Gauge/String Duality and D-Brane Inflation

Igor Klebanov

Department of Physics
Princeton University

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The AdS/CFT duality
Maldacena; Gubser, IK, Polyakov; Witten

- Relates 4-d conformal gauge theory to string theory on 5-d Anti-de Sitter space times a 5-d compact Einstein space. For the $\mathcal{N}=4$ SU(N) SYM theory this space is a 5-d sphere.

- When a gauge theory is strongly coupled, the radius of curvature of the dual AdS$_5$ and of the 5-d compact space becomes large:

\[
\frac{L^2}{\alpha'} \sim \sqrt{g_{YM}^2 N}
\]

- String theory in such a weakly curved background can be studied in the effective (super)-gravity approximation, which allows for a host of explicit calculations. Corrections to it proceed in powers of

\[
\frac{\alpha'}{L^2} \sim \lambda^{-1/2}
\]

- Feynman graphs instead develop a weak coupling expansion in powers of $\lambda$. At weak 't Hooft coupling the dual string theory becomes difficult. It is expected that we need to solve a sigma model with R-R backgrounds on a highly curved space.
Can the closed string side simplify for some purposes at weak coupling?

- My recent work with Dymarsky and Roiban reconsiders quiver gauge theory on a stack of D3-branes at the tip of a cone $R^6/\Gamma$ where the orbifold group $\Gamma$ breaks all the supersymmetry.

- At first sight, the gauge theory seems conformal because the planar beta functions for all single-trace operators vanish. The candidate string dual is $AdS_5 \times S^5/\Gamma$. Kachru, Silverstein; Lawrence, Nekrasov, Vafa; Bershadsky, Johanson

- However, dimension 4 double-trace operators made out of twisted single-trace ones, $f O_n O_{-n}$, are induced at one-loop. Their planar beta-functions have the form

$$ \beta_f = a \lambda^2 + 2 \gamma f \lambda + f^2 $$

$$ \beta_\lambda = 0 $$
• If $D = \gamma^2 - a < 0$, then there is no real fixed point for $f$.

• A class of $Z_k$ orbifolds with global SU(3) symmetry, that are freely acting on the 5-sphere, has the group action in the fundamental of SU(4)

$$r(\theta^n) = \text{diag}(\omega_k^n, \omega_k^n, \omega_k^n, \omega_k^{-3n})$$

$$\omega_k = e^{i\alpha_k}, \quad \alpha_k = \frac{2\pi}{k}$$

• Here is a plot of a one-loop SU(N)$^k$ gauge theory discriminant, $D$, and of the ground state closed string $m^2$ on the cone without the D-branes. $n=1, \ldots, k-1$ labels the twisted sector, and $x=n/k$.

• The simplest freely acting non-susy example is $Z_5$ where there are four induced double-trace couplings

$$\delta_{\text{trace}} \mathcal{L} = f_{8,1} O^{(ij)}_1 O^{(ij)}_{-1} + f_{8,2} O^{(ij)}_2 O^{(ij)}_{-2} + f_{1,1} O_1 O_{-1} + f_{1,2} O_2 O_{-2}$$

• For example, the SU(3) adjoints $O^{(ij)}_n$ are

$$\sum_{k=1}^{5} \left( \Phi^j_{k,k+2} \Phi^j_{k+2,k} - \frac{1}{3} \eta^{ij} \Phi^j_{k,k+2} \Phi^j_{k+2,k} \right) e^{i\alpha(k-1)}$$

$(\alpha=2\pi/5)$
• For more complicated orbifolds, crossing of eigenvalues of the discriminant matrix becomes important. The agreement with closed strings continues to hold.

• Generally, there are three twists that define a cube. The stability/instability regions agree between one-loop gauge theory and string theory.
• Any non-SUSY abelian orbifold contains unstable operators. This appears to remove all such orbifold quivers from a list of large N perturbatively conformal gauge theories.
• The one-loop beta functions destroy the conformal invariance precisely in those twisted sectors where there exist closed-string tachyons localized at the tip of $R^6/\Gamma$. Thus, a very simple correspondence emerges between perturbative gauge theory and free closed string on an orbifold. Why? Perhaps, in the presence of tachyons, the standard AdS/CFT decoupling argument may fail.
• The $AdS_5 \times S^5/\Gamma$ background is tachyon-free at large radius. Could it have some hidden instabilities? If not, then there is a transition from instability to stability as $\lambda$ is increased.
• What is the end-point of the RG flow?
• Condensation of localized tachyon smoothes out the tip of the cone. Adams, Polchinski, Silverstein

• The gauge theory on D3-branes at a smooth point is $\mathcal{N}=4$ SYM. Hence, a natural conjecture is that the gauge theory flows from the non-SUSY $SU(N)^k$ quiver gauge theory to the $\mathcal{N}=4$ $SU(N)$ SYM. Dymarsky, Franco, Roiban, IK (work in progress)
String Theoretic Approach to Confinement

- It is possible to generalize the AdS/CFT correspondence in such a way that the quark-antiquark potential is linear at large distance.
- A “cartoon” of the necessary metric is

\[ ds^2 = \frac{dz^2}{z^2} + a^2(z)(- (dx^0)^2 + (dx^i)^2) \]

- The space ends at a maximum value of \( z \) where the warp factor is finite. Then the confining string tension is

\[ \frac{a^2(z_{\text{max}})}{2\pi \alpha'} \]
Confinement in SYM theories

- Introduction of minimal supersymmetry ($\mathcal{N}=1$) facilitates construction of gauge/string dualities.
- A useful tool is to place D3-branes and wrapped D5-branes at the tip of a 6-d cone, e.g. the conifold.
- The 10-d geometry dual to the gauge theory on these branes is the warped deformed conifold (IK, Strassler)

\[
ds_{10}^2 = h^{-1/2}(t)\left(- (dx^0)^2 + (dx^i)^2\right) + h^{1/2}(t) ds_6^2
\]

- $ds_6^2$ is the metric of the deformed conifold, a simple Calabi-Yau space defined by the following constraint on 4 complex variables:

\[
\sum_{i=1}^{4} z_i^2 = \varepsilon^2
\]
• In the UV there is a logarithmic running of the gauge couplings. Surprisingly, the 5-form flux, dual to N, also changes logarithmically with the RG scale.  
  IK, Tseytlin

• What is the explanation in the dual SU(kM)xSU((k-1)M) SYM theory coupled to bifundamental chiral superfields $A_1, A_2, B_1, B_2$? A novel phenomenon, called a duality cascade, takes place: $k$ repeatedly changes by 1 as a result of the Seiberg duality  
  IK, Strassler

(diagram of RG flows from a review by M. Strassler)
• **Dimensional transmutation** in the IR. The dynamically generated confinement scale is

\[ \sim \varepsilon^{2/3} \]

• The pattern of **R-symmetry breaking** is the same as in the SU(M) SYM theory: \( Z_{2M} \to Z_2 \).

• In the IR the gauge theory cascades down to SU(2M) \( \times \) SU(M). The SU(2M) gauge group effectively has \( N_f = N_c \).

• The baryon and anti-baryon operators

\[
A = e^{i_1 \ldots i_{N_c}} A_{\alpha_1 i_1} \ldots A_{\alpha_{N_c} i_{N_c}} \\
B = e^{i_1 \ldots i_{N_c}} B_{\tilde{\alpha_1} a_1} \ldots B_{\tilde{\alpha}_{N_c} a_{N_c}}
\]

acquire expectation values and break the U(1) symmetry under which \( A_k \to e^{i\alpha} A_k \); \( B_l \to e^{-i\alpha} B_l \). Hence, we observe confinement without a mass gap: due to U(1)\textsubscript{baryon} chiral symmetry breaking there exist a Goldstone boson and its massless scalar superpartner.
• The KS solution is part of a moduli space of confining SUGRA backgrounds, **resolved warped deformed conifolds.** Gubser, Herzog, IK; Butti, Grana, Minasian, Petrini, Zaffaroni

• To look for them we need to use the PT ansatz:

\[
\begin{align*}
    ds_{10}^2 &= H^{-1/2} dx_m dx_m + e^x ds_6^2, \\
    ds_6^2 &= (e^g + a^2 e^{-g})(c_1^2 + c_2^2) + e^{-g} \sum_{i=1}^2 \left( e_i^2 - 2a e_i \varepsilon_i \right) + v^{-1}(\varepsilon_3^2 + dt^2)
\end{align*}
\]

• H, x, g, a, v, and the dilaton are functions of the radial variable t. The asymptotic near-AdS radial variable is \( r \sim \varepsilon^{2/3} e^{t/3} \)

• Additional radial functions enter into the ansatz for the 3-form field strengths. The PT ansatz preserves the SO(4) but breaks a \( \mathbb{Z}_2 \) charge conjugation symmetry, except at the KS point.
• BGMPZ used the method of SU(3) structures to derive the complete set of coupled first-order equations.

• A result of their integration is that the warp factor and the dilaton are related:

\[ H(t) = \tilde{H} \left( e^{-2\phi(t)} - 1 \right) \]

Dymarsky, IK, Seiberg

• The integration constant determines the 'modulus' U:

\[ \tilde{H} = \gamma U^{-2} \quad \text{where} \quad \gamma = 2^{10/3} (g_s M \alpha')^2 \varepsilon^{-8/3} \]

• At large \( t \) the solution approaches the KT 'cascade asymptotics':

\[
\gamma^{-1} H(t) = \frac{3}{32} e^{-4t/3} (4t - 1) - \frac{3}{32 \cdot 512} U^2 (256t^3 - 864t^2 + 1752t - 847)e^{-8t/3} + O \left( e^{-10t/3} \right)
\]

\[ a(t) = -2e^{-t} + U e^{-5t/3} (-t + 1) + \ldots \]
• The resolution parameter $U$ is proportional to the VEV of the operator

$$U = Tr \left( \sum_\alpha A_\alpha A_\alpha^\dagger - \sum_{\dot{\alpha}} B_{\dot{\alpha}} B_{\dot{\alpha}}^\dagger \right)$$

• This family of resolved warped deformed conifolds is dual to the `baryonic branch’ in the gauge theory (the quantum deformed moduli space):

$$A = i\Lambda_1^{2M} \zeta, \quad B = i\Lambda_1^{2M} / \zeta$$

• Various quantities have been calculated as a function of the modulus $U = \ln |\zeta|$.

• Here are plots of the string tension (a fundamental string at the bottom of the throat is dual to an `emergent’ chromo-electric flux tube) and of the dilaton profiles Dymarsky, IK, Seiberg
BPS Domain Walls

- A D5-brane wrapped over the 3-sphere at the bottom of the throat is the domain wall separating two adjacent vacua of the theory.
- Since it is BPS saturated, its tension cannot depend on the baryonic branch modulus. This is indeed the case. This fact provides a check on the choice of the UV boundary conditions, and on the numerical integration procedure necessary away from the KS point.
- Analytic proof?
Applications to D-brane Inflation

- The Slow-Roll Inflationary Universe (Linde; Albrecht, Steinhardt) is a very promising idea for generating the CMB anisotropy spectrum observed by the WMAP.

- Finding models with very flat potentials has proven to be difficult. Recent string theory constructions use moving D-branes. Dvali, Tye, ...

- In the KKLT/KKLMMT model, the warped deformed conifold is embedded into a string compactification. An anti-D3-brane is added at the bottom to break SUSY and generate a potential. A D3-brane rolls in the throat. Its radial coordinate plays the role of an inflaton.

  Kachru, Kallosh, Linde, Maldacena, McAllister, Trivedi
A related suggestion for D-brane inflation (A. Dymarsky, IK, N. Seiberg)

- In a flux compactification, the $U(1)_{\text{baryon}}$ is gauged. Turn on a Fayet-Iliopoulos parameter $\xi$.
- This makes the throat a resolved warped deformed conifold.
- The probe D3-brane potential on this space is asymptotically flat, if we ignore effects of compactification and D7-branes. The plots are for two different values of $U\sim \xi$.
- No anti-D3 needed: in presence of the D3-brane, SUSY is broken by the D-term $\xi$. Related to the `D-term Inflation’ Binetruy, Dvali; Halyo
Slow roll D-brane inflation?

- Effects of D7-branes and of compactification generically spoil the flatness of the potential. Non-perturbative effects introduce the KKLT-type superpotential

\[ W = W_0 + A(X)e^{-a\rho} \]

where X denotes the D3-brane position. In any warped throat D-brane inflation model, it is important to calculate \( A(X) \).
• The gauge theory on D7-branes wrapping a 4-cycle $\Sigma_4$ has coupling

$$\frac{1}{g^2} = \frac{V_{\Sigma_4}^w}{g_7^2} = \frac{T_3 V_{\Sigma_4}^w}{8\pi^2}$$

• The non-perturbative superpotential depends on the D3-brane location through the warped volume

$$V_{\Sigma_4}^w \equiv \int_{\Sigma_4} d^4\xi \sqrt{g^{\text{ind}}} h(X)$$

• In the throat approximation, the warp factor can be calculated and integrated over a 4-cycle explicitly. Baumann, Dymarsky, Klebanov, Maldacena, McAllister, Murugan (to appear).

• For a class of conifold embeddings

Arean, Crooks, Ramallo ($\omega_1 = z_1 + iz_2$, etc.)

the result is

$$A = A_0 \left( \frac{\mu^P - \prod_{i=1}^4 w_i^{p_i}}{\mu^P} \right)^{1/n}$$

$$P \equiv \sum_{i=1}^4 p_i$$
• This formula applies both to $n$ wrapped D7-branes, and to a wrapped Euclidean D3 ($n=1$).

• For the latter case, Ganor showed that $A$ has a simple zero when the D3-brane approaches the 4-cycle. Our result agrees with this.

• We have also carried out such calculations for 4-cycles within the Calabi-Yau cones over $Y^p,q$ with analogous results: $A(X)$ is proportional to the embedding equation raised to the power $1/n$. This appears to be a general rule for 4-cycles in the throat.
• The dependence of the non-perturbative superpotential on D3-brane position, and other compactification effects, give Hubble-scale corrections to the inflaton potential.

• Some `fine-tuning’ is generally needed to cancel different corrections to the D3-brane potential. This is currently under investigation with D. Baumann, A. Dymarsky, J. Maldacena, L. McAllister and P. Steinhardt.
Conclusions

• In the first part, we investigated non-SUSY orbifolds of AdS/CFT. At one loop, flow of double-trace couplings spoils conformal invariance even in the large N limit. There is a precise connection of this instability with presence of twisted sector closed string tachyons.

• Gauge/string dualities for confining gauge theories give a new geometrical view of such important phenomena as dimensional transmutation, chiral symmetry breaking, and quantum deformation of moduli space.

• Embedding gauge/string dualities into string compactifications offers new possibilities for physics beyond the SM, and for modeling inflation. In particular, D3-branes on resolved warped deformed conifolds may realize D-term inflation.

• Calculation of non-perturbative corrections to the inflaton potential is important for determining if these models can produce slow-roll inflation.