Gauge/String Duality and D-Brane Inflation

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The AdS/CFT duality

Maldacena; Gubser, IK, Polyakov; Witten

- Relates 4-d conformal gauge theory to string theory on 5-d Anti-de Sitter space times a 5-d compact Einstein space. For the $\mathcal{N}$=4 SU(N) SYM theory this space is a 5-d sphere.
- When a gauge theory is strongly coupled, the radius of curvature of the dual AdS_5 and of the 5-d compact space becomes large:

$$\frac{L^2}{\alpha'} \sim \sqrt{g_{\rm YM}^2 N}$$

- String theory in such a weakly curved background can be studied in the effective (super)-gravity approximation, which allows for a host of explicit calculations. Corrections to it proceed in powers of

$$\frac{\alpha'}{L^2} \sim \lambda^{-1/2}$$

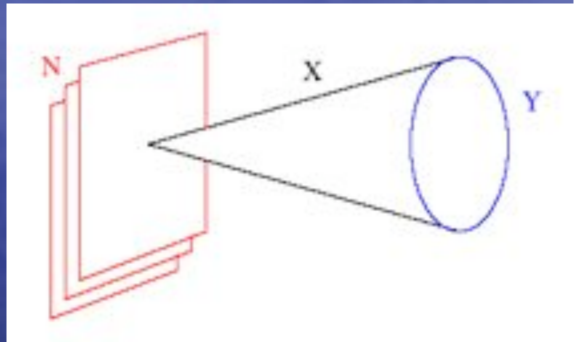
- Feynman graphs instead develop a weak coupling expansion in powers of λ. At weak `t Hooft coupling the dual string theory becomes difficult. It is expected that we need to solve a sigma model with R-R backgrounds on a highly curved space.

Can the closed string side simplify for some purposes at weak coupling ?

- My recent work with Dymarsky and Roiban reconsiders quiver gauge theory on a stack of D3-branes at the tip of a cone R^6/Γ where the orbifold group Γ breaks all the supersymmetry.
- At first sight, the gauge theory seems conformal because the planar beta functions for all single-trace operators vanish. The candidate string dual is $AdS_5 \times S^5/\Gamma$. Kachru, Silverstein; Lawrence, Nekrasov, Vafa; Bershadsky, Johanson
- However, dimension 4 double-trace operators made out of twisted single-trace ones, $f\, O_n\, O_{-n}$, are induced at one-loop. Their planar beta-functions have the form

$$\beta_f = a\,\lambda^2 + 2\,\gamma\, f\,\lambda + f^2$$

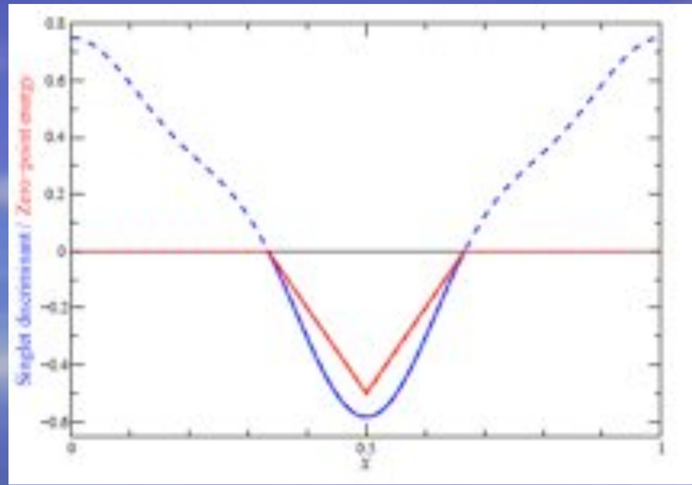
$$\beta_\lambda = 0$$



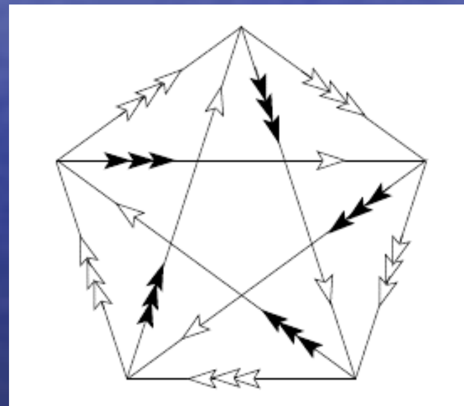
- If $D=\gamma^2 - a < 0$, then there is no real fixed point for f.
- A class of Z_k orbifolds with global SU(3) symmetry, that are freely acting on the 5-sphere, has the group action in the fundamental of SU(4)

$$r(g^n) = \mathrm{diag}(\omega_k^n, \omega_k^n, \omega_k^n, \omega_k^{-3n})$$

$$\omega_k = e^{i\alpha_k} \ , \qquad \alpha_k = \frac{2\pi}{k}$$



- Here is a plot of a one-loop $SU(N)^k$ gauge theory discriminant, D, and of the ground state closed string m^2 on the cone without the D-branes. n=1, …, k-1 labels the twisted sector, and x=n/k.
- The simplest freely acting non-susy example is Z_5 where there are four induced double-trace couplings



$$\delta_{2\ \mathrm{trace}}\mathcal{L} = f_{8,1} O_1^{\langle i\bar{j}\rangle} O_{-1}^{\langle j\bar{i}\rangle} + f_{8,2} O_2^{\langle i\bar{j}\rangle} O_{-2}^{\langle j\bar{i}\rangle} + f_{1,1} O_1 O_{-1} + f_{1,2} O_2 O_{-2}$$

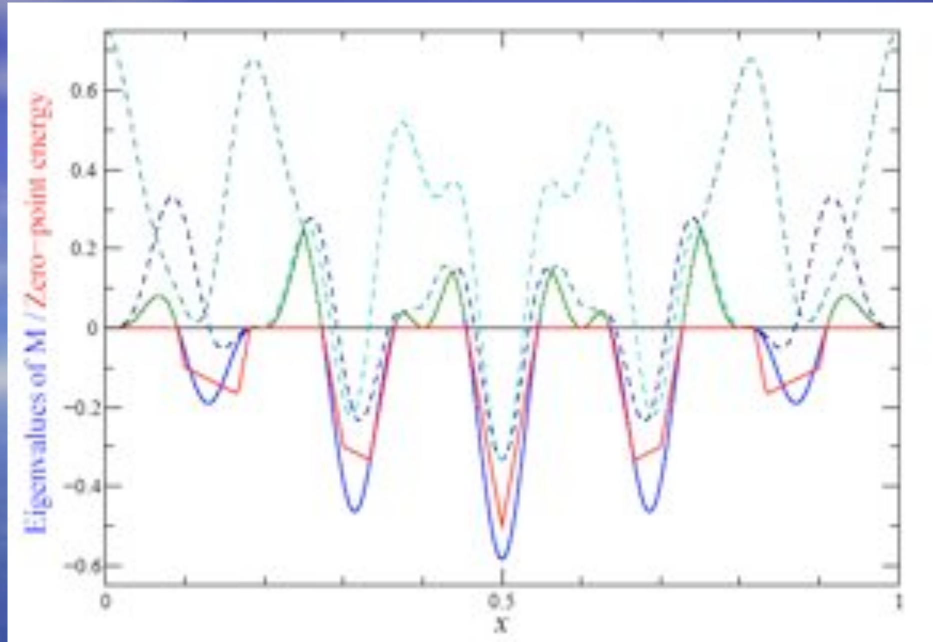
- For example, the SU(3) adjoints

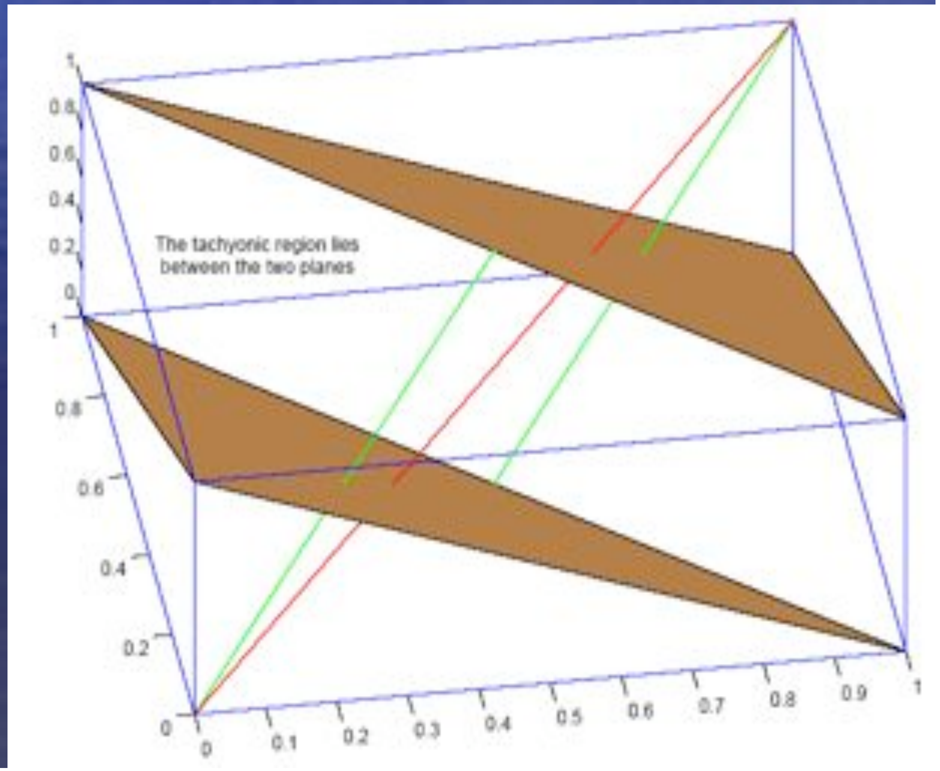
$O_n^{\langle i\bar{j}\rangle}$ are

$$\sum_{k=1}^{5} \left(\Phi^i_{k,k+2} \Phi^{\bar{j}}_{k+2,k} - \frac{1}{3} \eta^{i\bar{j}} \Phi^l_{k,k+2} \Phi^{\bar{l}}_{k+2,k} \right) e^{in\alpha(k-1)}$$

($\alpha=2\pi/5$)

- For more complicated orbifolds, crossing of eigenvalues of the discriminant matrix becomes important. The agreement with closed strings continues to hold.
- Generally, there are three twists that define a cube. The stability/instability regions agree between one-loop gauge theory and string theory.





- Any non-SUSY abelian orbifold contains unstable operators. This appears to remove all such orbifold quivers from a list of large N perturbatively conformal gauge theories.
- The one-loop beta functions destroy the conformal invariance precisely in those twisted sectors where there exist closed-string tachyons localized at the tip of R^6/Γ. Thus, a very simple correspondence emerges between perturbative gauge theory and free closed string on an orbifold. Why? Perhaps, in the presence of tachyons, the standard AdS/CFT decoupling argument may fail.
- The $AdS_5 \times S^5/\Gamma$ background is tachyon-free at large radius. Could it have some hidden instabilities? If not, then there is a transition from instability to stability as λ is increased.

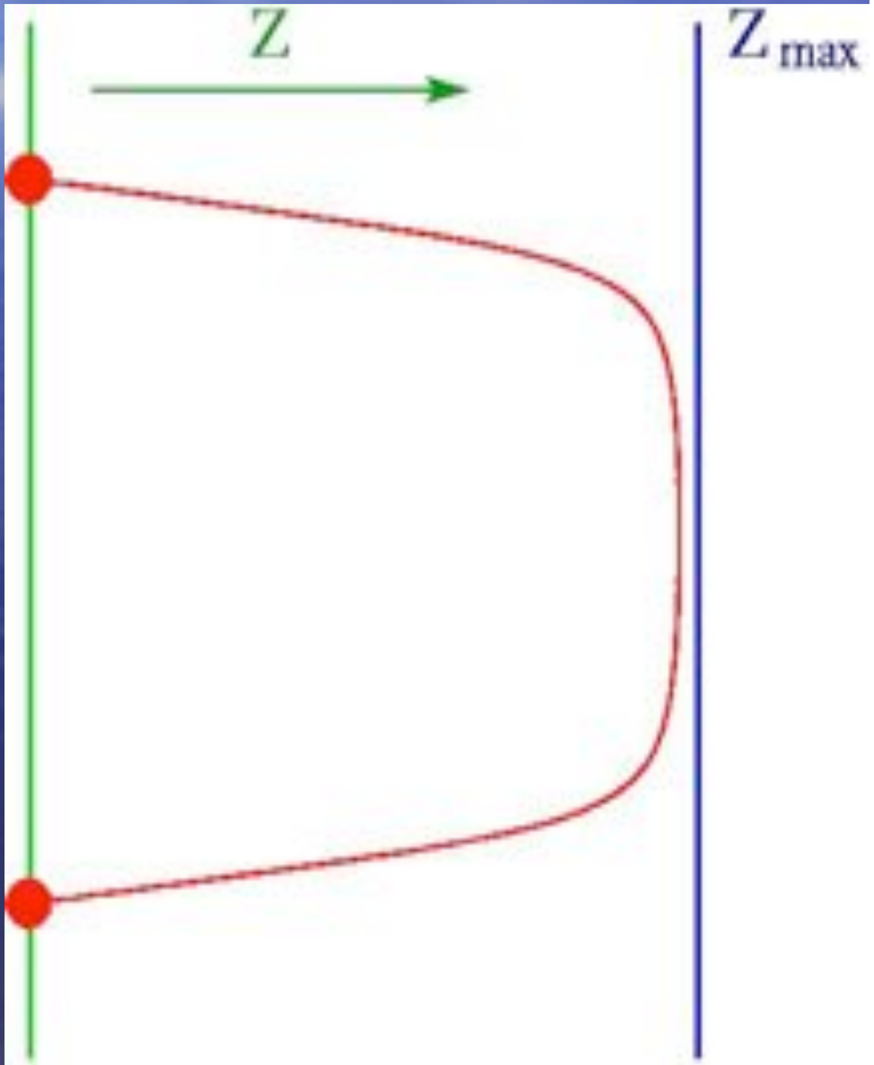
- What is the end-point of the RG flow?
- Condensation of localized tachyon smoothes out the tip of the cone. Adams, Polchinski, Silverstein
- The gauge theory on D3-branes at a smooth point is $\mathcal{N}$=4 SYM. Hence, a natural conjecture is that the gauge theory flows from the non-SUSY $SU(N)^k$ quiver gauge theory to the $\mathcal{N}$=4 SU(N) SYM. Dymarsky, Franco, Roiban, IK (work in progress)

String Theoretic Approach to Confinement

- It is possible to generalize the AdS/CFT correspondence in such a way that the quark-antiquark potential is linear at large distance.
- A "cartoon" of the necessary metric is

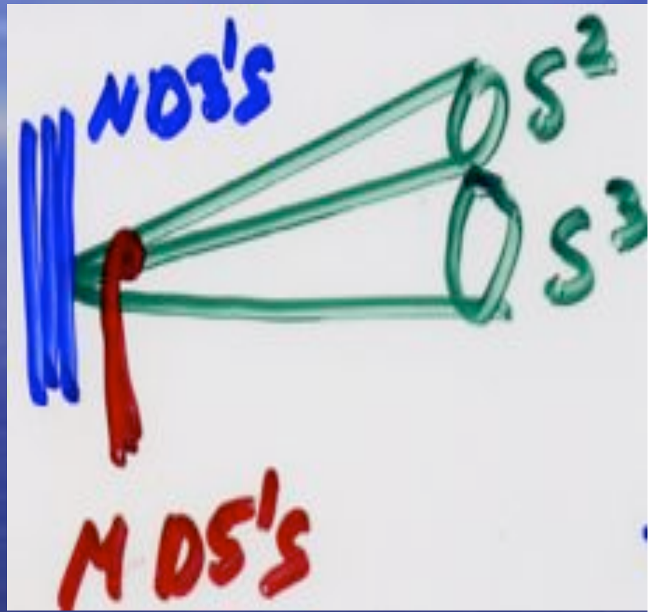
$$ds^2 = \frac{dz^2}{z^2} + a^2(z)\big(-(dx^0)^2 + (dx^i)^2\big)$$

- The space ends at a maximum value of z where the warp factor is finite. Then the confining string tension is $\frac{a^2(z_{\max})}{2\pi\alpha'}$



Confinement in SYM theories

- Introduction of minimal supersymmetry ($\mathcal{N}$=1) facilitates construction of gauge/string dualities.
- A useful tool is to place D3-branes and wrapped D5-branes at the tip of a 6-d cone, e.g. the conifold.
- The 10-d geometry dual to the gauge theory on these branes is the warped deformed conifold (IK, Strassler)

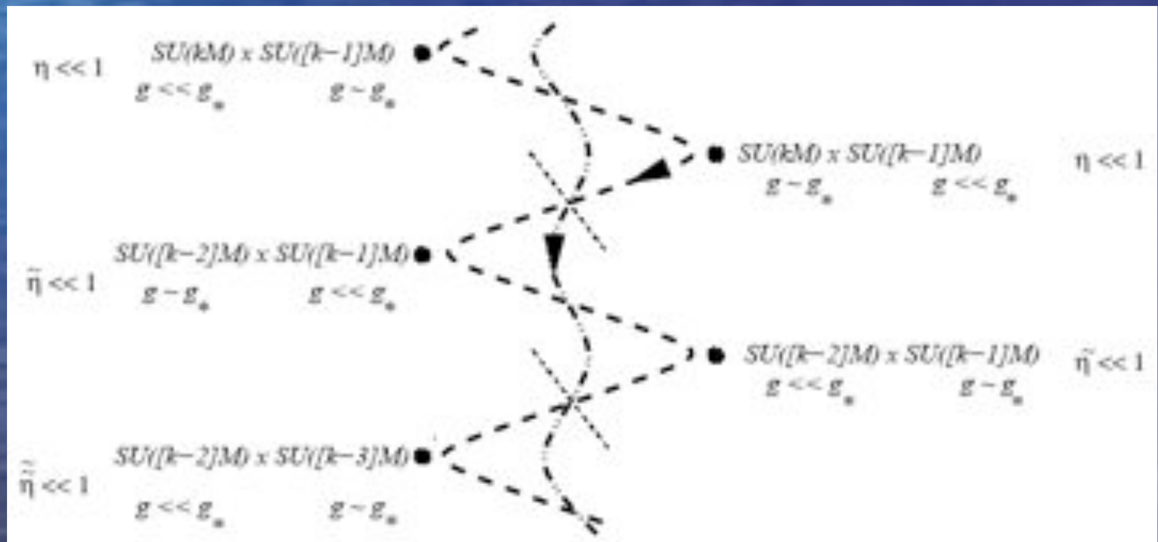


$$ds_{10}^2 = h^{-1/2}(t)\big(-(dx^0)^2 + (dx^i)^2\big) + h^{1/2}(t)ds_6^2$$

- ds_6^2 is the metric of the deformed conifold, a simple Calabi-Yau space defined by the following constraint on 4 complex variables:

$$\sum_{i=1}^{4} z_i^2 = \varepsilon^2$$

- In the UV there is a logarithmic running of the gauge couplings. Surprisingly, the 5-form flux, dual to N, also changes logarithmically with the RG scale. IK, Tseytlin
- What is the explanation in the dual SU(kM)xSU((k-1)M) SYM theory coupled to bifundamental chiral superfields A_1, A_2, B_1, B_2 ? A novel phenomenon, called a duality cascade, takes place: k repeatedly changes by 1 as a result of the Seiberg duality IK, Strassler
(diagram of RG flows from a review by M. Strassler)



- Dimensional transmutation in the IR. The dynamically generated confinement scale is

$$\sim \varepsilon^{2/3}$$

- The pattern of R-symmetry breaking is the same as in the SU(M) SYM theory: Z_{2M} -> Z_2.
- In the IR the gauge theory cascades down to SU(2M) x SU(M). The SU(2M) gauge group effectively has $N_f = N_c$.
- The baryon and anti-baryon operators Seiberg

$$\mathcal{A} = \epsilon^{i_1 \dots i_{N_c}} A^{a_1}_{\alpha_1 i_1} \dots A^{a_{N_c}}_{\alpha_{N_c} i_{N_c}}$$

$$\mathcal{B} = \epsilon_{i_1 \dots i_{N_c}} B^{i_1}_{\dot{\alpha}_1 a_1} \dots B^{i_{N_c}}_{\dot{\alpha}_{N_c} a_{N_c}}$$

acquire expectation values and break the U(1) symmetry under which A_k -> $e^{ia} A_k$; B_l -> $e^{-ia} B_l$. Hence, we observe confinement without a mass gap: due to $U(1)_{baryon}$ chiral symmetry breaking there exist a Goldstone boson and its massless scalar superpartner.

- The KS solution is part of a moduli space of confining SUGRA backgrounds, resolved warped deformed conifolds. Gubser, Herzog, IK; Butti, Grana, Minasian, Petrini, Zaffaroni
- To look for them we need to use the PT ansatz:

$$ds^2_{10} = H^{-1/2} dx_m dx_m + e^x ds^2_6,$$

$$ds^2_6 = (e^g + a^2 e^{-g})(e_1^2 + e_2^2) + e^{-g} \sum_{i=1}^{2} (\epsilon_i^2 - 2a e_i \epsilon_i) + v^{-1}(\tilde{\epsilon}_3^2 + dt^2)$$

- H, x, g, a, v, and the dilaton are functions of the radial variable t. The asymptotic near-AdS radial variable is $r \sim \varepsilon^{2/3} e^{t/3}$
- Additional radial functions enter into the ansatz for the 3-form field strengths. The PT ansatz preserves the SO(4) but breaks a Z_2 charge conjugation symetry, except at the KS point.

- BGMPZ used the method of SU(3) structures to derive the complete set of coupled first-order equations.
- A result of their integration is that the warp factor and the dilaton are related:

$$H(t) = \tilde{H}\left(e^{-2\phi(t)} - 1\right)$$

Dymarsky, IK, Seiberg

- The integration constant determines the \`modulus' U: $\tilde{H} = \gamma U^{-2}$ where $\gamma = 2^{10/3}(g_s M\alpha')^2\varepsilon^{-8/3}$
- At large t the solution approaches the KT \`cascade asymptotics': $a(t) = -2e^{-t} + Ue^{-5t/3}(-t+1) + \ldots$

$$\gamma^{-1}H(t) = \frac{3}{32}e^{-4t/3}(4t-1) - \frac{3}{32\cdot 512}U^2(256t^3 - 864t^2 + 1752t - 847)e^{-8t/3} + O\left(e^{-10t/3}\right)$$

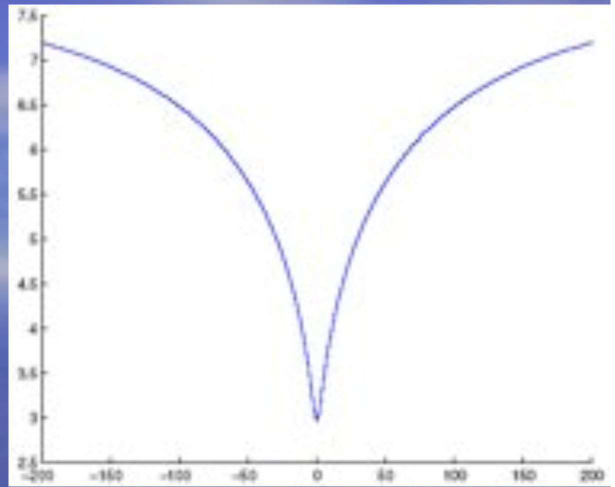
- The resolution parameter U is proportional to the VEV of the operator

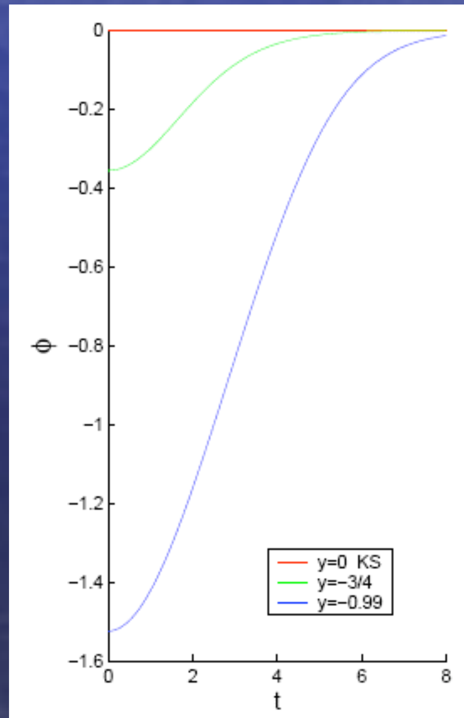
$$\mathcal{U} = \mathrm{Tr}\left(\sum_{\alpha} A_\alpha A_\alpha^\dagger - \sum_{\dot\alpha} B_{\dot\alpha}^\dagger B_{\dot\alpha}\right)$$

- This family of resolved warped deformed conifolds is dual to the `baryonic branch' in the gauge theory (the quantum deformed moduli space):

$$\mathcal{A} = i\Lambda_1^{2M}\zeta\,, \qquad \mathcal{B} = i\Lambda_1^{2M}/\zeta$$

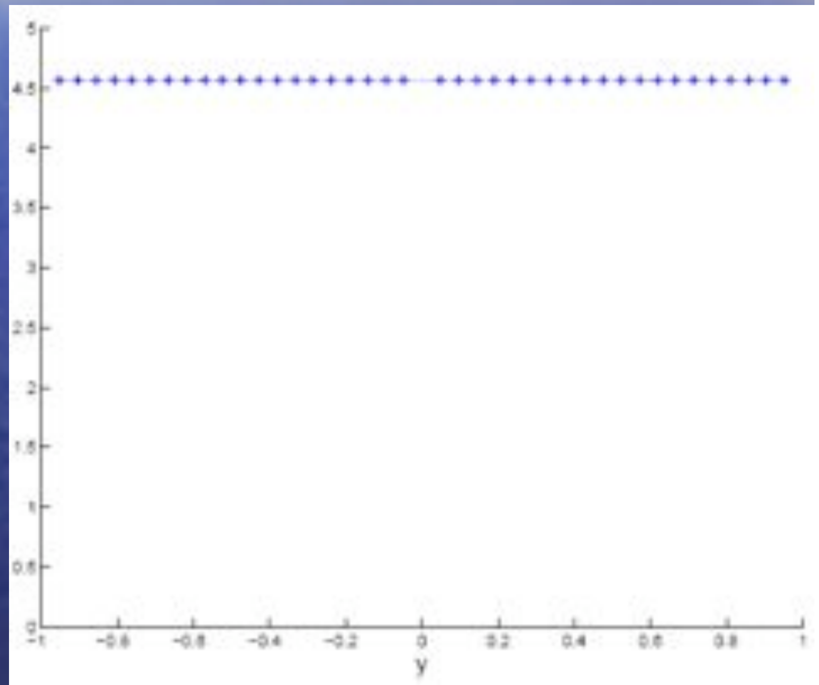
- Various quantities have been calculated as a function of the modulus U=ln |ζ|.
- Here are plots of the string tension (a fundamental string at the bottom of the throat is dual to an `emergent' chromo-electric flux tube) and of the dilaton profiles   Dymarsky, IK, Seiberg





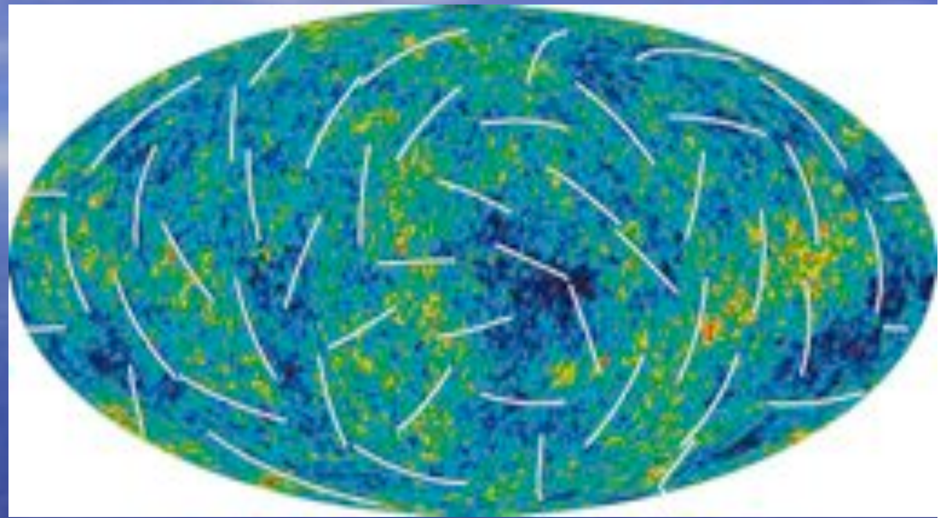
BPS Domain Walls

- A D5-brane wrapped over the 3-sphere at the bottom of the throat is the domain wall separating two adjacent vacua of the theory.
- Since it is BPS saturated, its tension cannot depend on the baryonic branch modulus. This is indeed the case. This fact provides a check on the choice of the UV boundary conditions, and on the numerical integration procedure necessary away from the KS point.
- Analytic proof?

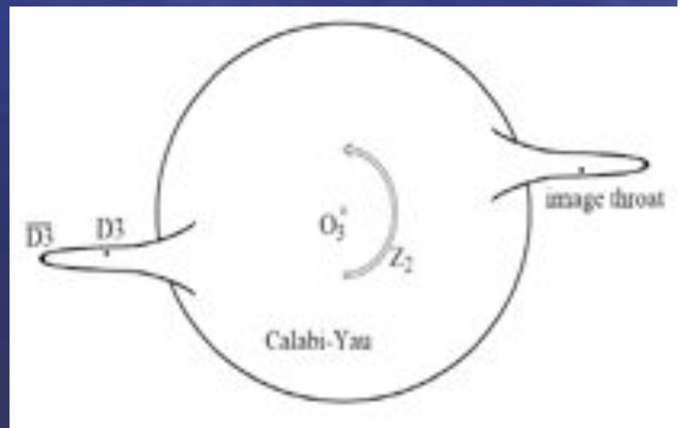


Applications to D-brane Inflation

- The Slow-Roll Inflationary Universe (Linde; Albrecht, Steinhardt) is a very promising idea for generating the CMB anisotropy spectrum observed by the WMAP.
- Finding models with very flat potentials has proven to be difficult. Recent string theory constructions use moving D-branes. Dvali, Tye, …

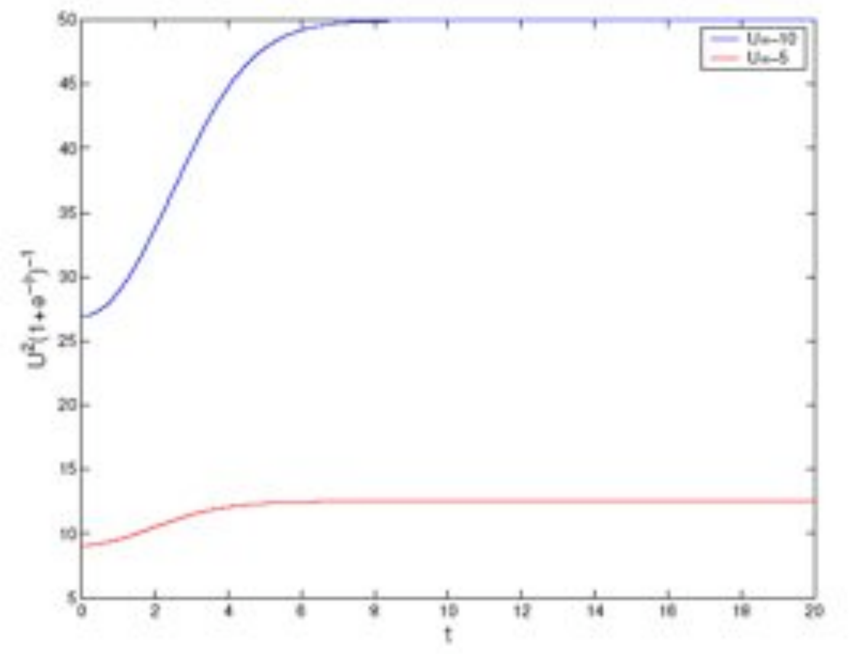


- In the KKLT/KKLMMT model, the warped deformed conifold is embedded into a string compactification. An anti-D3-brane is added at the bottom to break SUSY and generate a potential. A D3-brane rolls in the throat. Its radial coordinate plays the role of an inflaton.
  Kachru, Kallosh, Linde, Maldacena, McAllister, Trivedi



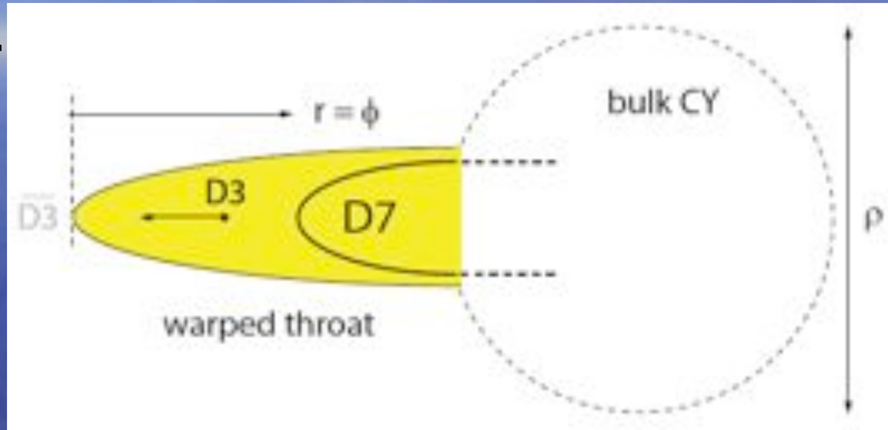
A related suggestion for D-brane inflation (A. Dymarsky, IK, N. Seiberg)

- In a flux compactification, the $U(1)_{baryon}$ is gauged. Turn on a Fayet-Iliopoulos parameter ξ .
- This makes the throat a resolved warped deformed conifold.
- The probe D3-brane potential on this space is asymptotically flat, if we ignore effects of compactification and D7-branes. The plots are for two different values of $U \sim \xi$.
- No anti-D3 needed: in presence of the D3-brane, SUSY is broken by the D-term ξ. Related to the `D-term Inflation' Binetruy, Dvali; Halyo



Slow roll D-brane inflation?

- Effects of D7-branes and of compactification generically spoil the flatness of the potential. Non-perturbative effects introduce the KKLT-type superpotential $W = W_0 + A(X)e^{-a\rho}$ where X denotes the D3-brane position. In any warped throat D-brane inflation model, it is important to calculate A(X).



- The gauge theory on D7-branes wrapping a 4-cycle Σ_4 has coupling $$\frac{1}{g^2} = \frac{V^w_{\Sigma_4}}{g_7^2} = \frac{T_3 V^w_{\Sigma_4}}{8\pi^2}$$
- The non-perturbative superpotential $\propto \exp\left(-\frac{T_3 V^w_{\Sigma_4}}{N_{D7}}\right)$

  depends on the D3-brane location through the warped volume $$V^w_{\Sigma_4} \equiv \int_{\Sigma_4} \mathrm{d}^4\xi \sqrt{g^{ind}}\, h(X)$$
- In the throat approximation, the warp factor can be calculated and integrated over a 4-cycle explicitly. Baumann, Dymarsky, Klebanov, Maldacena, McAllister, Murugan (to appear).
- For a class of conifold embeddings $$\prod_{i=1}^{4} w_i^{p_i} = \mu^P$$

  Arean, Crooks, Ramallo ($w_1 = z_1 + iz_2$, etc.) $$P \equiv \sum_{i=1}^4 p_i$$

  the result is $$A = A_0 \left(\frac{\mu^P - \prod_{i=1}^4 w_i^{p_i}}{\mu^P}\right)^{1/n}$$

- This formula applies both to n wrapped D7-branes, and to a wrapped Euclidean D3 ($n=1$).
- For the latter case, Ganor showed that A has a simple zero when the D3-brane approaches the 4-cycle. Our result agrees with this.
- We have also carried out such calculations for 4-cycles within the Calabi-Yau cones over $Y^{p,q}$ with analogous results: $A(X)$ is proportional to the embedding equation raised to the power $1/n$. This appears to be a general rule for 4-cycles in the throat.

- The dependence of the non-perturbative superpotential on D3-brane position, and other compactification effects, give Hubble-scale corrections to the inflaton potential.
- Some `fine-tuning' is generally needed to cancel different corrections to the D3-brane potential. This is currently under investigation with D. Baumann, A. Dymarsky, J. Maldacena, L. McAllister and P. Steinhardt.

Conclusions

- In the first part, we investigated non-SUSY orbifolds of AdS/CFT. At one loop, flow of double-trace couplings spoils conformal invariance even in the large N limit. There is a precise connection of this instability with presence of twisted sector closed string tachyons.
- Gauge/string dualities for confining gauge theories give a new geometrical view of such important phenomena as dimensional transmutation, chiral symmetry breaking, and quantum deformation of moduli space.
- Embedding gauge/string dualities into string compactifications offers new possibilities for physics beyond the SM, and for modeling inflation. In particular, D3-branes on resolved warped deformed conifolds may realize D-term inflation.
- Calculation of non-perturbative corrections to the inflaton potential is important for determining if these models can produce slow-roll inflation.