Emergent Geometry:

Towards a proof of the AdS/CFT correspondence

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Strings 2006

Based on : hep-th/0507203

hep-th/0509015 with D. Correa, S. Vazquez

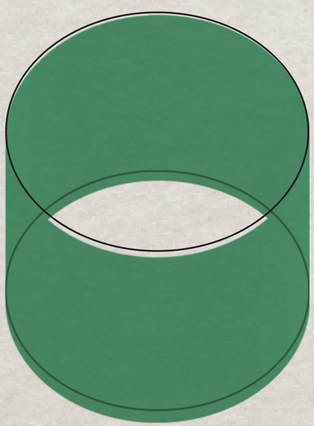
hep-th/0511104 with D. Correa, hep-th/0605220 with R. Cotta

Works in progress with R. Corrado, S. Vazquez

The AdS/CFT correspondence

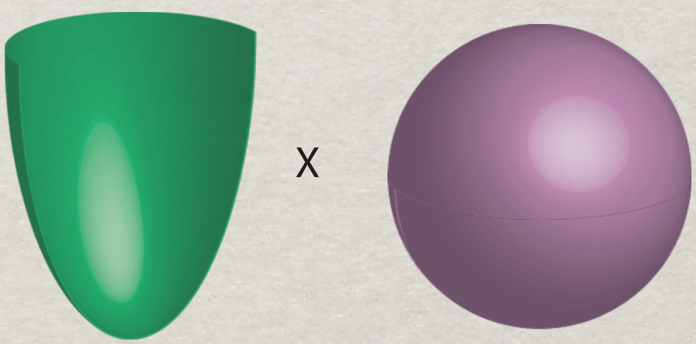
Conjectured exact quantum duality between:

N=4 SYM on a sphere



Type IIB Superstring on

$AdS_5 \times S^5$



We have a lot of reasons why the correspondence should work.

The big question is: how does it work?

More precisely: where does geometry come from?

This talk will present a proposal for the origin of geometry that passes many consistency tests and seems to give a good expansion of the N=4 SYM or other SCFT's at strong coupling.

What we're after

- Some background independence: we need to encode many geometries /topologies. Not just the ground state of the system.
- Focus on BPS dynamics to help tame quantum corrections.
- Universal description of the origin of strings for all states.

BPS approach

- 1/8 BPS states: Chiral ring dynamics.
- Leads to Improved effective low energy dynamics.

Want to study states that respect 1/8 SUSY (Chiral ring)

BPS bound: Energy = Angular momentum

Via operator state correspondence:

$$\mathcal{O}(0) \sim |\mathcal{O}\rangle$$

Any local gauge invariant operator is constructed from traces of fields and derivatives.

$$\partial^{[n]}\phi^i \sim a^\dagger_{i,[n]}$$

Spherical harmonic expansion on sphere

In free field limit all states that correspond to chiral ring can be written in terms of the 3 s-wave complex scalar components: X, Y, Z

This is like dimensional reduction on sphere.

BPS argument: dimensional reduction is an accurate description of the BPS dynamics (even at strong coupling). Semiclassical is often exact.

$$Z^j = X^{2j-1} + iX^{2j}$$

$$S_{sc} = \int dt\, \mathrm{tr} \left(\sum_{a=1}^{6} \frac{1}{2}(D_t X^a)^2 - \frac{1}{2}(X^a)^2 - \sum_{a,b=1}^{6} \frac{1}{8\pi^2} g_{YM}^2 [X^a, X^b][X^b, X^a] \right)$$

Contributes to E but not to J

Set it to zero by hand (BPS condition)

Now we want to quantize these configurations

Effective dynamics is a gauged Matrix quantum mechanics of commuting matrices

$$[X^i, X^j] = 0$$

(Minisuperspace approximation)

Can diagonalize all matrices simultaneously, by gauge transformations.

Associate a 6-vector per eigenvalue $\vec{x}_j \simeq (X^i_{jj})$

Eigenvalues are coordinates of particles (a la BFSS)

Classically the eigenvalue dynamics is free.

Going to eigenvalue vatiables is like going to spherical coordinates.

Measure term from going to eigenvalue basis that affects the effective laplacian (angular variables are dropped)

$$\mu^2 = \prod_{i<j} |\vec{x}_i - \vec{x}_j|^2$$

$$H = \sum_i -\frac{1}{2\mu^2}\nabla_i \mu^2 \nabla_i + \frac{1}{2}|\vec{x}_i|^2$$

Quantum truncation of classical H to mini-superspace

- Terms we set to zero (commutators) are D,F terms of potential. This means we are reducing to dynamics on moduli space of vacua.

- This is well know to be given by N particles in 6 flat dimensions. (Lessons from M(atrix) theory)

Eigenvalue Distributions:

One can find ground state, and absorb square root of the measure in wave functions:

~ free fermion description of hermitian matrix models

$$\psi_0 \sim \exp(-\sum \vec{x}_i^2/2)$$

$$\hat{\psi} = \mu\psi$$

$$|\hat{\psi}_0^2| \sim \mu^2 \exp(-\sum x_i^2) = \exp\left(-\sum \vec{x}_i^2 + 2\sum_{i<j} \log|\vec{x}_i - \vec{x}_j|\right)$$

Square of wave function tells us which configurations are dominant.

Interpret collection of eigenvalues as positions of particles in 6d.

N Bosons in 6d with logarithmic repulsive interactions.

$$|\hat{\psi}_0^2| \sim \mu^2 \exp(-\sum x_i^2) = \exp\left(-\sum \vec{x}_i^2 + 2\sum_{i<j} \log|\vec{x}_i - \vec{x}_j|\right)$$

We want to study the thermodynamics of this ensemble in the saddle point approximation.
(Large N limit, replace sums by integrals)
Introduce a density of bosons (eigenvalues)

Density of bosons is a singular configuration. Symmetries of ensemble suggest the following density of "eigenvalues"

$$\rho = N \frac{\delta(|\vec{x}| - r_0)}{r_0^{2d-1} Vol(S^{2d-1})}$$

We get a round five sphere

$$r_0 = \sqrt{\frac{N}{2}}$$

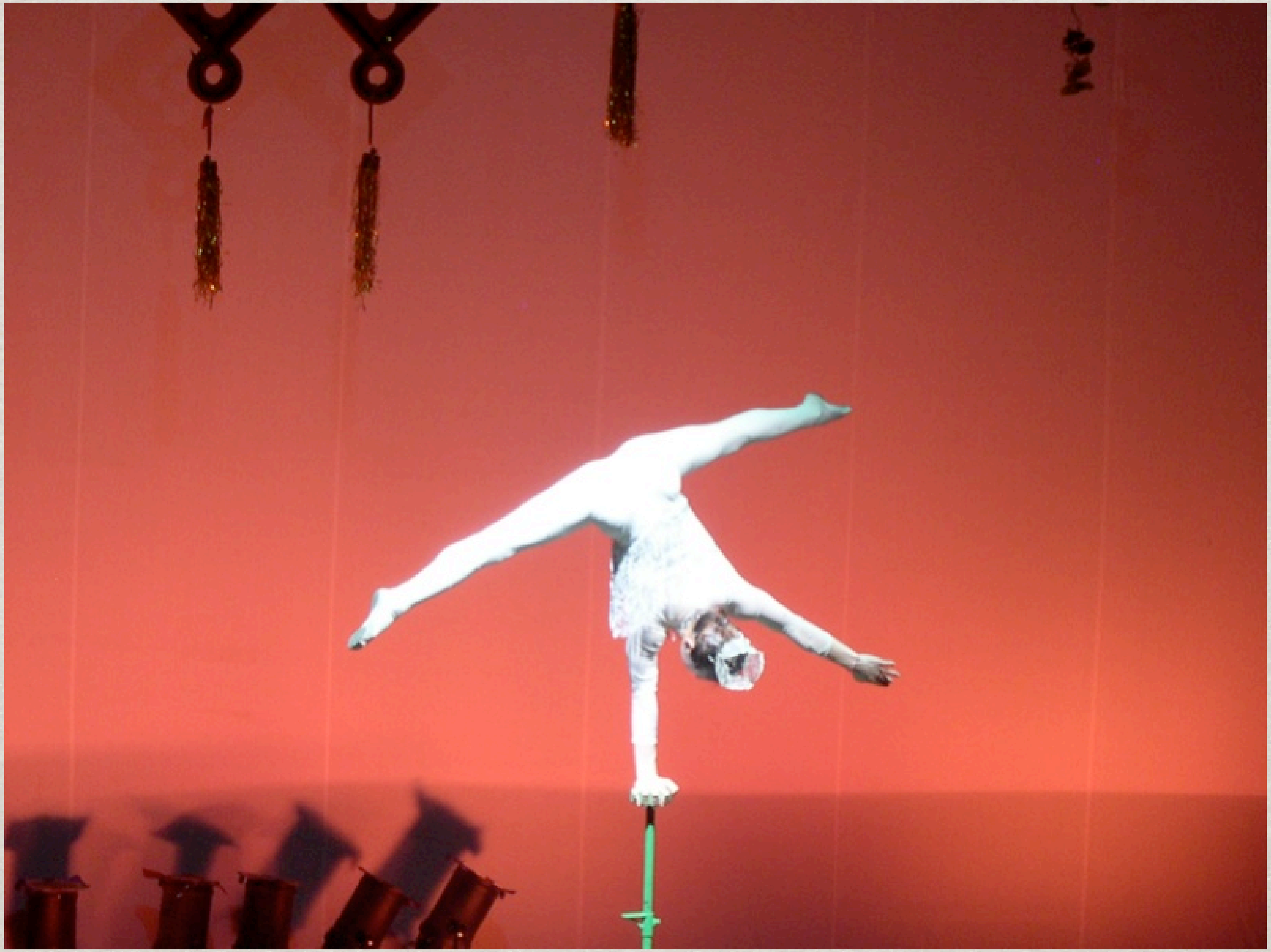
Other interesting wave functions: consider coherent states by traces of the complex commuting matrices (use AdS/CFT dictionary of Gubser, Klebanov, Polyakov, Witten). These just deform the distribution of eigenvalues.

- Deformation of geometry of eigenvalues parallels deformation of gravity by a classical “coherent state”.

- We identify the eigenvalue distribution with gravity. (Geometry is emergent)

- BPS wave functions are holomorphic, multiplying the ground state wave function.
- This is the same as holomorphic quantization of the moduli space of vacua (complex Kahler manifold)
- Different topologies of eigenvalue distributions lead to different spacetime topologies (explicit in Lin, Lunin, Maldacena case)
- Generalizes to many other SCFT's

Hope something special happens



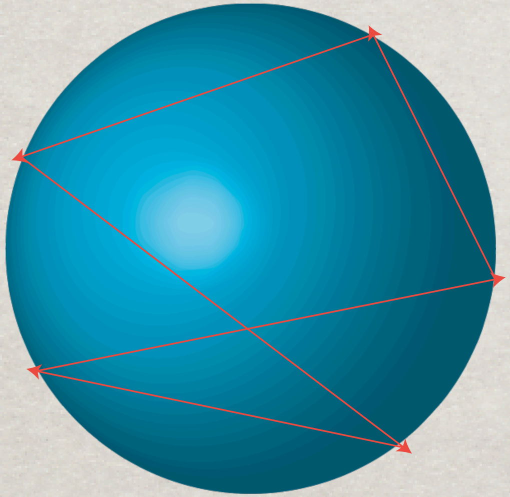
Approximations that lead to commuting matrices improve at strong coupling! Off-diagonal modes become heavy.

$$S_{sc} = \int dt\, \mathrm{tr} \left(\sum_{a=1}^{6} \frac{1}{2}(D_t X^a)^2 - \frac{1}{2}(X^a)^2 - \sum_{a,b=1}^{6} \frac{1}{8\pi^2} g_{YM}^2 [X^a, X^b][X^b, X^a] \right)$$

$$E_{osc} = \sqrt{1 + \frac{1}{2\pi^2} g_{YM}^2 |\vec{x}_j - \vec{x}_{j'}|^2}$$

With the typical radius, the energies scale like square root of the ‘t Hooft coupling. Diagonal modes are slow degrees of freedom in a Born-Oppenheimer approx.

Can also suggest origin of string scale: off-diagonal modes are massive and can be represented by lines joining eigenvalues (points on the sphere): STRING BITS. Need to dress them with gravity (eigenvalues).



One can also verify string tension
(D.B, D. Correa, S. Vazquez)

One can calculate string energies for BMN states:

$$O_k \sim \sum_{l=0}^{J} \exp(ikl/J)\mathrm{tr}(Z^{l-1}[Y,Z]Z^{J-l-1}[X,Z])$$

Diagonal modes

Off-diagonal modes

One treats the off-diagonal modes as free fields.

One calculates the energy of the BMN state assuming the off-diagonal modes don't affect the diagonal ones to first order.

The calculation of energies can be done in a saddle point and stationary phase approximation.

$$E \sim \frac{\langle\psi_k|H^{total}|\psi_k\rangle}{\langle\psi_k|\psi_k\rangle}$$

Details of state description

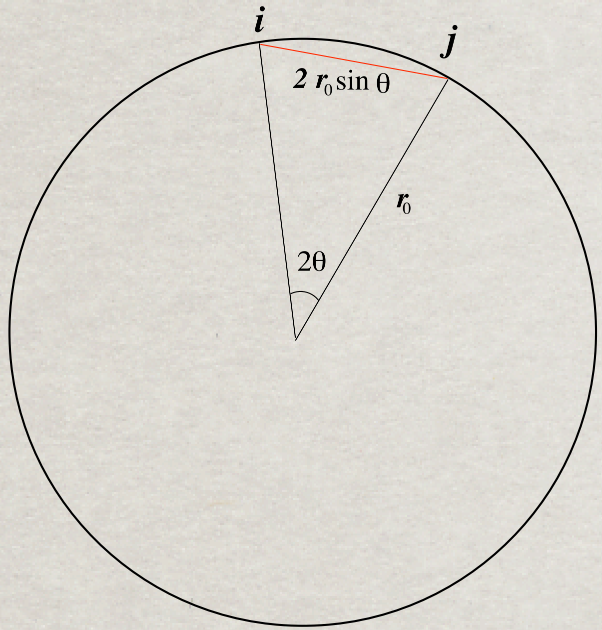
$$|\psi_k\rangle \sim \sum_{l=0}^{J} \exp(ikl/J) \sum_{j,j'} z_j^l Y_{j'}^{\dagger j} z_{j'}^{J-l} X_j^{\dagger j'} \hat{\psi}_0 |0\rangle_{od} \, ,$$

Manipulations of formulae lead to the following expression

$$\langle E^{osc}\rangle = \frac{\int \prod dx^i |\hat{\psi}_0|^2 \sum_{j,j'} |\sum_l \exp(ikl/J) z_j^l z_{j'}^{J-l}|^2 2\sqrt{1+\frac{g_{YM}^2}{2\pi^2}|\vec{x}_j - \vec{x}_{j'}|^2}}{\int \prod dx^i |\psi_0|^2 \sum_{j,j'} |\sum_l \exp(ikl/J) z_j^l z_{j'}^{J-l}|^2} \, .$$

Localizes on sphere

Localizes on |z| maximal (diameter), and particular phase between z, z'



Result localizes to a string bit
of particular fixed length

$$\theta = k/2J$$

$$\langle E^{osc} \rangle = 2\sqrt{1 + \frac{g_{YM}^2 N}{\pi^2} \sin^2(k/2J)}$$

Matches BMN string calculation to all orders in the 't Hooft coupling exactly, by taking J large k fixed. (Small angles)

- Matches the formula of Santambrogio and Zanon for all loop anomalous dimensions.

- It also matches the conjectures from the ``all loop Bethe ansatz" dispersion relation (Arutyunov, Beisert, Dippel, Staudacher) (Ref. Staudacher talk)

- Surprisingly, this also matches the recent classical Nambu-Goto calculation (Hofman, Maldacena), including the full geometrical interpretation. (Ref. Maldacena talk)

- The origin of strings by string-bits is robust for other BPS geometries (different eigenvalue distributions)

- Incomplete picture because off-diagonal modes are treated as free fields.

- Suggests breakdown of all-loop Bethe ansatz for small J, but it's not clear if it happens and where it happens. Number of magnons ~ number of off-diagonal modes.

- One should be able to change the number of string bits to make smooth slow strings moving on the sphere (or other geometries)

Strategy for other backgrounds AdS x X

Study ``Chiral ring states" in the dual CFT. These are half BPS in N=1

Argue that F and D terms vanish.
Checked explicitly for some orbifolds in different works with D. Correa, R. Cotta.

Generic reduction of BPS dynamics to the moduli-space of vacua of the SCFT

This moduli space problem gives M points on a CY cone geometry. One can try to write an effective theory of particles in this geometry.

There is an effective repulsion between the particles, just as in N=4 SYM. This is because of "volume of the gauge orbit effect". This also produces repulsion from tip of cone (another degeneration of gauge orbit). Same ideas as in N=4 apply.

The important regime for the dynamics of (short, low energy) string bits is when two D-branes collide away from the tip of the cone.
This low energy effective dynamics is in the same universality class as N=4 SYM.

One can imagine the same type of description of the BPS geometries: holomorphic quantization of the moduli space multiplying gound state wave function.

Check: Chiral ring is exactly the holomorphic coordinate ring of the moduli space of vacua.

One can do other checks, get many expected results.

Conclusion

- Reduction in low energy degrees of freedom to eigenvalues.
- Non-trivial repulsion of eigenvalues gives geometry as thermodynamic saddle point of wave function.
- Off-diagonal modes give string bits for all geometries.
- The approach gives a strong coupling expansion that is quantitative (can reproduce giant magnons, BMN limits to all order in 't Hooft coupling).
- If AdS/CFT true for N=4 SYM, "the other cases follow"