Transplanckian Physics and the CMB: A Status Report

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Basic Questions

- We all hope LHC will give insight into string theory.
- But is it our only hope for contact with data?
- Can string/planck-scale physics leave an imprint on the CMB?
- Can observations of CMB give insight/information about string/planck-scale physics?
Why Think Possible?

- Expansion Factor at least \((e^{60})(10^{28}) = 10^{54}\)
  Size of Universe = \(10^{10}\) light-years = \(10^{61}\) Planck Lengths

- Ripples we see began sub-Planckian; Planckian imprint?

- Only way to know:
  Calculate perturbations standard way.
  See if string/quantum gravity modifies in significant way.
Outline

I. Brief Review of Standard Calculation.

II. Possible Modifications; Affect on CMB.

III. Observational opportunities.
I. Spectrum: Standard Results

- Perturb metric/scalar fields.

- Calculate equation of motion.

- Calculate power in quantum fluctuations: 2-pt function
I. Spectrum: Standard Results

- Consider Tensor Modes: Perturb Metric.
  \[ ds^2 = a^2(\eta)(d\eta^2 - (\delta_{ij} + h_{ij})dx^i dx^j) \]

- Expand in Traceless Symmetric Modes:
  \[ h_{ij}(\eta, x) = h_+ e_{ij}^+ + h_\times e_{ij}^\times \]

- Equations of Motion:
  \[ u_k'' + (k^2 - a''/a)u_k = 0 \]
  \[ (u_k = a(\eta)h_k) \]
I. Spectrum: Standard Results

- Quantum Mechanics:

\[ \hat{u}(\eta, x) = 1/(2\pi)^{3/2} \int d^3 k [\hat{a}_k(\eta) e^{ikx} + \hat{a}_k^+(\eta) e^{-ikx}] \]

Express operators in terms of basis at a fixed reference time.

\[ \hat{a}_k(\eta) = \alpha_k(\eta)\hat{a}_k(\eta) + \beta_k(\eta)\hat{a}_{-k}^+(\eta) \]

\[ \hat{a}_k(\eta) \rightarrow (\alpha_k + \beta_k^*)(\eta)\hat{a}_k(\eta) = u_k(\eta)\hat{a}_k(\eta) \]

- Measure of Fluctuation: Power Spectrum/Two Point Fcn

\[ P(k) = \frac{k^3}{2\pi^2 a^2} \langle 0 | \hat{u}_k(\eta)\hat{u}_k^+(\eta) | 0 \rangle_{\text{LateTime}} = \frac{k^3}{2\pi^2} \left| \frac{u_k}{a} \right|^2 \]
I. Spectrum: Standard Results

- Tensor Perturbations:
  \[ u_k'' + \left( k^2 - \frac{a''}{a} \right) u_k = 0 \]

- Power Spectrum:
  \[ P^{1/2}(k) = \sqrt{\frac{k^3}{2\pi^2}} \left| \frac{u_k}{a} \right|_{k=aH} \]

- Boundary Conditions: Arbitrarily early times/short scales modes behave as in Minkowski space:
  \[ u_k(\eta) \rightarrow \frac{1}{\sqrt{2k}} e^{-ik\eta} \]
I. Spectrum: Standard Results

- Mode Solutions:

- Early time: $a''/a \ll k^2 \rightarrow$ Oscillation.
- Late time: $a''/a \gg k^2 \rightarrow$ Freeze out.
I. Spectrum: Standard Results

Density Perturbations: CMB
I. Spectrum: Standard Results

- **Tensor Perturbations:**
  \[ u_k'' + \left( k^2 - \frac{a''}{a} \right) u_k = 0 \]

- **Power Spectrum:**
  \[ P^{1/2}(k) = \frac{k^3}{2\pi^2} \left| \frac{u_k}{a} \right|_{k=aH} \]

- **Boundary Conditions: Early times/short scales—Minkowskian:**
  \[ u_k(\eta) \to \frac{1}{\sqrt{2k}} e^{-ik\eta}, \eta \to -\infty \]

- **Assumption: Standard QFT applies unmodified on arbitrarily short scales. What if it doesn’t?**
II. Imprints of modified Short Scale Physics?

- No complete, first principles calculation.
- Parameterize Possibilities via:
  - Modified Dynamics
  - Modified Boundary Conditions
II. SHORT SCALE IMPRINTS

A Brief History

- **Modify Dispersion Relations:**

- **Short Scale Noncommutativity:**
  - Chu, BRG, Shiu hep-th/0011241
  - Lizzi, Mangano, Miele, Peloso hep-th/0203119
  - Tsujikawa, Maartens, Brandenberger hep-th/0307016

- **Introduce minimum length:**
  - Kempf and Niemeyer astro-ph/0103225
  - Easther, BRG, Shiu, Kinney hep-th/0104102, hep-th/0110226

- **Higher Order Operators in Dynamics:**
  - Kaloper, Kleban, Lawrence, Shenker hep-th/0201158
  - Shiu, Wasserman hep-th/0203113
II. SHORT SCALE IMPRINTS
A Brief History

- **Modify Short Scale Boundary Conditions:**
  - Easther, BRG, Shiu, Kinney hep-th/0104102, hep-th/0110226
  - U. Danielsson, hep-th/0203198
  - Easther, BRG, Kinney, Shiu hep-th/0204129
  - Goldstein, Lowe hep-th/0208167
  - Martin, Brandenberger hep-th/0305161

- **Controlled Effective Field Theories:**
  - Burgess, Cline, Lemieux, Holman hep-th/0210233
  - Burgess, Cline, Holman hep-th/0306079
  - Kaloper, Kaplinghat hep-th/0307016
  - Schalm, Shiu, Van der Schaar hep-th/0401164
  - BRG, Schalm, Shiu, Van der Schaar hep-th/0411217
  - BRG, Parikh, Van der Schaar, JHEP 0604 (2006) 057
II. Philosophy

- **Question**: How far do we need to deviate from conventional physics to yield a (potentially) observable imprint on the CMB?

- **Question**: Is the required deviation remotely sensible?

- **Related Approach**: Let CMB speak for itself.
II. Parameterized Modifications: Representative Examples

I. Modify Dynamics and Boundary Conditions: (Easther, BRG, Kinney, Shiu)

- Dynamics: String Uncertainty principle.
  \[ [x, p] = i \frac{h}{2\pi} (1 + (L_{\text{string}}^2) p^2); \Delta x_{\text{min}} \sim L_{\text{string}} \]

- Boundary Conditions: Modes physical iff wavelength larger than string length. Set boundary conditions there. Choose state of zero particle number wrt pos/neg frequency solutions to new EOM.
- Found: \(O(H/M)\) modulation to \(P(k)\).

II. Modify Only Boundary Conditions: (Danielsson)

- Boundary Conditions: Same prescription as I, wrt pos/neg frequency solutions to standard EOM.
- Found: \(O(H/M)\) modulation to \(P(k)\). Same origin as I; easier to analyse.

III. Modify Only Dynamics: (Kaloper, Kleban, Lawrence, Shenker)

- Dynamics: Include next order operators in EFT derivative expansion.
- Found: \(O((H/M)^2)\) shift to \(P(k)\). (Smaller effect; no iconic modulation.)
II. Parameterized Modifications:

Representative Examples: Estimates

- **I. / II. $O(H/M)$:**
  - $a_{\text{NEW}} = a_{\text{ORIGINAL}} + O(H/M) a_{\text{ORIGINAL}}$
  - $P(k)$: Cross term in 2-point function picks up $O(H/M)$ contribution.
  - Generic modification to vacuum/boundary data (will return to this).

  **For realistic $H/M$, potentially observable.**

- **III. $O((H/M)^2)$:**
  - $P(k)$ picks up $O((H/M)^2)$ contribution (will return to this).

  **For realistic $H/M$, unlikely to be observable.**
II. Calculating Modifications:
Boundary Conditions

- **Usual Case:**
  \[ \tilde{u}(\eta, x) = 1/(2\pi)^{3/2} \int d^3k [\tilde{a}_k(\eta)e^{ikx} + \tilde{a}_k^+(\eta)e^{-ikx}] \]
  \[ \tilde{a}_k(\eta) = \alpha_k(\eta)\tilde{a}_k(\eta_i) + \beta_k(\eta)\tilde{a}_k^+(\eta_i) \]
  \[ \tilde{a}_k(\eta) \rightarrow (\alpha_k + \beta_k^*)(\eta)\tilde{a}_k(\eta_i) = u_k(\eta)\tilde{a}_k(\eta_i) \]
  \[ \beta_k(\eta) = 0, \text{ as } \eta \rightarrow -\infty, u_k(\eta) \rightarrow \frac{1}{\sqrt{k}} e^{-ik\eta} \]

- **Modify:**
  \[ \beta_k(\eta_k) = 0, \text{ when } \frac{k}{a(\eta_k)} = M_{\text{string}} \Rightarrow \eta_k = -\frac{M}{Hk}; |A_k|^2 - |B_k|^2 = 1 \]
  \[ u_{\text{mod}}^k(\eta) = A_ku_k(\eta) + B_ku_k^*(\eta) \]
  \[ |A_k|^2 = (1 - (1 - 2ki\eta_k)^{-1})^{-1} \]

- **Spectrum:**
  DeSitter Space: Shift (Danielsson)
  General Slow Roll: Modulation (Easther, BRG, Kinney, Shiu)
  \[ P_k = \left(\frac{H}{2\pi}\right)^2 \left(1 - \frac{H}{M_{\text{st}}} \sin\left(\frac{2}{H/M_{\text{st}}}\right)\right) \]
II. Short Scale Modifications to CMB

- \( L_{\text{string}} = 100L_{\text{planck}} \), power law inflation, exponent = 500
- Roughly normalized to CMB
- Size of deviation: \( O\left( \frac{H}{M_{\text{string}}} \right) \)}
II. Impact on Observations

\[ \Delta C_l / C_l \]
II. Criticisms/Responses

- **Large backreaction I:** (Tanaka; Goldstein, Lowe):
  - Issue: $E(\text{fluctuations})$—relative to BD vacuum—on order of $V(\text{field})$.
  - Energy of modes with wavelengths less than Hubble radius, at end of inflation, relative to BD vacuum large—collapse.
  - Resolution: $(M_{\text{string}})^4 (H/M_{\text{string}})^2 < (M_{\text{planck}})^2 H^2$, i.e. $M_{\text{string}} < M_{\text{planck}}$

- **Large backreaction II:** (Kleban, Kaloper, Lawrence, Shenker, Susskind)
  - What are modes doing “before” creation? (Looking inside box.)
  - Need to modify modes up to $e^{65} M_{\text{string}}$. Large backreaction.
  - Responses: Only think outside the box; Ignorance of subplanck Physics; Degree of fine tuning; Look for signature.

- **Boundary Effective Field Theory:** Control Backreaction
II. Boundary Effective Field theory and the CMB

- **Encode Boundary Conditions in Boundary Effective Field Theory**
  (Schalm, Shiu, Van der Schaar; Poratti; BRG, Schalm, Shiu, Van der Schaar)

  - Boundary data: Specified on spacelike hypersurface, \( t = t_0 \), for all modes.
  - Previous approach: Specified on fixed energy surface:
    New Physics Hypersurface (NPH)—different times for diff modes.

- \( S = S_{\text{Boundary}} + S_{\text{BULK}} \)
  - Choose \( S_{\text{boundary}} \) to encode BD boundary conditions.
  - One-loop corrections:
    \( S_{\text{BULK}}: O((H/M)^2) \) (dim 6 op.)
    \( S_{\text{BOUNDARY}}: O(H/M) \). (3d QFT, dim 4 op.)

- Contribution to \( P(K) \):

\[
P(k) = P(k)_{BD} \left( 1 + \beta \frac{k}{a(t_0)M} \sin\left( \frac{2k}{a(t_0)H} \right) \right)
\]

\[
S_{\text{Bound}}^{BD} = \int_{t=t_0} d^3 x \sqrt{g}^{(3)} \left( -\frac{1}{2} \mu_{BD} \phi^2 \right)
\]

\[
\mu_{BD}^{Corrected} = \mu_{BD} + \frac{\beta}{M} \left( \frac{k^2}{a(t_0)^2} \right)
\]
II. Boundary Effective Field theory and the CMB

- Comparison:
  - Boundary EFT: explicit scale dependence (at fixed $t$, can distinguish diff k modes).
  - NPH: scale dependence only from background ($H(K)$)

- EFT: Larger k, larger amplitude (large k, closer to boundary).
- NPH: Larger k, smaller amplitude (large k, late exit, small H).

\[ P(k) = P(k)_{BD} \left( 1 + \beta \frac{k}{a(t_0)M} \sin \left( \frac{2k}{a(t_0)H} \right) \right) \]

\[ P(k) = P(k)_{BD} \left( 1 - \frac{H}{M_{st}} \sin \left( \frac{2}{H / M_{st}} \right) \right) \]
II. Parameterized Modifications:

Final Example: Generic Deviation from Thermality

(BRG, Parikh, Van der Schaar)

- **Black Holes are not precisely thermal** (Parikh, Wilczek)
  - Radiation constituents have $M < M_{\text{black hole}} \Rightarrow$ truncated thermal tail.
  - $T = T(r_{\text{horizon}})$; $r_{\text{horizon}}$ changes as radiation emitted $\Rightarrow$ violation of strict thermality.

- **BD Vacuum:**
  - BD vacuum *is* precisely thermal.
  - Physically: DeSitter Vacuum state **NOT** precisely thermal:
    - ✓ Truncated thermal tail no thermal emission with $M > M_{\text{nariai Black Hole}}$
    - ✓ DeSitter Horizon backreacts from emission as in black hole case.

\[
\frac{E}{T} = e^{-\frac{2\pi E}{H}} \to e^{\frac{2\pi E}{H}(1 + \frac{EH}{8\pi M_{\text{Planck}}^2})}
\]

- **Implication:** BD vacuum receives universal correction.

- **Result:**

\[
P_{BD}(k) \to P_{BD}(k)(1 + \frac{1}{4e^\pi}(\frac{H(k)}{M_{\text{Planck}}})^2)
\]
## II. Summary of Corrections

<table>
<thead>
<tr>
<th>Correction Type</th>
<th>Change to Spectrum</th>
</tr>
</thead>
<tbody>
<tr>
<td>New Physics Energy Hypersurface</td>
<td>$O(H/M_{\text{string}})$</td>
</tr>
<tr>
<td>Boundary Effective Field Theory</td>
<td>$O(H/M_{\text{string}})$</td>
</tr>
<tr>
<td>Bulk Effective Field Theory</td>
<td>$O((H/M_{\text{string}})^2)$</td>
</tr>
<tr>
<td>Universal Thermal Correction</td>
<td>$O((H/M_{\text{Planck}})^2)$</td>
</tr>
</tbody>
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III. OBSERVATIONS

- Can these corrections be seen/measured? (Bergstrom, Danielsson; Elgaroy, Hannestad; Okamoto, Lim; Martin, Ringeval; Verde, et. al.; Spergel, et. al.; Easther, Kinney, Peiris).

- Tight connection between observing tensor mode and TP Physics (Easther, Kinney, Peiris)

- Representative case (EKP): NPH
  - High Scale Inflation/Large Tensor Mode Signal ($r=.15$) Transplanck signature detectable for $H/M > .004$
  - Low Scale Inflation/Small Tensor Mode Signal ($r=.00013$) Transplanck signature detectable for $H/M > .03$
Conclusions

- Modifications to vacuum can yield effects sensitive to first order variation from standard vacuum choice.
- Conclusions sensitive to assumptions about initial conditions and overall model.
- Overall Point: Cosmological window on Planckian Physics is a promising alternative approach—by refining these rough ideas we just might one day make experimental contact.
- Stringy effects may be closer to the surface in cosmology than in particle physics.