HETEROTIC STRINGS AND FLUXES

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Strings 06, Beijing
Overview

- Review of torsional constraints
- M-theory dual
- FSY geometries
- Explicit examples and gauge fields
- Moduli stabilization
- What’s next…

K.B., M. Becker, J.X.Fu, L.S.Tseng and S-T.Yau, to appear


K.B. and K.Dasgupta, ``Heterotic strings with torsion,'' hep-th/0209077

K.B., M. Becker, P.S.Green, K.Dasgupta and E.Sharpe, ``Compactifications of heterotic strings on non-Kaehler complex manifolds, hep-th/0310058

- J. X. Fu and S-T.Yau, ``The theory of superstring with flux on non-Kaehler manifolds and the complex Monge-Ampere equation,'' hep-th/0604063

- K.B. and L.S. Tseng, ``Heterotic flux compactifications and their moduli,'', hep-th/0509131
Review of torsional constraints

\[ ds^2 = ds_4^2 + ds_X^2 \]

Properties of $X$

1) Complex and hermitian: \( J_{\bar{a}b} = ig_{\bar{a}b} \)

2) Conformally balanced: \( d \left( \| \Omega \|^2 J \wedge J \right) = 0 \)

3) Hermitian Yang-Mills: \( F_{2,0} = F_{0,2} = F_{ab}J^{ab} = 0 \)

4) Anomaly cancellation: \( i \partial \bar{\partial} J = \frac{\alpha'}{8} \left( tr R \wedge R - tr F \wedge F \right) \)

\[ H = i \left( \bar{\partial} - \partial \right) J \quad \phi = \phi_0 - \frac{1}{2} \log \| \Omega \| \]
M-theory dual

M-theory on $K3 \times K3$ $\rightarrow$ M-theory on $K3 \times T^4 / \mathbb{Z}_2$ $\rightarrow$ Type IIB on $K3 \times T^2 / \mathbb{Z}_2$

where $\mathbb{Z}_2 = \Omega(-1)^{F_L} I_{89}$

$\uparrow$

Type I on $K3 \times f_7 T^2$ $\leftarrow$ Heterotic on $K3 \times f_7 T^2$
M-theory Bianchi identity

\[ 0 < N + \frac{1}{2} \int_{CY_4} G \wedge G = \frac{\chi}{24} \]

\[ G = \ast G \]

Non-trivial fluxes exist on the heterotic side only if the background is a \( T^2 \) bundle over a K3 base while a \( T^4 \) base leads to trivial solutions only...
FSY Geometries

\[ ds^2 = e^{2\phi} \underbrace{ds^2}_{\text{Base}} + (dx + \alpha_1)^2 + (dy + \alpha_2)^2 \]

\[ \alpha = \alpha_1 + i\alpha_2 \]

\[ \omega = \omega_1 + i\omega_2 = d\alpha \]

\[ \omega \wedge J_s = 0 \]
1) **Complex and Hermitian**

\[ J = e^{2\phi} J_S + \frac{i}{2} \theta \wedge \bar{\theta} \]

Kaehler form on the base

\[ \theta = dz + \alpha \]

2) **Holomorphic 3-form**

\[ \Omega = \Omega_S \wedge \theta \]

holomorphic (2,0) form on S

3) **Stable bundles**

Conformally balanced
4) **Anomaly cancellation**

\[ i \partial \bar{\partial} J = \frac{\alpha'}{8} (tr R \wedge R - tr F \wedge F) \]

\[
\frac{2i}{\alpha'} \partial \bar{\partial} e^{2\phi} \wedge J^s - \partial \bar{\partial} \left[ e^{-2\phi} tr \left( \bar{\partial} B \wedge \partial B^\dagger g_s^{-1} \right) \right] - 8 \partial \bar{\partial} \phi \wedge \partial \bar{\partial} \phi + \psi J_s^2 / 2 = 0
\]

\[ \bar{\partial} B = \omega_A \]

**source term**

\[
\psi J_s^2 = \frac{1}{\alpha'} \left( \| \omega_s \|^2 + \| \omega_A \|^2 \right) J_s^2 - \frac{1}{2} \left( tr R_s \wedge R_s - tr F \wedge F \right)
\]

1) \( \int_{K3} \psi = 0 \quad \implies \quad \text{Topological constraint} \)

2) Solution space is finite \( \implies \text{Elliptic condition.} \)
Explicit examples and gauge fields

\[ c_2(E) - \frac{1}{2} c_1^2(E) + \int (\| \omega_S \|^2 + \| \omega_A \|^2) \frac{J_s^2}{2} = 24 \]

**Gauge fields**

\[
\begin{align*}
\omega_S &= m \Omega_S \\
\omega_A &= \sum_{I=1}^{19} n^I K_I
\end{align*}
\]

1) \( F_{2,0} = F_{0,2} = F_{a\bar{b}} J^{\bar{a}b} = 0 \) \( \iff \) stable bundles

2) \( F \) has no components along the fiber. Mostly we will work with stable bundles on K3

\[ \dim M = 2rc_2(E) - (r - 1)c_1^2(E) - 2r^2 + 2 \geq 0 \]
Example: the $E_8 \times E_8$ heterotic with an $SU(N)$ bundle.

\[ c_2(E) + 4(m_1^2 + m_2^2) - \sum_{I,J} d_{IJ} n^I n^J = 24 \]

\[ c_2(E) \geq N - \frac{1}{N} \]

**\( N=2 \)\hfill E_8 \supset SU(2) \times E_7**

**\( N=3 \)\hfill E_8 \supset SU(3) \times E_6**
$N=4,5$

$E_8 \supset SU(4) \times SO(10)$

$E_8 \supset SU(5) \times SU(5)$

$(r, c_2(E)) = (4, 4)$

$(m_1, m_2) = (\pm 2, \pm 1)$

$\dim M = 2$

$248 = (15, 1) \oplus (1, 45) \oplus (4, 16) \oplus (\bar{4}, \bar{16}) \oplus (6, 20)$

$\dim M = 2$

$c_3(E) = 0$
Moduli Stabilization

deformations: \( J \rightarrow J + \delta J \)

\[
H = i \left( \bar{\partial} - \partial \right) J
\]

\[
d\delta J = 0
\]

\[
\omega \wedge J = 0 \quad \Rightarrow \quad \omega \wedge \delta J = 0
\]

Example:

\( \omega = nK^{(1,1)}_{-} \)

\( \omega = m\Omega_{S} \)

Superpotential

\[
W = \int (H + idJ) \wedge \Omega
\]
Conclusion

1) moduli fields

2) examples with no moduli

3) susic cycles

4) \[ ds^2_X = e^{2\phi} ds^2_{K3} + |dz + \alpha|^2 \]

M-theory on \( K3 \times K3 \)