Analysis of QCD via Supergravity

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based on : T. Sakai and S.S.

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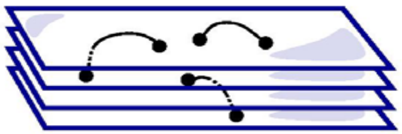
What is nice?

- QCD is one of our favorite QFT.
- We can use
  * knowledge of QCD
  * experimental data

  to test the conjectured gauge/gravity duality.
- A new way to analyze QCD.
  → application to phenomenology
- Anyway, it is fun !

Outline

String theory



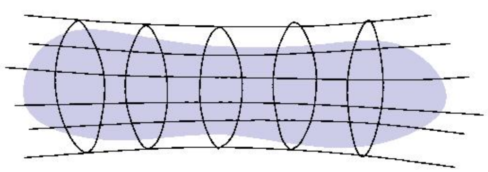
① →

QCD

$$\mathcal{L} = -\frac{1}{4g^2}\mathrm{Tr}\, F_{\mu\nu}^2 + i\bar{\psi}_i \not{D} \psi^i$$

② ↓

Sugra



③ →

Hadron physics

$q\bar{q}$ $\quad$ qqq

$\pi, \rho, a_1, \cdots$ $\quad$ $p, n, \cdots$

① Construction of QCD

★ building blocks

QCD = YM + quarks

- <u>Yang–Mills</u>

  fermion

  D4–brane on S^1 with $\lambda(x^\mu,\tau) = -\lambda(x^\mu,\tau+2\pi M_{\rm KK}^{-1})$

  4 dim $\mathcal{N}=4$ super YM → (~~SUSY~~, $E \ll M_{\rm KK}$) → 4 dim pure YM

  Corresponding SUGRA solution is known. [Witten 1998]

- <u>quarks</u>

  Add probe D8–branes.

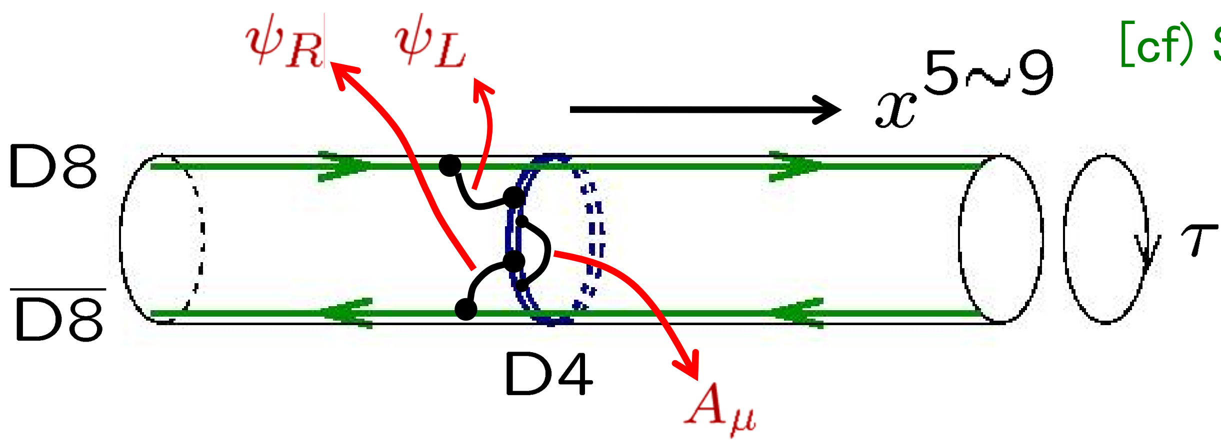
  cf) KMMW model : D4 + probe D6

  [Kruczenski–Mateos–Myers–Winter 2003]

★ the brane configuration

	x^0	x^1	x^2	x^3	τ	x^5	x^6	x^7	x^8	x^9
D4 $\times N_c$	○	○	○	○	○	–	–	–	–	–
D8-$\overline{\text{D8}}$ $\times N_f$	○	○	○	○	–	○	○	○	○	○

[cf) S.S.–Takahashi 2004]



	D4 $U(N_c)$	D8 $U(N_f)_L$	$\overline{\text{D8}}$ $U(N_f)_R$
A_μ	adjoint	1	1
ψ_L	N_c	N_f	1
ψ_R	N_c	1	N_f

➡ 4 dim $U(N_c)$ QCD with N_f massless quarks

② Supergravity description

★ Probe approximation

We assume $N_c \gg N_f$

Only D4−brane is replaced with the SUGRA solution.
D8-$\overline{\text{D8}}$ pairs are treated as probes.

cf)

The metric of the corresponding D4 solution is

$\lambda = g_{\rm YM}^2 N_c$

$$ds^2 = \frac{\lambda l_s^2}{3}\left[\frac{4}{9}M_{\rm KK}^2\, u^{3/2}(\eta_{\mu\nu}dx^\mu dx^\nu + f(u)d\tau^2) + u^{-3/2}\left(\frac{du^2}{f(u)} + u^2 d\Omega_4^2\right)\right]$$

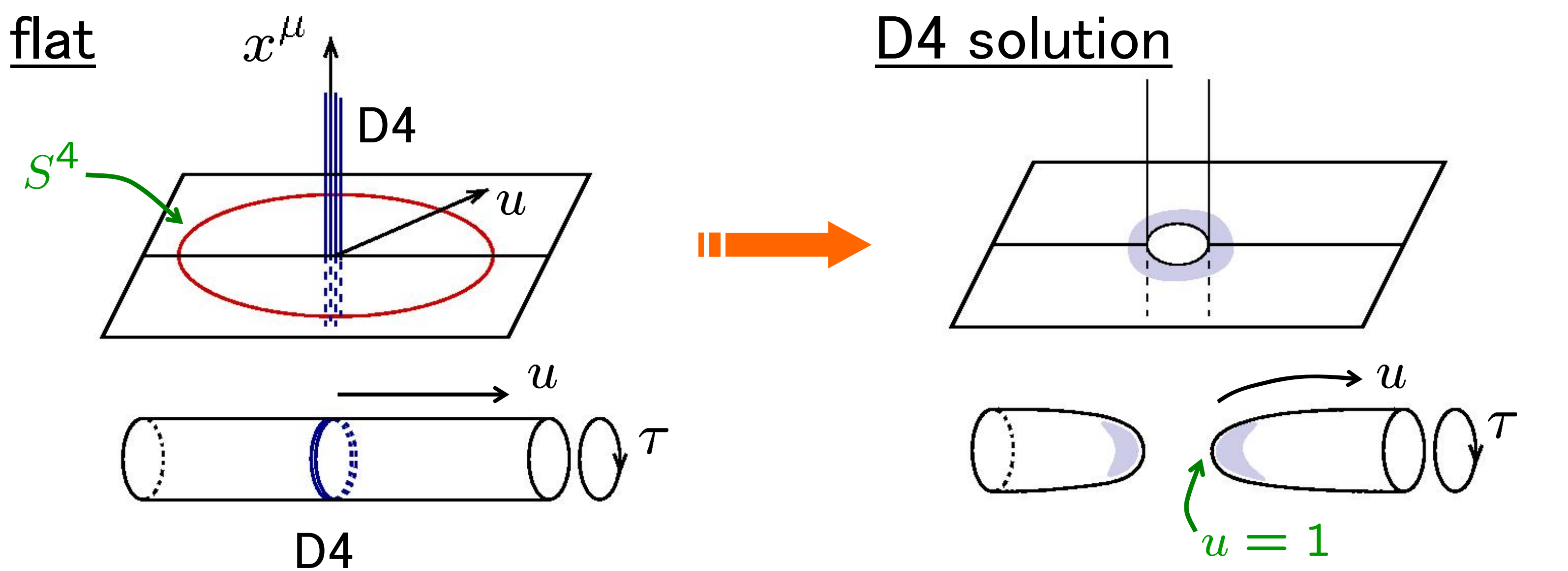
$\mathbf{R}^{1,3}$, S^1, S^4

$(\tau \sim \tau + 2\pi M_{\rm KK}^{-1})$

where

$$f(u) \equiv 1 - 1/u^3$$

radial direction, bounded as $u \geq 1$



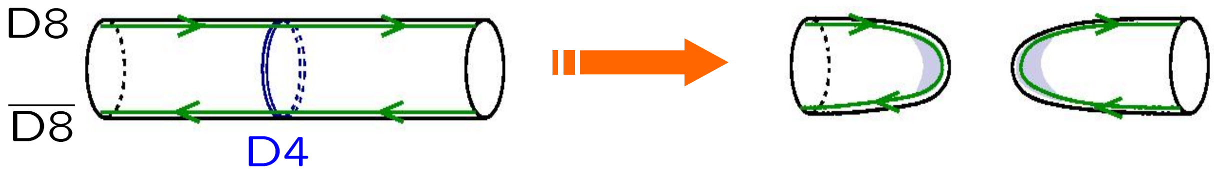
③ Exploration

<u>highlights</u>

- Chiral symmetry breaking
- Unification of mesons
- Construction of baryons
- mass & couplings
- vector meson dominance

★ Chiral symmetry breaking

We replace D4 with the SUGRA sol.



D8 and $\overline{\text{D8}}$ must be connected

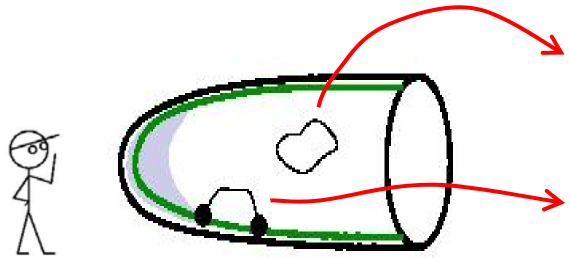
in the D4 background.

➡ interpreted as the chiral symmetry breaking !

$$U(N_f)_L \times U(N_f)_R \to U(N_f)_V$$

$U(N_f)_L$	$U(N_f)_R$	$U(N_f)_V$
↕	↕	↕
D8	$\overline{\text{D8}}$	connected D8

★ Where are mesons?



closed string → glueballs

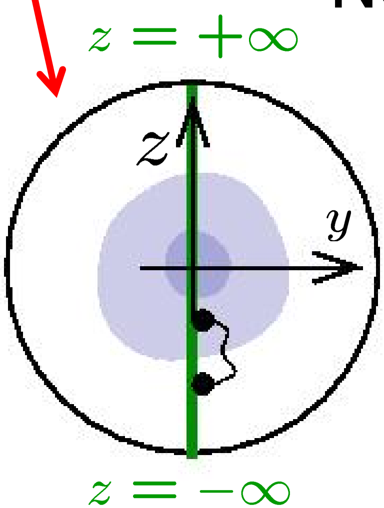
open string → mesons

The effective theory is a 9 dim $U(N_f)$ gauge theory

$$A_\mu(x^\mu, z, \theta^i),\quad A_z(x^\mu, z, \theta^i),\quad A_i(x^\mu, z, \theta^i)$$

(θ^i: S^4)

Note: <u>There is an $SO(5)$ symmetry ↷ S^4</u>



$(u^3 = 1 + y^2 + z^2)$

※ Today, we only consider the $SO(5)$ singlet states, for simplicity.

(mesons in realistic QCD live in this sector)

➡ reduced to **5 dim $U(N_f)$ gauge theory**

$$A_\mu(x^\mu, z),\quad A_z(x^\mu, z)$$

D8-brane action

- $$S_{D8}^{DBI} \simeq -T\int d^9x\, e^{-\phi}\sqrt{-\det(g_{MN}+2\pi\alpha' F_{MN})}$$
$$\sim \int d^9x\, e^{-\phi}\sqrt{-g}\, g^{MN}g^{PQ}F_{MP}F_{NQ}+\cdots$$

Inserting the SUGRA solution,

$$K(z) \equiv 1+z^2$$

$$\Rightarrow \quad S_{D8}^{DBI} \sim \kappa \int d^4x dz\, \mathrm{Tr}\left(\frac{1}{2}K(z)^{-1/3}F_{\mu\nu}^2 + M_{KK}^2 K(z) F_{\mu z}^2\right)$$

$$\kappa \equiv \frac{\lambda N_c}{108\pi^3}$$

- $$S_{D8}^{CS} \simeq \int_9 C\wedge \mathrm{Tr}e^{F/2\pi} \sim \int_9 dC_3 \wedge \frac{1}{3!(2\pi)^3}\omega_5(A)+\cdots$$

CS 5-form
$d\omega_5(A) = \mathrm{Tr}\, F^3$

D4 charge

$$\frac{1}{2\pi}\int_{S^4} dC_3 = N_c \quad \Rightarrow \quad S_{D8}^{CS} \sim \frac{N_c}{24\pi^2}\int_5 \omega_5(A)$$

This 5 dim YM-CS theory is considered as the effective theory of mesons.

[cf) Son-Stephanov 2003]

mode expansion

$$A_\mu(x^\mu, z) = \sum_n B_\mu^{(n)}(x^\mu)\psi_n(z)$$
$$A_z(x^\mu, z) = \sum_n \varphi^{(n)}(x^\mu)\phi_n(z)$$

some complete sets

- To diagonalize kinetic & mass terms of $B_\mu^{(n)}, \varphi^{(n)}$,

★ choose $\{\psi_n\}_{n\geq 1}$ as eigen functions satisfying

$$-K^{1/3}\partial_z(K\partial_z\psi_n) = \lambda_n\psi_n \,, \qquad \int dz\, K^{-1/3}\psi_n\psi_m = \delta_{nm}$$

★ choose $\{\phi_n\}_{n\geq 1}$ as $\phi_n(z) = \partial_z\psi_n(z)$ $\quad (K \equiv 1 + z^2)$

→ $\int dz\, K\phi_n\phi_m = \lambda_n\delta_{nm}$

- In addition, we have one more normalizable mode

$$\phi_0(z) = \frac{c}{K(z)}$$

$$\left(\begin{array}{l} \int dz\, K\phi_0\phi_n \propto \int dz\, \partial_z\psi_n = 0 \\ \int dz\, K\phi_0^2 < \infty \end{array} \right)$$

Using these, we obtain

$F_{\mu\nu}^{(n)} \equiv \partial_\mu B_\nu^{(n)} - \partial_\nu B_\mu^{(n)}$

$$S_{\rm D8}^{\rm DBI} \sim \sum_{n\geq 1} \int d^4x \,{\rm Tr} \left[\frac{1}{2} F_{\mu\nu}^{(n)2} + \lambda_n M_{\rm KK}^2 \left(B_\mu^{(n)} - \partial_\mu \varphi^{(n)} \right)^2 \right] + \int d^4x \,{\rm Tr}\, \partial_\mu \varphi^{(0)2}$$

$+$ (interaction terms)

eaten

massless scalar meson

massive vector meson

In summary,

$$A_z(x^\mu, z) = \varphi^{(0)}(x^\mu)\phi_0(z) + \varphi^{(1)}(x^\mu)\phi_1(z) + \varphi^{(2)}(x^\mu)\phi_2(z) + \cdots$$

eaten　eaten

$$A_\mu(x^\mu, z) = B_\mu^{(1)}(x^\mu)\psi_1(z) + B_\mu^{(2)}(x^\mu)\psi_2(z) + \cdots$$

π　ρ　a_1

$\pi, \rho, a_1, \cdots$ are unified in the 5 dim gauge field !

★ Comments on J^{PC}

- Parity : $(x^1, x^2, x^3, z) \to (-x^1, -x^2, -x^3, -z)$
- Charge conjugation : $A_M \to -A_M^T$ and $z \to -z$

	$\varphi^{(0)}$	$B_\mu^{(\mathrm{odd})}$	$B_\mu^{(\mathrm{even})}$
J^{PC}	0^{-+} pseudo-scalar	1^{--} vector	1^{++} axial-vector

★ Consistent with our interpretation.

$$\pi \sim \varphi^{(0)}, \quad \rho \sim B^{(1)}, \quad a_1 \sim B^{(2)}, \quad \text{etc.}$$

★ Prediction: vector and axial−vector mesons appear alternately.

1^{--}	$\rho(770)$ $\rho(1450)$ $\rho(1700)$ $\rho(1900)^{\triangle}$ $\rho(2150)^{\triangle}$
1^{++}	$a_1(1260)$ $a_1(1640)^{\triangle}$

($\triangle$ ... not established)

★ mass spectrum

The mass of the n−th meson is given by

$$(\text{mass})^2 = \lambda_n M_{\text{KK}}^2$$

where λ_n is the eigen value of the equation

$$-K^{1/3}\partial_z(K\partial_z\psi_n) = \lambda_n\psi_n$$

$$\int dz\, K^{-1/3}\psi_n\psi_m = \delta_{nm}$$

★ Let us solve this numerically,
to see if our model is too bad or not.

We should not be too serious,
since the approximation is very crude.
$\left(\begin{array}{l} N_c \gg N_f \sim \mathcal{O}(1) \\ E \ll M_{\text{KK}},\ m_q = 0, \\ \text{etc.} \end{array} \right)$

Results : $\lambda_n \simeq 0.669,\ 1.57,\ 2.87,\ \cdots$

$$\frac{\lambda_2}{\lambda_1} \simeq \frac{1.57}{0.669} \simeq 2.35$$ (our model)

+7 %

$$\frac{m_{a_1}^2}{m_\rho^2} \simeq \frac{(1230\mathrm{MeV})^2}{(776\mathrm{MeV})^2} \simeq 2.51$$ (experiment)

$$\frac{\lambda_3}{\lambda_1} \simeq \frac{2.87}{0.669} \simeq 4.29$$ (our model)

−17 %

$$\frac{m_{\rho(1450)}^2}{m_\rho^2} \simeq \frac{(1465\mathrm{MeV})^2}{(776\mathrm{MeV})^2} \simeq 3.56$$ (experiment)

★ chiral symmetry

We have implicitly assumed

$$A_M(x^\mu, z) \to 0 \ , \qquad (z \to \pm\infty)$$

$M = 0 \sim 3, z$

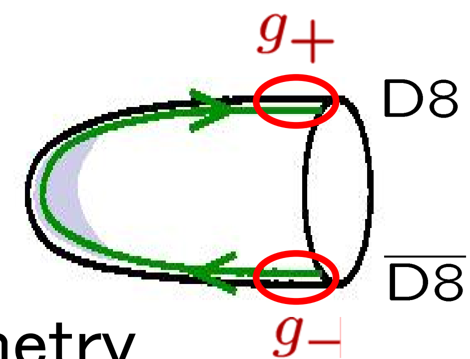
Residual gauge symmetry

constant

$$g(x^\mu, z) \in U(N_f) \ \text{s.t.}\ g(x^\mu, z) \to g_\pm \ , \ (z \to \pm\infty)$$

This is interpreted as

$$(g_+, g_-) \in U(N_f)_L \times U(N_f)_R$$



- If you want to 'gauge' the chiral symmetry,

$$g_\pm \Longrightarrow g_\pm(x^\mu)$$

the corresponding gauge fields appear as

$$A_\mu(x^\mu, z) \to A_{\pm\mu}(x^\mu) \ , \qquad (z \to \pm\infty)$$

- Define $$U(x^\mu) \equiv P\exp\left\{-\int_{-\infty}^{\infty} dz' A_z(x^\mu, z')\right\}$$

  → transforms as

$$U(x^\mu) \to g_+ U(x^\mu) g_-^{-1} \qquad \left(g_\pm \equiv \lim_{z\to\pm\infty} g(x^\mu, z) \right)$$

  → interpreted as the pion field

$$U(x^\mu) = e^{2i\pi(x^\mu)/f_\pi}$$

- It is also useful to define

$$\xi_\pm^{-1}(x^\mu) \equiv P\exp\left\{-\int_0^{\pm\infty} dz' A_z(x^\mu, z')\right\}$$

$$\left(U(x^\mu) = \xi_+^{-1}(x^\mu)\xi_-(x^\mu) \right)$$

  → transforms as

$$\xi_\pm(x^\mu) \to h(x^\mu)\xi_\pm(x^\mu) g_\pm^{-1} \qquad \left(h(x^\mu) \equiv g(x^\mu, z=0) \right)$$

$h(x^\mu)$: "hidden" local symmetry

[Bando-Kugo-Uehara-Yamawaki-Yanagida 1985]

★ Two useful gauge choices

- $A_z \simeq \pi$ gauge (→ simple interaction)

$$A_z(x^\mu, z) = \pi(x^\mu)\phi_0(z) \quad \text{(higher modes are gauged away.)}$$
$$A_\mu(x^\mu, z) = A_{+\mu}(x^\mu)\psi_+(z) + A_{-\mu}(x^\mu)\psi_-(z) + \sum_{n\geq 1} B_\mu^{(n)}(x^\mu)\psi_n(z)$$

$$\psi_+(z) \to \begin{cases} 1 & z \to +\infty \\ 0 & z \to -\infty \end{cases} \quad \psi_-(z) \to \begin{cases} 0 & z \to +\infty \\ 1 & z \to -\infty \end{cases}$$ (appropriately chosen)

- $A_z = 0$ gauge (→ manifest chiral symmetry)

$$A_z(x^\mu, z) = 0$$

← obtained by the gauge tr. with $g^{-1}(x^\mu, z) = P\exp\left\{-\int_0^z dz' A_z(x^\mu, z')\right\}$

Pion appears in the boundary condition

$$A_\mu(x^\mu, z) \to A_{\pm\mu}^{\xi_\pm}(x^\mu), \quad (z \to \pm\infty) \qquad (A_\mu^g \equiv gA_\mu g^{-1} + g\partial_\mu g^{-1})$$

The mode exp. consistent with this b.c. is

$$A_\mu(x^\mu, z) = A_{+\mu}^{\xi_+}(x^\mu)\psi_+(z) + A_{-\mu}^{\xi_-}(x^\mu)\psi_-(z) + \sum_{n\geq 1} B_\mu^{(n)}(x^\mu)\psi_n(z)$$

★ effective action

Inserting the mode exp. into the D8−brane action

$$S_{\rm D8} \simeq S_{\rm D8}^{\rm DBI} + S_{\rm D8}^{\rm CS}$$

we obtain the effective action for the mesons written in terms of π (or $\xi_\pm$), $B_\mu^{(n)}$ and $A_{\pm\mu}$.

- Using the $A_z = 0$ gauge,
  the effective action of the pion field is obtained as

$$S_{\rm D8}^{\rm DBI} \simeq \int d^4x \left[\frac{f_\pi^2}{4} {\rm Tr}(U^{-1}\partial_\mu U)^2 + \frac{1}{32 e_S^2} {\rm Tr}[U^{-1}\partial_\mu U, U^{-1}\partial_\nu U]^2 \right] + \cdots$$

$$f_\pi^2 = \frac{4\kappa}{\pi} M_{\rm KK}^2 \qquad e_S^{-2} \simeq 2.51 \cdot \kappa \qquad \left(\kappa \equiv \frac{\lambda N_c}{108\pi^3} \right)$$

This is the Skyrme model action.
The coefficients are consistent with the experimental data, if we set $\kappa \sim 0.007$, $M_{\rm KK} \sim 1$ GeV

★ Baryon

- Skyrme proposed [Skyrme 1961]

  Baryon $\simeq$ Soliton in Skyrme model (Skyrmion)

- Baryons in the AdS/CFT context is constructed as wrapped D-branes. [Witten 1998]

  In our case, Baryon $\simeq$ D4-brane wrapped on S^4

- In our case, a Skyrmion can be lifted to an instanton in the 5 dim YM theory, and the instanton on the D8 is equivalent the wrapped D4.

$\therefore$ Skyrmion $\simeq$ instanton on D8 $\simeq$ Wrapped D4 ■

★ KSRF relation

- Working in the $A_z \simeq \pi$ gauge , the 3 pt coupling

$$S_{\rm D8}^{\rm DBI} \sim \int dx^4 \left[\cdots + 2 g_{\rho\pi\pi} \mathrm{Tr}(\rho_\mu [\pi, \partial^\mu \pi]) + \cdots \right] \quad (\rho_\mu \equiv B_\mu^{(1)})$$

is calculated as $g_{\rho\pi\pi} \simeq 0.415 \cdot \kappa^{-1/2}$

$$\frac{2 g_{\rho\pi\pi}^2 f_\pi^2}{m_\rho^2} \simeq 0.654 \quad \text{(our model)}$$

$$\simeq 1.02 \quad \text{(experiment)}$$

$$\simeq 1 \quad \text{(KSRF)}$$

[Kawarabayashi–Suzuki 1966]
[Riazuddin–Fayyazddin 1966]

Note: $$\sum_{n=1}^{\infty} \frac{2 g_{\rho^n\pi\pi}^2 f_\pi^2}{m_{\rho^n}^2} = \frac{2}{3} \qquad \left(\rho_\mu^n \equiv B_\mu^{(2n-1)} \right)$$

[cf) Da Rold–Pomarol 2005]

★ including photon

- ele−mag gauge field $A_\mu^{\rm em}$ is introduced by setting

$$\mathcal{V}_\mu \equiv \frac{1}{2}(A_{+\mu} + A_{-\mu}) = e\,Q\,A_\mu^{\rm em}$$

charge matrix

$$Q = \frac{1}{3}\begin{pmatrix} 2 & \\ & -1 \end{pmatrix}$$

for $N_f = 2$

- We obtain

$$S_{\rm D8}^{\rm DBI} \sim \int dx^4 \left[\cdots + 2g_{\rho\pi\pi}{\rm Tr}(\rho_\mu[\pi, \partial^\mu \pi]) - 2g_\rho {\rm Tr}(\rho_\mu \mathcal{V}^\mu) + \cdots \right]$$

with $g_\rho \simeq 2.11 \cdot \kappa^{1/2} M_{\rm KK}^2$

$$\rightarrow \quad \frac{g_\rho g_{\rho\pi\pi}}{m_\rho^2} \simeq 1.31 \quad \text{(our model)}$$

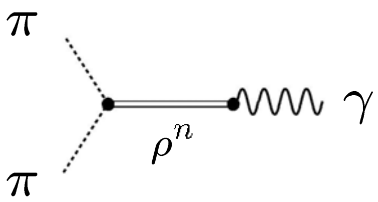
$$\simeq 1.20 \quad \text{(experiment)}$$

Note: $$\sum_{n=1}^{\infty} \frac{g_{\rho^n} g_{\rho^n\pi\pi}}{m_{\rho^n}^2} = 1$$

[cf) Son−Stephanov 2003]

★ pion form factor

- pion form factor is computed as



$$F_\pi(k^2) = \sum_{n\geq 1} \frac{g_{\rho^n} g_{\rho^n \pi\pi}}{k^2 + m_{\rho^n}^2}$$

"vector meson dominance"

[Gell-Mann –Zachariasen 1961]

[Sakurai 1969]

- charge radius

$$F_\pi(k^2) \simeq 1 - \frac{1}{6}\langle r^2\rangle^{\pi^\pm} k^2 + \cdots$$

$$\Rightarrow \quad \langle r^2\rangle^{\pi^\pm} = 6 \sum_{n\geq 1} \frac{g_{\rho^n} g_{\rho^n \pi\pi}}{m_{\rho^n}^4} \simeq 11.0\, M_{\mathrm{KK}}^{-2}$$

If we fix M_{KK} by fitting the ρ meson mass,
we obtain

$$\langle r^2\rangle^{\pi^\pm} \simeq (0.690\ \mathrm{fm})^2 \quad \text{(our model)}$$
$$\simeq (0.672\ \mathrm{fm})^2 \quad \text{(experiment)}$$

★ WZW term

- We can calculate the intrinsic parity violating interactions from the CS–term.

Working in the $A_z = 0$ gauge, we obtain

$$S^{\rm CS}_{\rm D8} \simeq -\frac{N_c}{48\pi^2}\int_4 Z - \frac{N_c}{240\pi^2}\int_5 {\rm Tr}(gdg^{-1})^5 + \mathcal{O}\left(B^{(n)}\right)$$

$$\begin{aligned} Z = \; & {\rm Tr}[(A_- dA_- + dA_- A_- + A_-^3)(U^{-1}A_+U + U^{-1}dU) - {\rm p.c.}] \\ & + {\rm Tr}[dA_- dU^{-1} A_+ U - {\rm p.c.}] + {\rm Tr}[A_-(dU^{-1}U)^3 - {\rm p.c.}] \\ & + \frac{1}{2}{\rm Tr}[(A_- dU^{-1}U)^2 - {\rm p.c.}] + {\rm Tr}[UA_-U^{-1}A_+ dU dU^{-1} - {\rm p.c.}] \\ & - {\rm Tr}[A_- dU^{-1} U A_- U^{-1} A_+ U - {\rm p.c.}] + \frac{1}{2}{\rm Tr}[(A_- U^{-1}A_+U)^2] \end{aligned}$$

$$A_+ \leftrightarrow A_-, \quad U \leftrightarrow U^{-1}$$

→ reproduces the well–known formula. [Witten 1983]

★ including vector mesons in WZW term

Working in the $A_z \simeq \pi$ gauge, we obtain

$$S_{\mathrm{D8}}^{\mathrm{CS}} \simeq \frac{N_c}{4\pi^2 f_\pi^2} \int_4 \mathrm{Tr}\left[\pi\, dB^{(n)} dB^{(m)} c_{nm} + \mathcal{O}\left((B_\mu^{(n)})^3\right),\right]$$

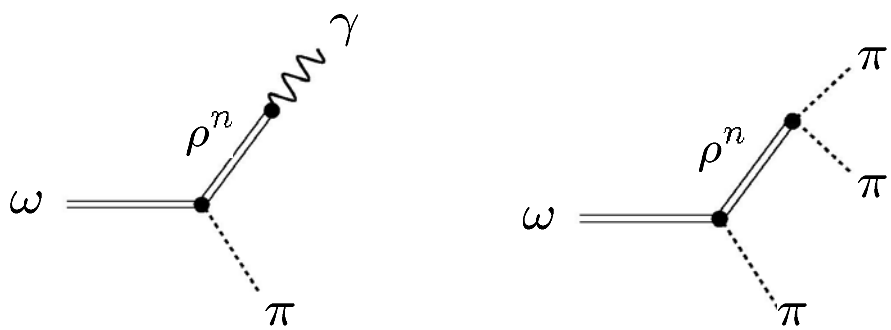
with $$c_{nm} = \frac{1}{\pi} \int dz\, K^{-1} \psi_n \psi_m$$

Moreover, one can show

- All the terms including $A_\pm$ are cancelled. (complete vector meson dominance)
- Terms with more than one pion field vanish.

★ $\omega \to \pi^0\gamma$ and $\omega \to \pi^0\pi^+\pi^-$

The relevant diagrams for these decay are



Exactly the same as the GSW model !

[Gell-Mann –Sharp–Wagner 1962]

- Furthermore, we find

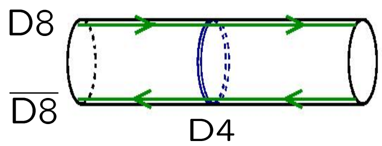
$$\Gamma(\omega \to \pi^0\gamma) = \frac{N_c^2}{3}\frac{\alpha}{64\pi^4 f_\pi^2}\left(\sum_{n=1}^{\infty}\frac{c_{1,2n-1}g_{\rho^n}}{m_{\rho^n}^2}\right)^2 |\mathbf{p}_\pi|^3 = \frac{N_c^2}{3}\frac{\alpha}{64\pi^4 f_\pi^2} {g_{\rho\pi\pi}}^2 |\mathbf{p}_\pi|^3$$

➡ reproduces the proposal given by Fujiwara et al !

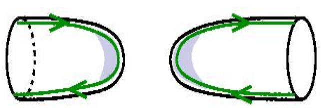
[Fujiwara–Kugo–Terao–Uehara–Yamawaki 1985]

Summary

String theory



Sugra + probe D8



$$S_{\text{D8}} \sim \kappa \int d^4x dz \,\text{Tr}\left(\frac{1}{2}K^{-1/3}F_{\mu\nu}^2 + KF_{\mu z}^2\right) + \frac{N_c}{24\pi^2}\int_5 \omega_5(A)$$

QCD

$$\mathcal{L} = -\frac{1}{4g^2}\text{Tr}\, F_{\mu\nu}^2 + i\overline{\psi}_i \not{D}\psi^i$$

Hadron physics

$q\overline{q}$ qqq

$\pi, \rho, a_1, \cdots$ $p, n, \cdots$