Giant Magnons

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Introduction

- Understand the spectrum of operators of N=4 SYM in the planar limit.
- ‘t Hooft limit → spin chains
- AdS/CFT spin chains→ string worldsheets
- Simplifications in a large J limit.
- One finds well defined elementary excitations that propagate along the chain: Magnons
- We will find the description of these magnons on the string theory side, at large ‘t Hooft coupling.

Large J limit

Mann, Polchinski
Rej, Serban, Staudacher
Zarembo, Arutynov,
Tseytlin,
….

- $J = J_{56}$ in SO(6)
- J → Infinity
- Δ- J = fixed , $\lambda = g^2 N$ = fixed
- O = $$\sum_l e^{ipl} (\ldots ZZZ\underset{l}{W}ZZZ\ldots.)$$
- p = 2 π n/J = fixed , p ~ p + 2 π
- No finite volume effects
- No P_{total} =0
- Not the pp-wave (or BMN) limit

$$....ZZZWZZZ..$$

$$....ZZZ\partial ZZZZ...$$

$$....ZZZ\partial^2 ZZZ... \to ...ZZ\partial ZZZZ\partial ZZZZ...$$

We have “impurities” that propagate along the “string” of Zs.

Finite number o f elementary impurites: 8 bosons and 8 fermions

Single impurity spectrum

$$\varepsilon = \Delta - J = \sqrt{1 + \frac{\lambda}{\pi^2}\sin^2\left(\frac{p}{2}\right)}$$

Beisert, Dippel Staudacher (Santambrogio Zanon, Berenstein, Correa, Vazquez)

Beisert

Proof

Full N=4 superconformal group → Subgroup that leaves Z invariant: PSU(4|4) → $SU(2|2)^2$

Impurity with p=0 is a BPS state.

Nonzero p → central extension → states are still BPS.
Central extension ~ $(1 - e^{ip})$. Related to terms of the form $[\psi, Z]$ in the supercharges.

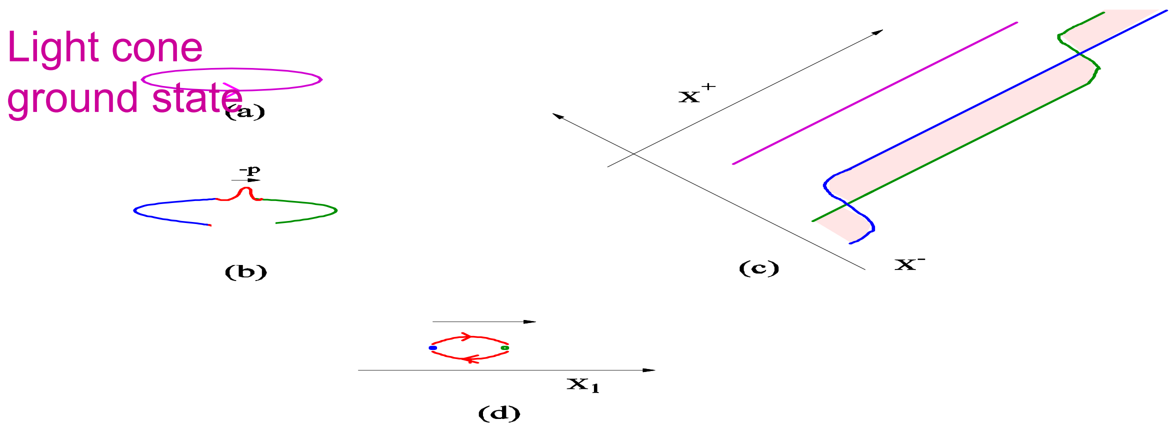
Note that p is a quasi-momentum: periodicity in p

We will now analyze the single impurity problem at large λ using string theory

$$f(\lambda) = \lambda$$

Strings in flat space

- Light cone gauge X^+, X^- + transverse
- Large p_-

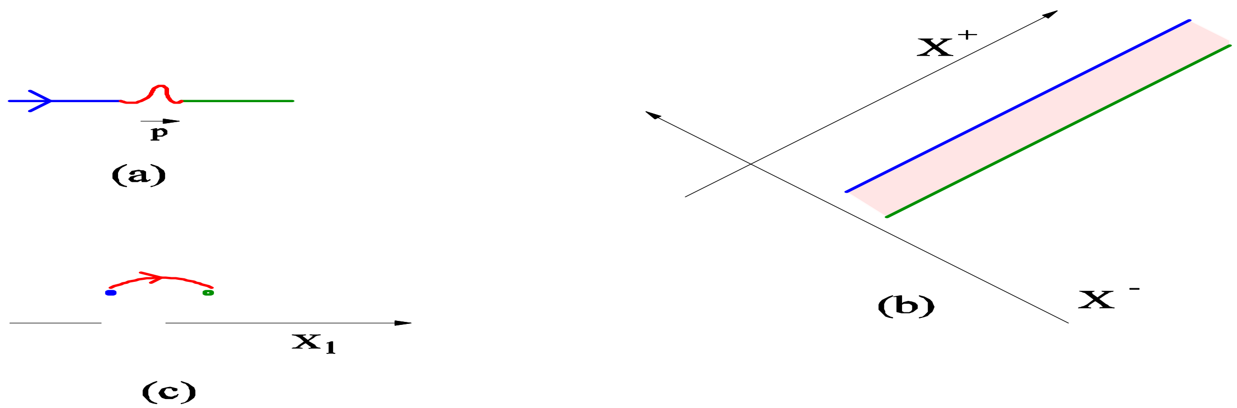


$$\partial_\sigma X^- = T_{01}$$

$$\Rightarrow$$

$$\Delta X^- = p$$

Infinite momentum→ infinite string in light cone gauge

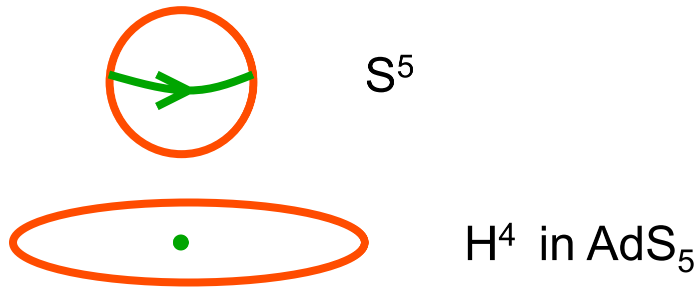


The string endpoints in spacetime look like light-like D-branes.

Strings in AdS_5 x S_5

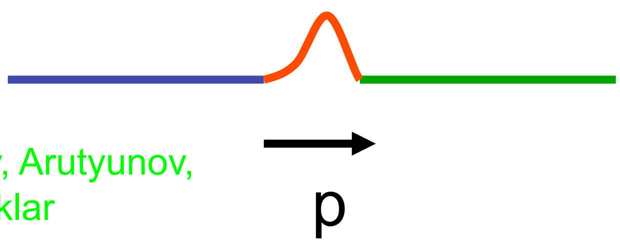
$\Delta = E = J = \infty$. Ground state

Pointlike string moving along $\varphi = t$



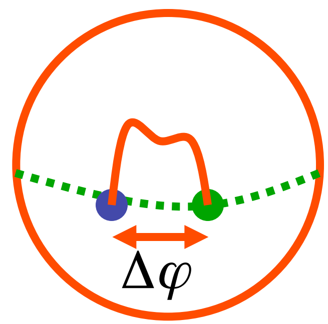
Choose $X^- = t - \varphi$

Worldsheet in light cone gauge



Frolov, Arutyunov, Zamaklar

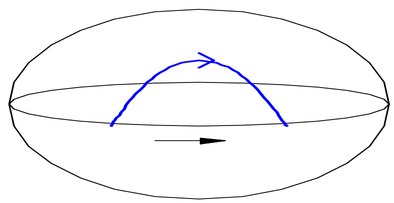
Spacetime picture



$$p = \Delta\varphi$$

p periodic = angle periodic

Find the solution with lowest energy for a given momentum p



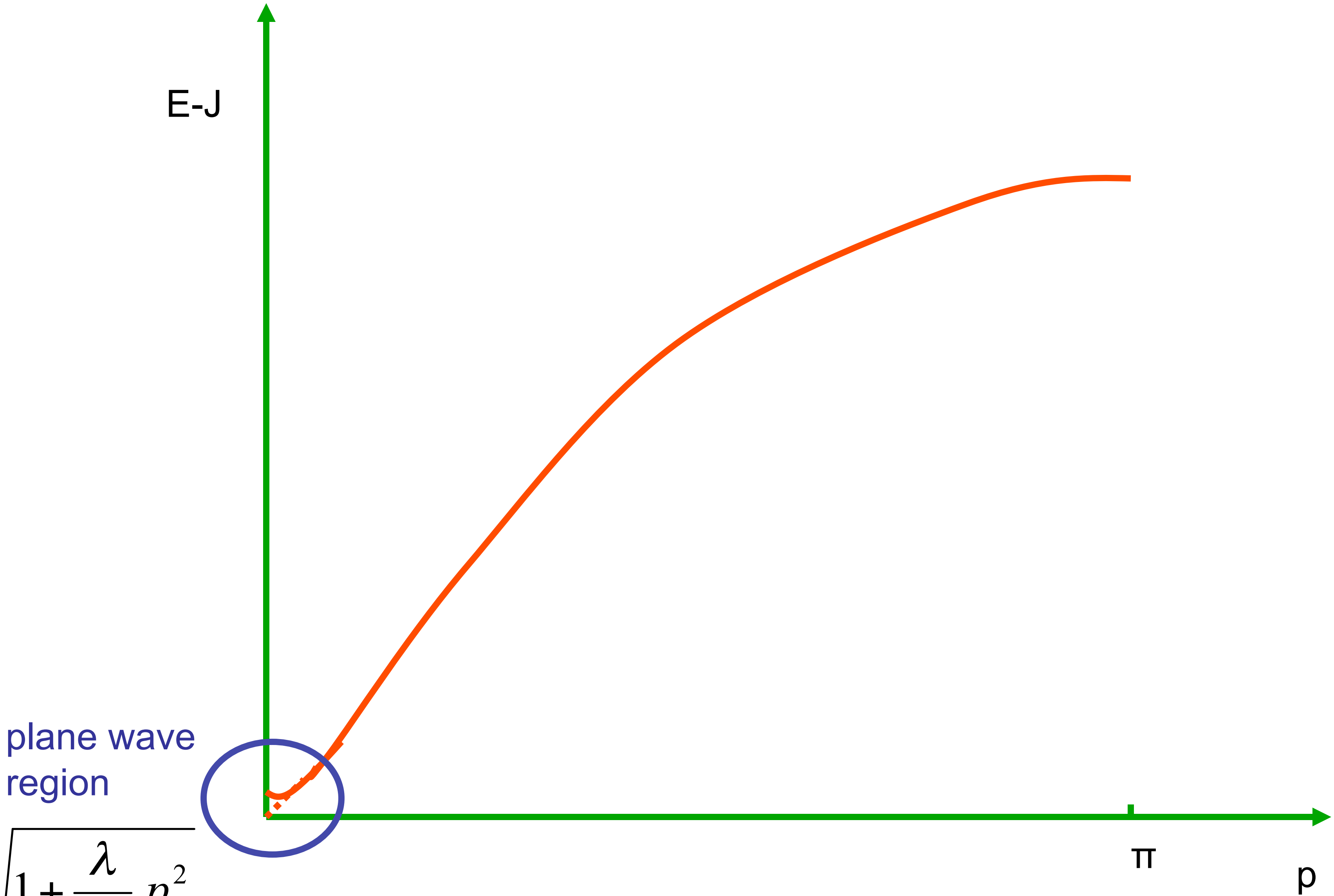
Compute the energy

$$\varepsilon = E - J = \frac{\sqrt{\lambda}}{\pi}\left|\sin\left(\frac{p}{2}\right)\right|$$

Agrees with the large λ limit of:

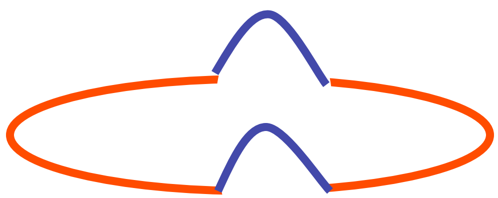
$$\varepsilon = \Delta - J = \sqrt{1 + \frac{\lambda}{\pi^2}\sin^2\left(\frac{p}{2}\right)}$$

Large λ dispersion relation

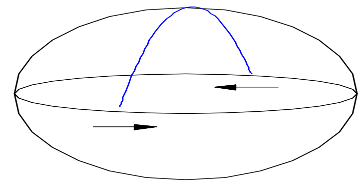


$$\varepsilon = \sqrt{1 + \frac{\lambda}{\pi^2} p^2}$$

Spinning string on S^2 inside S^5



worldsheet



spacetime

Gubser, Klebanov, Polyakov

For very large J we approximate it as a two magnon solution. Each magnon has momentum p=π

$$E - J = 2\frac{\sqrt{\lambda}}{\pi^2}$$

Solution in other coordinates

Project the S^5 on to a disk.

Lin, Lunin, J.M.

(a)

(b)

Energy is the length of the string.

Symmetries: Same as in the gauge theory $SU(2|2)^2$ + central extension

Central extension → String winding charge (physical states → no net winding charge)

S-Matrix

- Ground state: Chain of Zs
- Impurities = 8 bosons + 8 fermions = single BPS multiplet of the (extended) symmetries.
- Define asymptotic scattering S-matrix 2 $\rightarrow$ 2 impurities.

Staudacher

Beisert

Structure of the S matrix

- $S_{ab,cd} = M_{ab,cd}\ S_0$
- M is a known 16^2 x 16^2 matrix fixed by symmetries
- S_0 is an unknown phase
- This is true both in the gauge theory and string theory (same symmetries)
- Integrability → factorized scattering (obeys the Yang Baxter equation)
- $S_0(p_1, p_2, \lambda)$ is all we need to know to solve planar N=4 SYM!
- Feed the S-matrix into Bethe ansatz → get spectrum.

Beisert, Staudacher

Direct computation of S_0 at large coupling

- Classical scattering of two magnons
- Determined by the classical time delay in the two magnon classical solution
- Use:

Classical strings on S^2 x R

Pohlmeyer
Mikhailov

=

classical sine gordon theory. (only classical)

Sine gordon solition = single magnon

Energy of the string solution ≠ energy in sine gordon

Result using the classical sine gordon theory we get

$$S_0 = e^{i\delta}$$

$$\delta = -\frac{\sqrt{\lambda}}{\pi}\left(\cos\frac{p}{2} - \cos\frac{p'}{2}\right)\log\left[\frac{\sin^2\frac{p-p'}{2}}{\sin^2\frac{p+p'}{2}}\right]$$

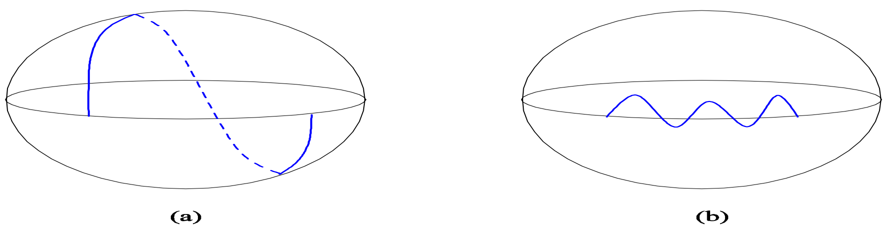
Same as the large λ limit of the “string S-matrix” of Arutyunov, Frolov and Staudacher

Bound states

Sine gordon theory has bound states. In the classical limit these are so called “breather” solutions = time dependent non-dissipative solutions

We can produce the explicit time dependent solutions for the string theory.

Mikhailov



More energy than a pair of states with half the momentum.

Semiclassical quantization gives

$$\varepsilon = \sqrt{n^2 + \frac{4\lambda}{\pi^2}\sin^2 p}$$

We can view this as the superposition of two magnons with momenta

$$p_1 = \frac{p}{2} + iq$$

$$p_2 = \frac{p}{2} - iq$$

Classically stable.

We expect that they are stable in the full theory due to integrability.

They should appear as poles in the phase of the S matrix

BPS bound states

Dorey
Arutyunov,
Frolov, Zamaklar

Bound state of n magnons.
Come from poles in the matrix structure of the S-matrix.
In string theory, similar to the magnons we described but with extra angular momentum in the SO(4) directions of the S^5

$$\varepsilon = \sqrt{n^2 + \frac{\lambda}{\pi^2}\sin^2\left(\frac{p}{2}\right)}$$

So for n=1 these solutions give precisely the formula for the energy.
No quantum corrections to first order in $1/\sqrt{\lambda}$

Minahan, Tirziu,
Tseytlin

Summary

- Simple class of observables at large J which allow a direct comparison between gauge theory and string theory.
- Identified magnons & matched dispersion relation at strong coupling. Periodicity in momentum = geometrical angle.
- Matched the energy of a spinning string
- Found the phase of the S matrix at strong coupling. Agreed with AFS.

Future

- Compute S_0
- Promising route: Use a crossing symmetry equation

Janik,
(Beisert)

Crossing symmetry equation

Janik, Beisert

- Equation based on crossing symmetry.
- $S_0(\bar{1}, 2)\, S_0(1,2) = f(1,2)$
- Kinematics → torus ($p \sim p + 2\pi$; $\theta = \theta + 2\pi i$)
- Think of the equation on C^2
- Initial goal: Find a meromorphic solution, i.e. a solution with no branch cuts or essential singularities.
- There exists no such solution. J.M., Neitzke, Swanson
- Understand better what is the allowed analytic structure!
- There are many solutions if one allows branch cuts and/ or essential singularities.

- To do: Select the correct solution….