combinations Z_1, Z_2, Z_3, Z_4, Z_5 of $h_{\mu\nu}$ and other fields determine G_1, G_2, G_3, G_4, G_5
- Z_1, Z_2, Z_3, Z_4, Z_5 obey a system of coupled ODEs
- Their (quasinormal) spectrum determines singularities of the correlator

Classification of fluctuations and universality

$$ds^2 = \frac{r^2}{R^2}\left(-f(r)dt^2 + dx^2 + dy^2 + dz^2\right) + \frac{R^2}{r^2 f}dR^2$$

$\delta g_{\mu\nu} \sim e^{-i\omega t + iqz} h_{\mu\nu}(r)$ O(2) symmetry in x-y plane

Shear channel: h_{tx} h_{zx} h_{ty} h_{zy} ⇨ Z_1

Sound channel: h_{tt} h_{tz} h_{zz} $h_{xx} + h_{yy}$ ⇨ Z_2

Scalar channel: h_{xy} $h_{xx} - h_{yy}$ ⇨ Z_3

Other fluctuations (e.g. $\delta\varphi_1 , \ldots \delta\varphi_n$) may affect sound channel

But not the shear channel ➡ universality of η/s

Bulk viscosity and the speed of sound in $\mathcal{N}=2^*$ SYM

$\mathcal{N}=2^*$ is a "mass-deformed" $\mathcal{N}=4$ (Pilch-Warner flow)

- Ø Finite-temperature version: A.Buchel, J.Liu, hep-th/0305064
- Ø The metric is known explicitly for $m/T \ll 1$
- Ø Speed of sound and bulk viscosity:

$$v_s = \frac{1}{\sqrt{3}}\left(1 - \frac{\left[\Gamma\left(\frac{3}{4}\right)\right]^4}{3\pi^4}\left(\frac{m_f}{T}\right)^2 - \frac{1}{18\pi^4}\left(\frac{m_b}{T}\right)^4 + \cdots\right)$$

$$\frac{\zeta}{\eta} = \beta_f^{\Gamma}\frac{\left[\Gamma\left(\frac{3}{4}\right)\right]^4}{3\pi^3}\left(\frac{m_f}{T}\right)^2 + \frac{\beta_b^{\Gamma}}{432\pi^2}\left(\frac{m_b}{T}\right)^4 + \cdots$$

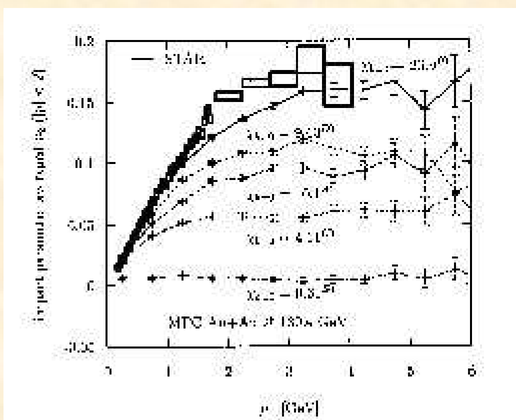
$$\frac{\zeta}{\eta} = -\kappa\left(v_s^2 - \frac{1}{3}\right)$$

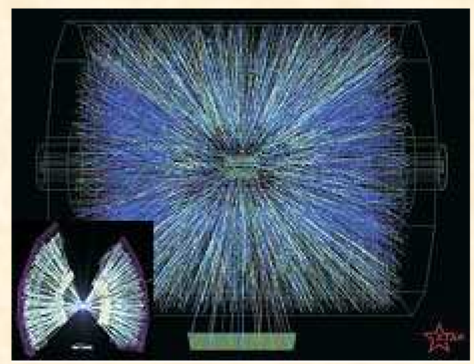
Relation to RHIC

Ø IF quark-gluon plasma is indeed formed in heavy ion collisions

Ø IF a hydrodynamic regime is unambiguously proven to exist

Ø THEN hydrodynamic MODELS describe experimental results for e.g. elliptic flows well, provided $\eta/s \sim 1/4\pi$





v Bulk viscosity and speed of sound results are potentially interesting

Epilogue

- AdS/CFT gives insights into physics of thermal gauge theories in the nonperturbative regime
- Generic hydrodynamic predictions can be used to check validity of AdS/CFT
- General algorithm exists to compute transport coefficients and the speed of sound in any gravity dual
- Model-independent statements can presumably be checked experimentally