HETEROTIC STRINGS AND FLUXES

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Strings 06, Beijing

Overview

- Review of torsional constraints
- M-theory dual
- FSY geometries
- Explicit examples and gauge fields
- Moduli stabilization
- What's next…

Literature

- K.B., M. Becker, J.X.Fu, L.S.Tseng and S-T.Yau, ``Anomaly cancellation and smooth non-Kaehler solutions in heterotic string theory," hep-th/0604137.
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- J. X. Fu and S-T.Yau, ``The theory of superstring with flux on non-Kaehler manifolds and the complex Monge-Ampere equation," hep-th/0604063
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Review of torsional constraints

$$ds^2 = ds_4{}^2 + ds_X^2$$

Properties of X

1) Complex and hermitian: $J_{a\bar{b}} = i g_{a\bar{b}}$

2) Conformally balanced: $d\left(\|\Omega\|^2 J \wedge J\right) = 0$

3) Hermitian Yang-Mills: $F_{2,0} = F_{0,2} = F_{a\bar{b}} J^{a\bar{b}} = 0$

4) Anomaly cancellation: $i\partial\bar{\partial}J = \frac{\alpha'}{8}\left(trR \wedge R - trF \wedge F\right)$

$$H = i\left(\bar{\partial} - \partial\right)J \qquad \phi = \phi_0 - \frac{1}{2}\log\|\Omega\|$$

M-theory dual

M-theory on $K3 \times K3$

M-theory on $K3 \times T^4 / \mathbb{Z}_2$

Type IIB on $K3 \times T^2 / \mathbb{Z}_2$ where $\mathbb{Z}_2 = \Omega(-1)^{F_L} I_{89}$

Type I on $K3 \times_f T^2$

Heterotic on $K3 \times_f T^2$

M-theory Bianchi identity

$$0 < N + \frac{1}{2}\int_{CY_4} G \wedge G = \frac{\chi}{24}$$

$$\Uparrow$$

$$G = {}^{*}G$$

Non-trivial fluxes exist on the heterotic side only if the background is a T^2 bundle over a K3 base while a T^4 base leads to trivial solutions only...

FSY Geometries

$$ds_X{}^2 = e^{2\phi}\,\underbrace{ds_S^2}_{Base} + \left(dx + \alpha_1\right)^2 + \left(dy + \alpha_2\right)^2$$

$$\alpha = \alpha_1 + i\alpha_2$$

$$\omega = \omega_1 + i\omega_2 = d\alpha$$

$$\omega \wedge J_S = 0$$

$$\frac{\omega_i}{2\pi\sqrt{\alpha'}} \in H^2\left(S, \mathbb{Z}\right)$$

1) Complex and Hermitian

$$J = e^{2\phi} J_S + \frac{i}{2}\theta \wedge \bar{\theta}$$

Kaehler form on the base

$$\theta = dz + \alpha$$

2) Holomorphic 3-form

$$\Omega = \Omega_S \wedge \theta$$

holomorphic (2,0) form on S

Conformally balanced

3) Stable bundles

4) <u>Anomaly cancellation</u> $$i\partial\bar{\partial}J = \frac{\alpha'}{8}\left(trR \wedge R - trF \wedge F\right)$$

$$\frac{2i}{\alpha'}\partial\bar{\partial}e^{2\phi} \wedge J_S - \partial\bar{\partial}\left[e^{-2\phi}tr\left(\bar{\partial}B \wedge \partial B^{\dagger} g_S^{-1}\right)\right] - 8\partial\bar{\partial}\phi \wedge \partial\bar{\partial}\phi + \boxed{\psi J_S^2/2} = 0$$

$$\boxed{\bar{\partial}B = \omega_A}$$

source term

$$\boxed{\psi J_S^2 = \frac{1}{\alpha'}\left(\|\omega_S\|^2 + \|\omega_A\|^2\right)J_S^2 - \frac{1}{2}\left(trR_S \wedge R_S - trF \wedge F\right)}$$

1) $\int_{K3}\psi = 0$ $\Longrightarrow$ <u>Topological constraint</u>

2) Solution space is finite $\Longrightarrow$ <u>Elliptic condition.</u>

Explicit examples and gauge fields

$$c_2(E) - \frac{1}{2}c_1^2(E) + \int \left(\| \omega_S \|^2 + \| \omega_A \|^2 \right) \frac{J_S^{\,2}}{2} = 24$$

$$\boxed{\omega_S = m\Omega_S}$$

$$\boxed{\omega_A = \sum_{I=1}^{19} n^I K_I}$$

<u>Gauge fields</u>

1) $F_{2,0} = F_{0,2} = F_{a\bar{b}} J^{a\bar{b}} = 0 \Longrightarrow$ stable bundles

2) F has no components along the fiber. Mostly we will work with stable bundles on K3

$$\dim M = 2rc_2(E) - (r-1)c_1^2(E) - 2r^2 + 2 \geq 0$$

Example: the $E_8 \times E_8$ heterotic with an SU(N) bundle.

$$c_2(E) + 4\left(m_1^2 + m_2^2\right) - \sum_{IJ} d_{IJ} n^I n^J = 24$$

$$c_2(E) \geq N - \frac{1}{N}$$

N=2 $\quad E_8 \supset SU(2) \times E_7$

N=3 $\quad E_8 \supset SU(3) \times E_6$

N=4,5

$$E_8 \supset SU(4) \times SO(10)$$

$$E_8 \supset SU(5) \times SU(5)$$

$\leftarrow$ **GUT**

$$(r, c_2(E)) = (4,4)$$

$$(m_1, m_2) = (\pm 2, \pm 1)$$

$$\dim M = 2$$

$$248 = \underbrace{(15,1)}_{\dim M \,=\, 2} \oplus (1,45) \oplus \underbrace{(4,16) \oplus (\bar{4},\overline{16})}_{c_3(E) = 0} \oplus (6,20)$$

Moduli Stablization

deformations: $J \rightarrow J + \delta J$

$$H = i\left(\bar{\partial} - \partial\right)J \quad \Longrightarrow \quad \boxed{d\delta J = 0}$$

$$+$$

$$\omega \wedge J = 0 \quad \Longrightarrow \quad \boxed{\omega \wedge \delta J = 0}$$

Example:

$$\omega = nK^{-}_{(1,1)}$$

$$\omega = m\Omega_S$$

Superpotential

$$\boxed{W = \int (H + idJ) \wedge \Omega}$$

Conclusion

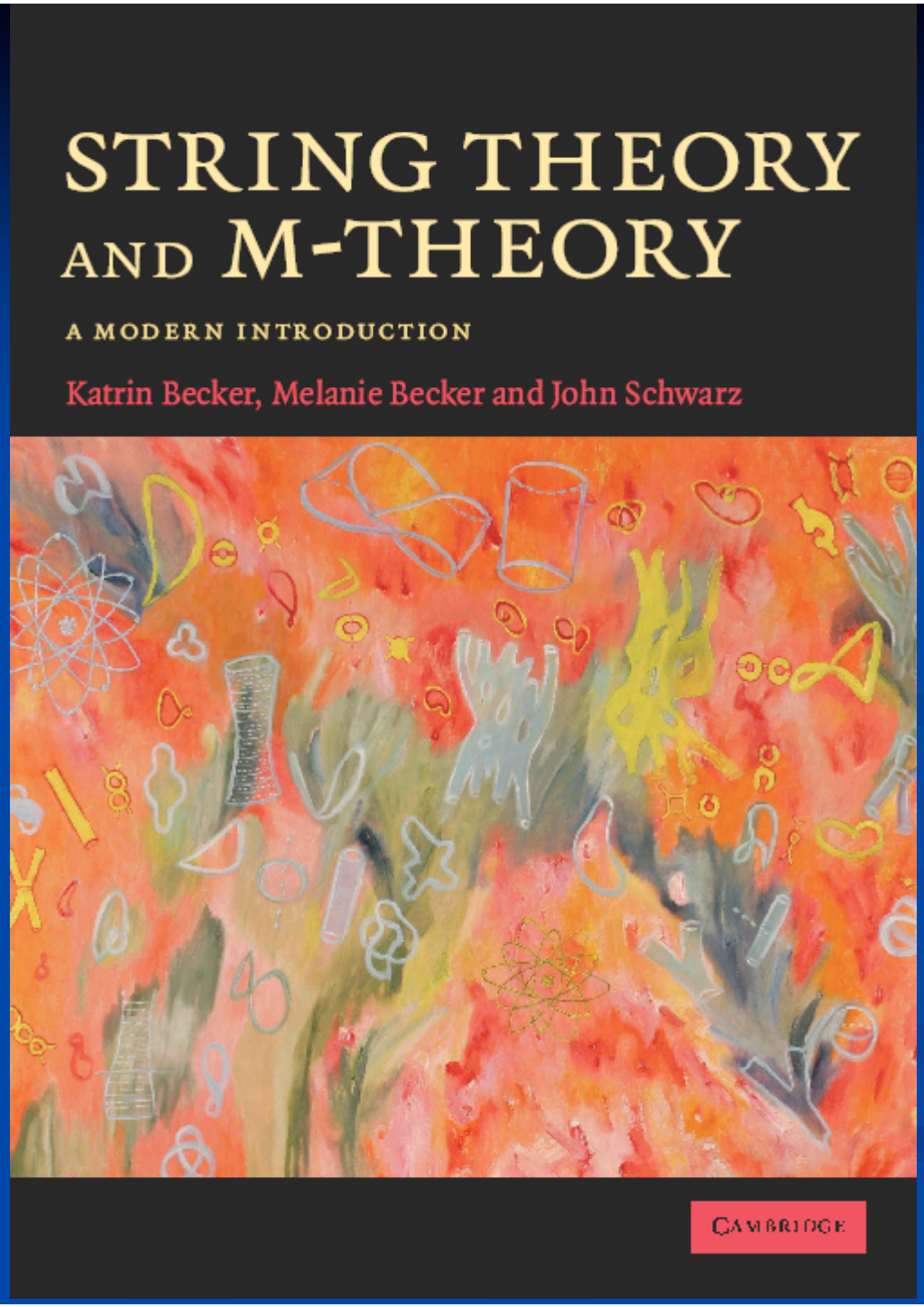
1) moduli fields

2) examples with no moduli

3) susic cycles

4) $$ds_X{}^2 = e^{2\phi} ds_{K3}^2 + |dz + \alpha|^2$$

M-theory on $K3 \times K3$


STRING THEORY
AND M-THEORY
A MODERN INTRODUCTION
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