Microscopics of the black hole - black string transition

OR

Developing the story of fractionation

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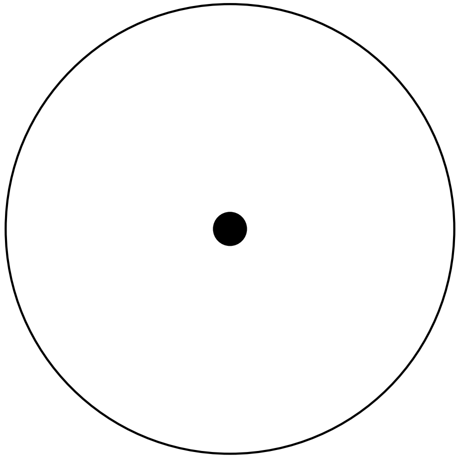
Work with
Borun D. Chowdhury and Stefano Giusto



$$S_{bek} = \frac{A}{4G} = 2\pi\sqrt{n_1 n_5 n_p}$$

$$S_{micro} = 2\pi\sqrt{n_1 n_5 n_p}$$

Strominger-Vafa '96

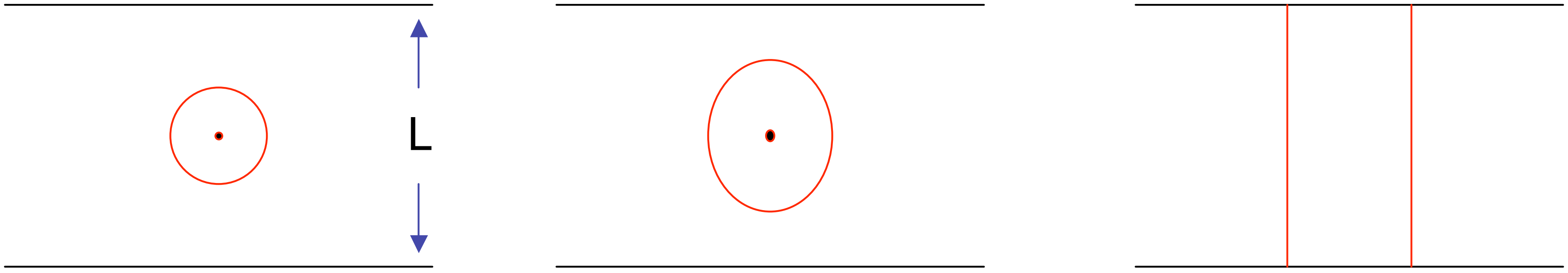
3-charge extremal
D1-D5-P black hole

We understand something about the quantum structure of black holes …

There are many other properties of black holes/black branes
Can we understand these microscopically?

That will teach us more about the nature of quantum gravity …

The black hole - black string transition



Small mass:
Black hole

Large mass:
Black string

Tension

$$\mathcal{T} = -\frac{1}{L}\int T_{zz} = (\frac{\partial M}{\partial L})_S$$

Small black hole $\mathcal{T} \approx 0$

Uniform black string $\mathcal{T}\frac{L}{M} = const.$

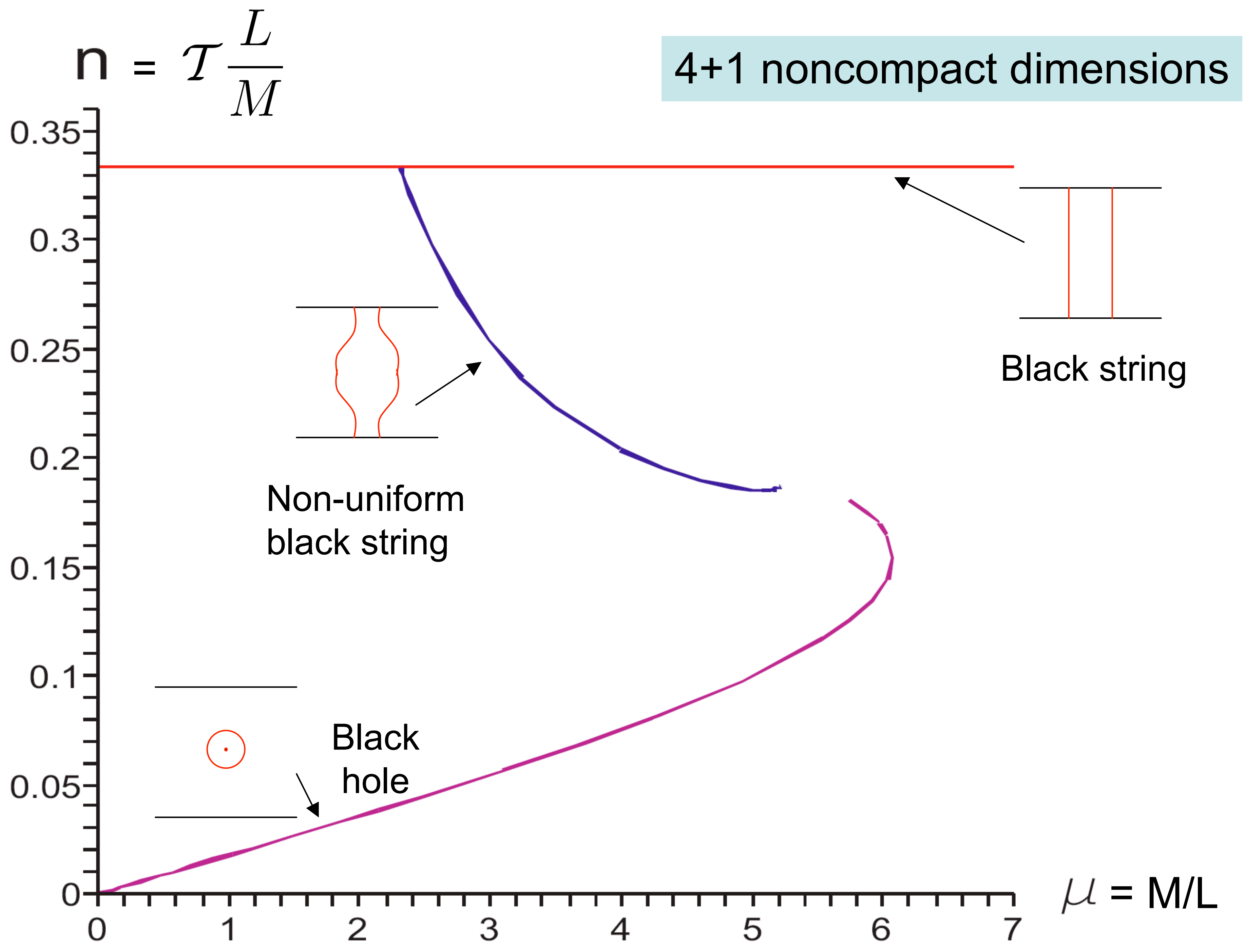
Numerical Work by

Gregory+Laflamme, Gubser, Kol,
Harmark+Obers, Wiseman, Kleihaus+Kunz+Radu,
Hobvedo+Myers

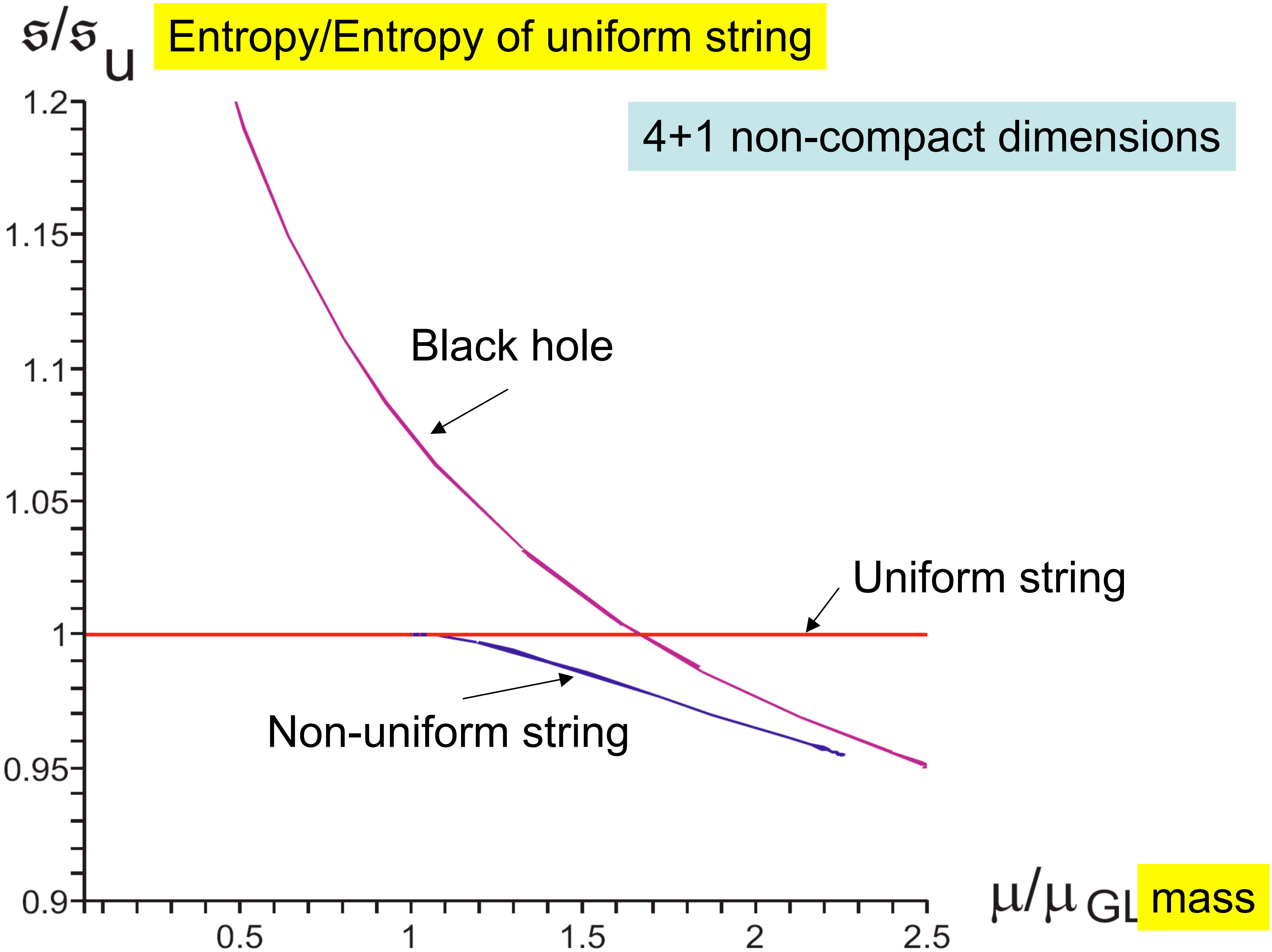
and many others …….

Has established a picture for the phase diagram
of this *black hole - black string* transition

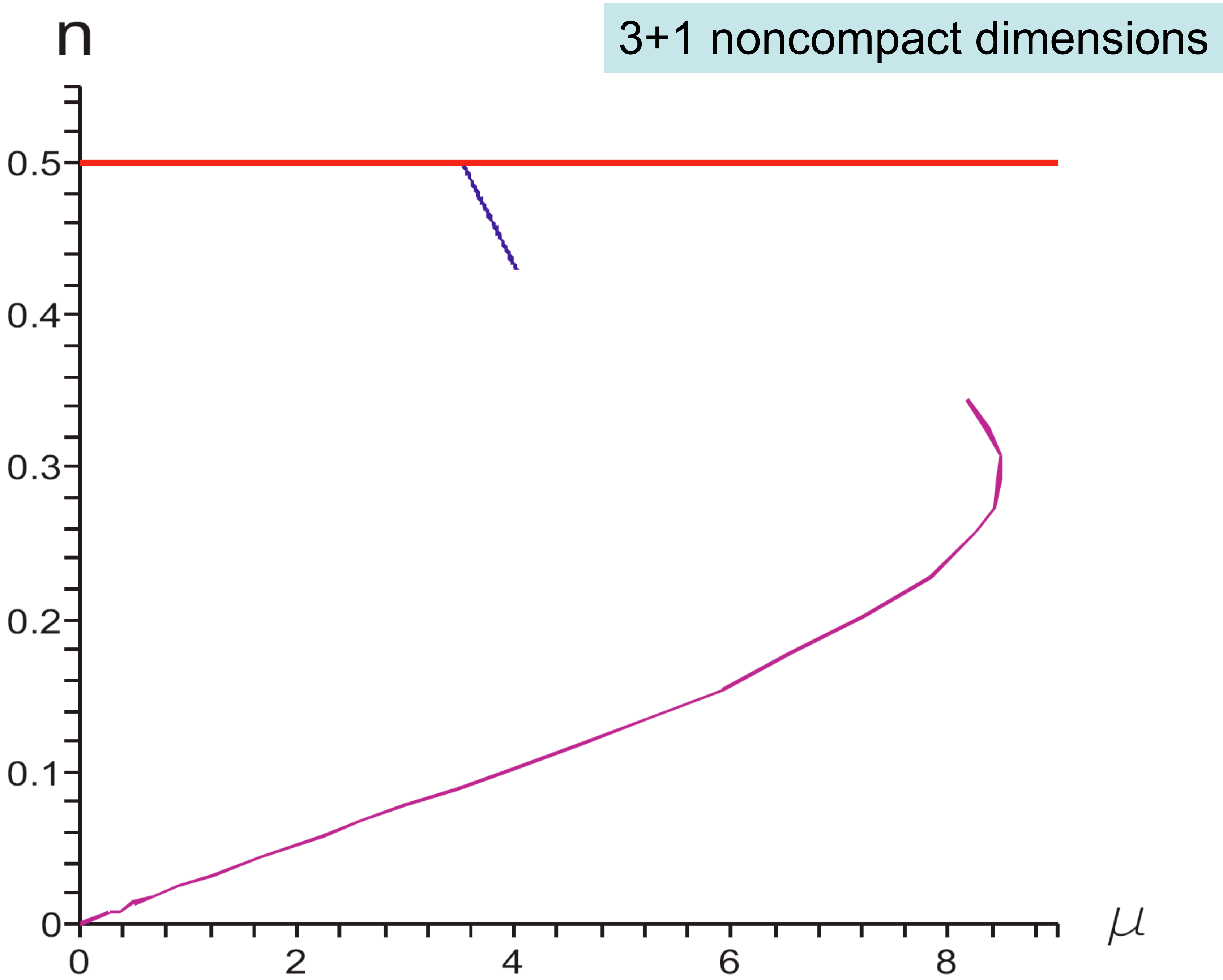
(We will borrow many graphs depicting numerical work from these authors…)



n = $\mathcal{T}\frac{L}{M}$
4+1 noncompact dimensions
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
Black string
Non-uniform
black string
Black
hole
0
1
2
3
4
5
6
7
μ = M/L

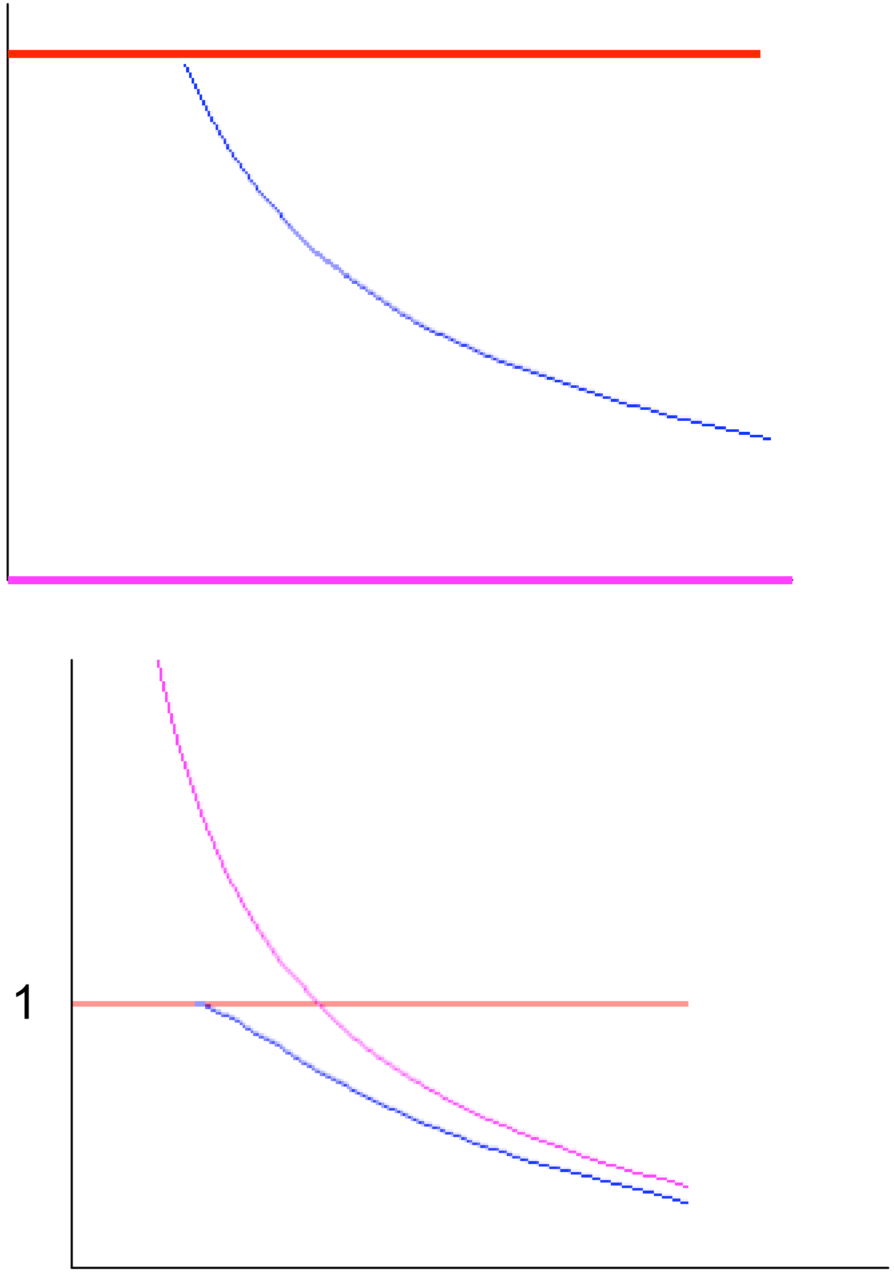


s/s u
Entropy/Entropy of uniform string
4+1 non-compact dimensions
Black hole
Uniform string
Non-uniform string
1.2
1.15
1.1
1.05
1
0.95
0.9
0.5
1
1.5
2
2.5
μ/μ GL
mass

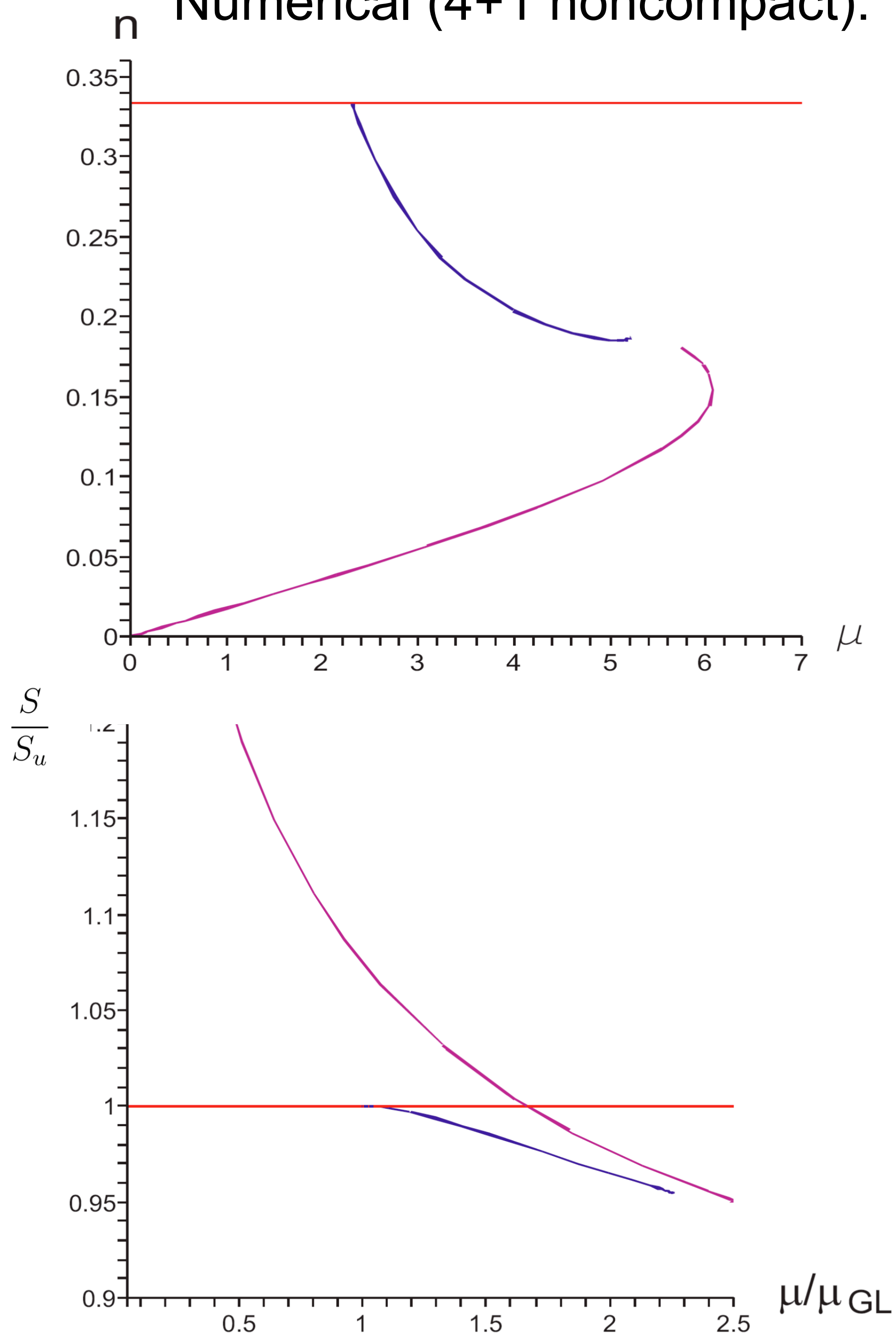


3+1 noncompact dimensions
n
0.5
0.4
0.3
0.2
0.1
0
0
2
4
6
8
μ

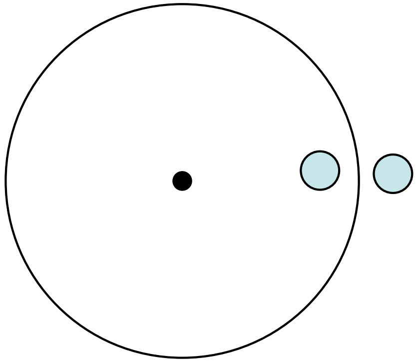
Microscopic (3+1 noncompact):



Numerical (4+1 noncompact):

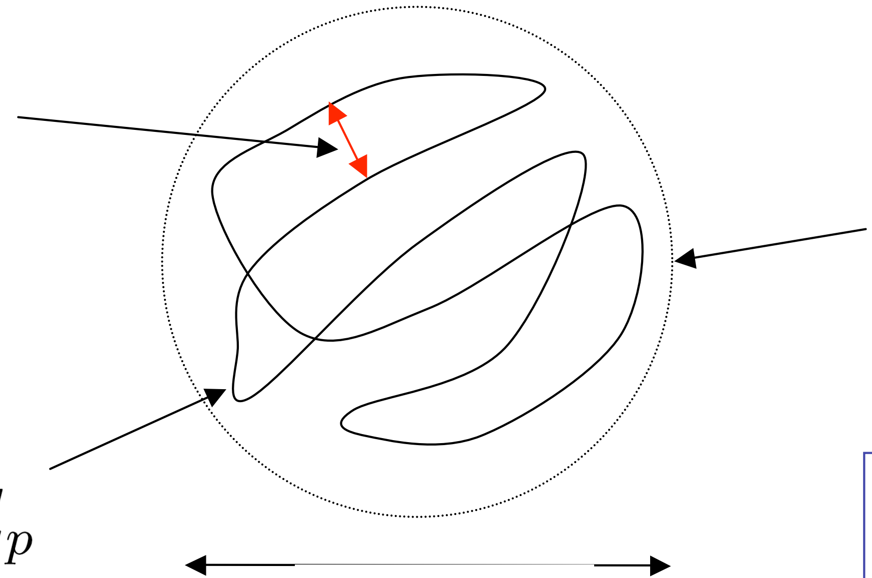


Motivation: Information loss if radiation happens in 'empty space around horizon'



But D1-D5 extremal bound state:

Charges $n_1 \sim n_5 \sim n$



R^4 corrections need to be studied

Natural scale of 'nonlocal quantum gravity' effects: l_p

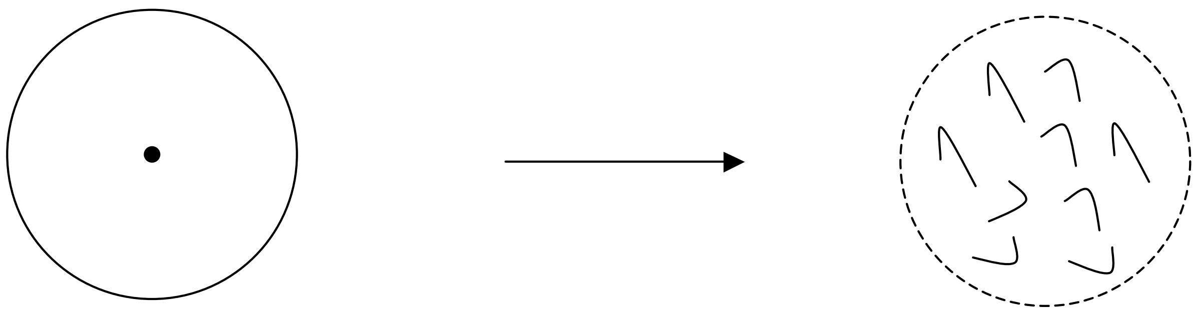
But a black hole is made from a large number of quanta (branes) $(N_1, N_2, N_3, \ldots)$

$$l_p \longrightarrow N^{\alpha} l_p \quad ??$$

Different kinds of branes *'fractionate each other'*

Fractional branes have low tension (Planck scale) $/(N_1 N_2 N_3)$

These low tension 'floppy fractional branes' stretch over long distances (Schwarzschild radius of black hole)



'Fuzzball'

??

Plan

Start with neutral black hole/black string, add charges by ‘boosting + duality’.

This gives a near-extremal system. Numerically computed phase diagrams can be easily converted to give the near-extremal case (Harmark+Obers).

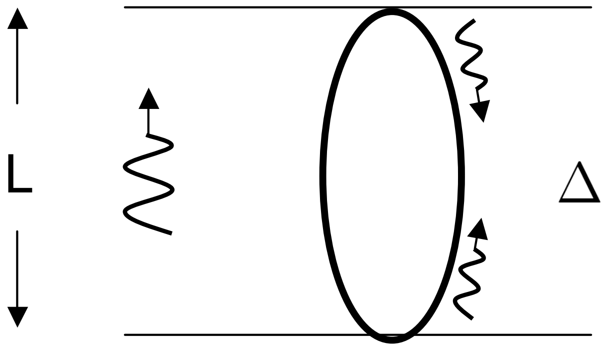
But for near extremal systems we have a microscopic brane description.

We will make one assumption about ‘fractionation’.

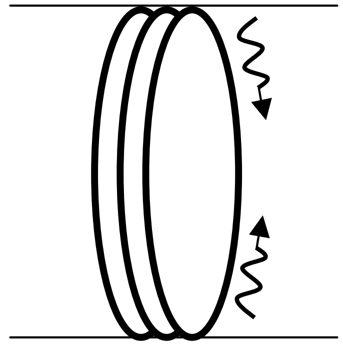
→

Compute phase diagram from microscopic brane description

Fractionation and Entropy



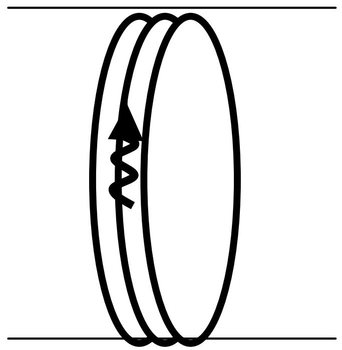
$$\Delta E = \frac{4\pi}{L}$$



$$\Delta E = \frac{4\pi}{n_1 L}$$

P $\overline{\text{P}}$ pairs are *fractionated* by the winding Charge n_1

2 large charges (extremal):

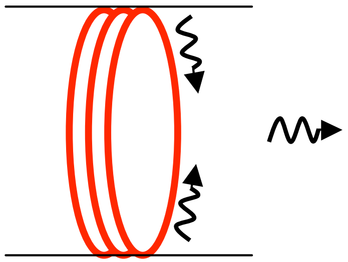


$$S = 2\sqrt{2}\pi\sqrt{n_1 n_p}$$

D1-D5-P: $M_{9,1} \to M_{4,1} \times T^4 \times S^1$

3 large charges (extremal): $S = 2\pi\sqrt{n_1 n_5 n_p}$ Strominger Vafa

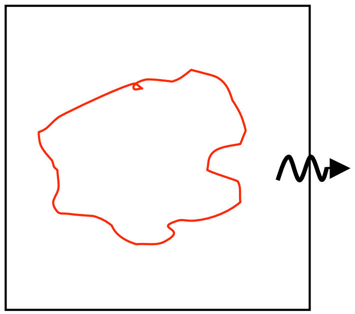
2 large charges + E : $S = 2\pi\sqrt{n_1 n_5}(\sqrt{n_p} + \sqrt{\bar{n}_p})$



$$= 2\pi\sqrt{n_1 n_5}(2\sqrt{\frac{E}{2m_p}})$$

Callan Maldacena

1 large charge + E :

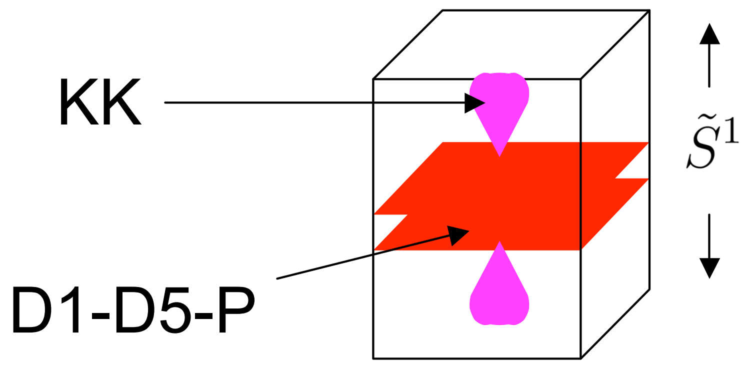


$$S = 2\pi\sqrt{n_5}(\sqrt{n_1} + \sqrt{\bar{n}_1})(\sqrt{n_p} + \sqrt{\bar{n}_p})$$

$$= 2\pi\sqrt{n_5}(\frac{E}{\sqrt{m_1 m_p}})$$

(Maldacena '96)

Similar relations exist for $M_{9,1} \to M_{3,1} \times T^4 \times S^1 \times \tilde{S}^1$



4 large charges (extremal): $$S = 2\pi\sqrt{n_1 n_5 n_p n_{kk}}$$

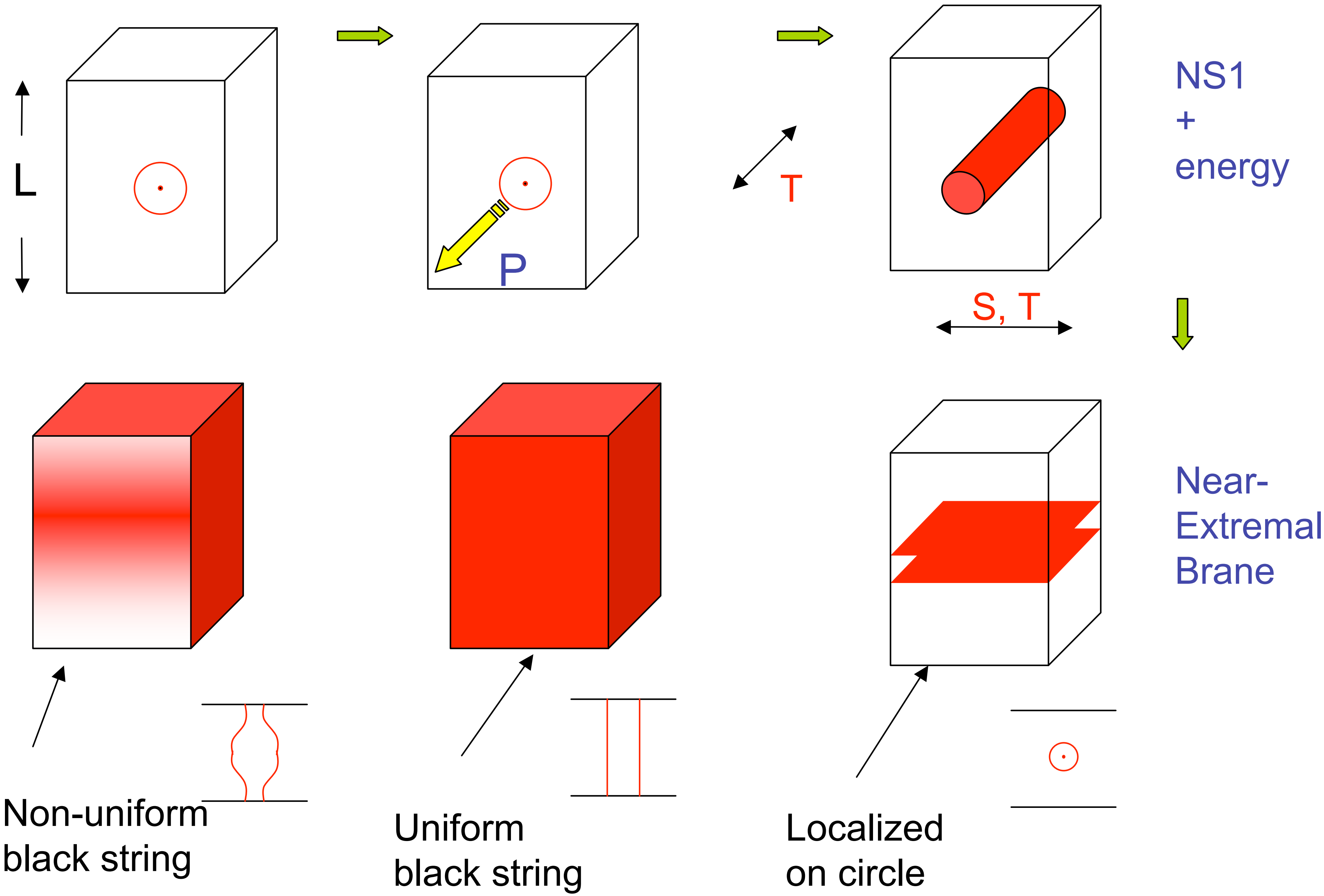
3 large charges + E : $$S = 2\pi\sqrt{n_1 n_5 n_{kk}}(\sqrt{n_p} + \sqrt{\bar{n}_p})$$

2 large charges + E :

$$S = 2\pi\sqrt{n_1 n_5}(\sqrt{n_p} + \sqrt{\bar{n}_p})(\sqrt{n_{kk}} + \sqrt{\bar{n}_{kk}})$$

$$= 2\pi\sqrt{n_1 n_5}\left(\frac{E}{\sqrt{m_p m_{kk}}}\right)$$

Adding charges: Neutral to near extremal: *Hassan+Sen '91*



Compactify: $M_{9,1} \rightarrow M_{3,1} \times T^4 \times S^1 \times \tilde{S}^1$

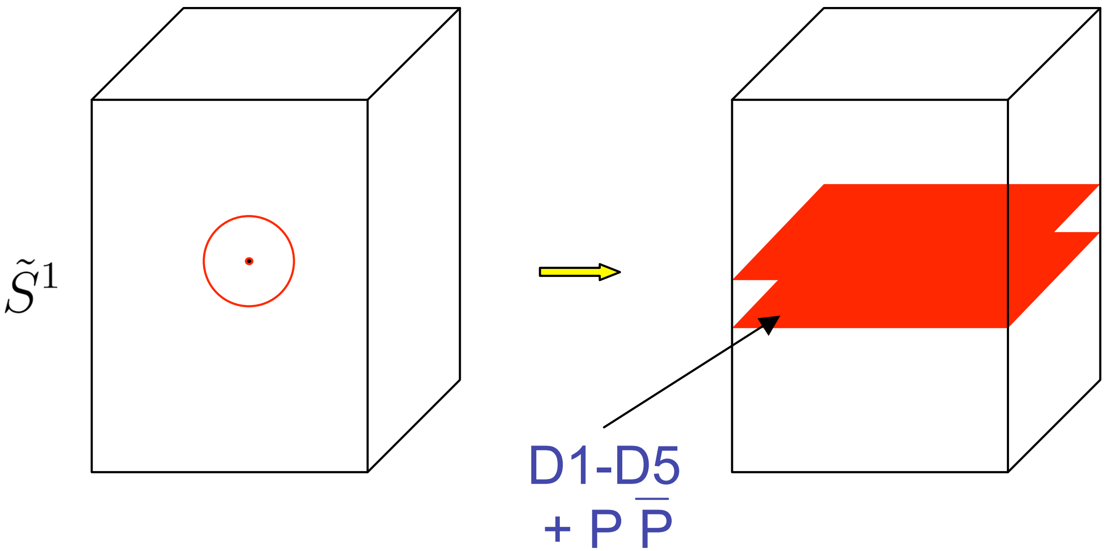
Let $\tilde{S}^1$ be large.

Then we effectively have a black hole in 4+1 non-compact dimensions

(only $T^4 \times S^1$ compact)

Add D1-D5 charges by `boosting+ duality'

→ Near extremal D1-D5



$$S = 2\pi\sqrt{n_1 n_5}(\sqrt{n_p} + \sqrt{\bar{n}_p})$$
$$= 2\pi\sqrt{N}\ (2\sqrt{\frac{E}{2m_p}})$$

$$N = n_1 n_5$$

$$M_{9,1} \to M_{3,1} \times T^4 \times S^1 \times \tilde{S}^1$$

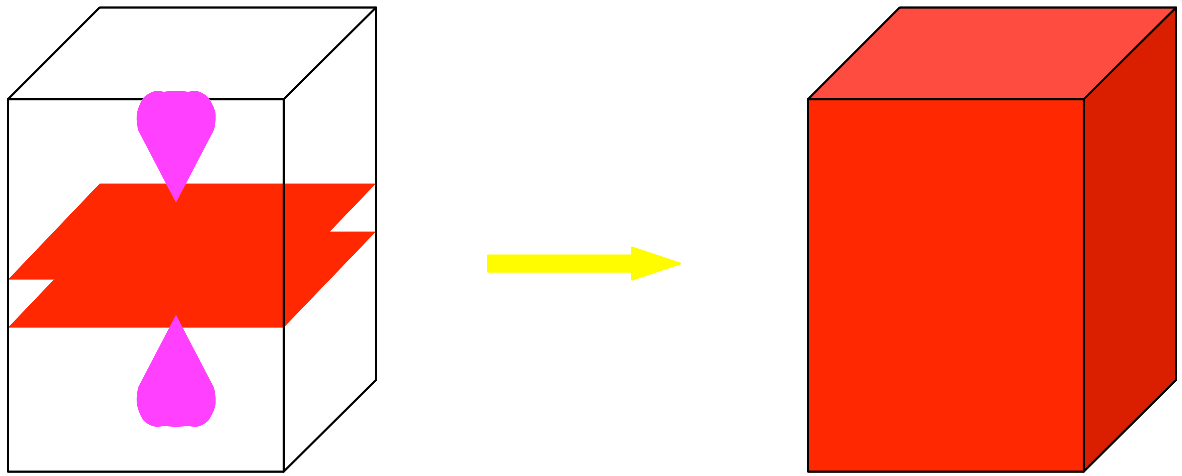
Suppose we could excite all charges appropriate to this compactification

We have 2 charges D1-D5 in 3+1 non-compact dimensions

$$S = 2\pi\sqrt{n_1 n_5}(\sqrt{n_p} + \sqrt{\bar{n}_p})(\sqrt{n_{kk}} + \sqrt{\bar{n}_{kk}})$$
$$= 2\pi\sqrt{N}\frac{E}{m_p m_{kk}}$$

$$N = n_1 n_5$$

D1-D5
+ P $\overline{\text{P}}$
+ KK $\overline{\text{KK}}$



Assumption:

A part N_1 of the D1-D5 effective string fractionates the P P charges

The remainder $N - N_1$ fractionates the P $\overline{\text{P}}$ + KK $\overline{\text{KK}}$ charges

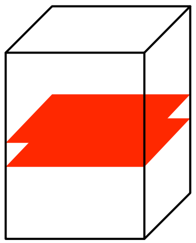
(Suggested by study of supertube excitations,
Giusto + SDM + Srivastava '06)

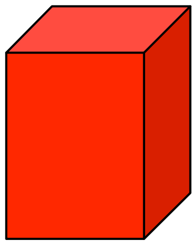
Energy E_1 goes to the P $\overline{\text{P}}$ excitations

Energy $E - E_1$ goes to the P $\overline{\text{P}}$ + KK $\overline{\text{KK}}$ excitations

$$S = 2\pi\sqrt{N_1}\ (2\sqrt{\frac{E_1}{2m_p}}) \quad + \quad 2\pi\sqrt{(N - N_1)}\ \frac{(E - E_1)}{m_p m_{kk}}$$

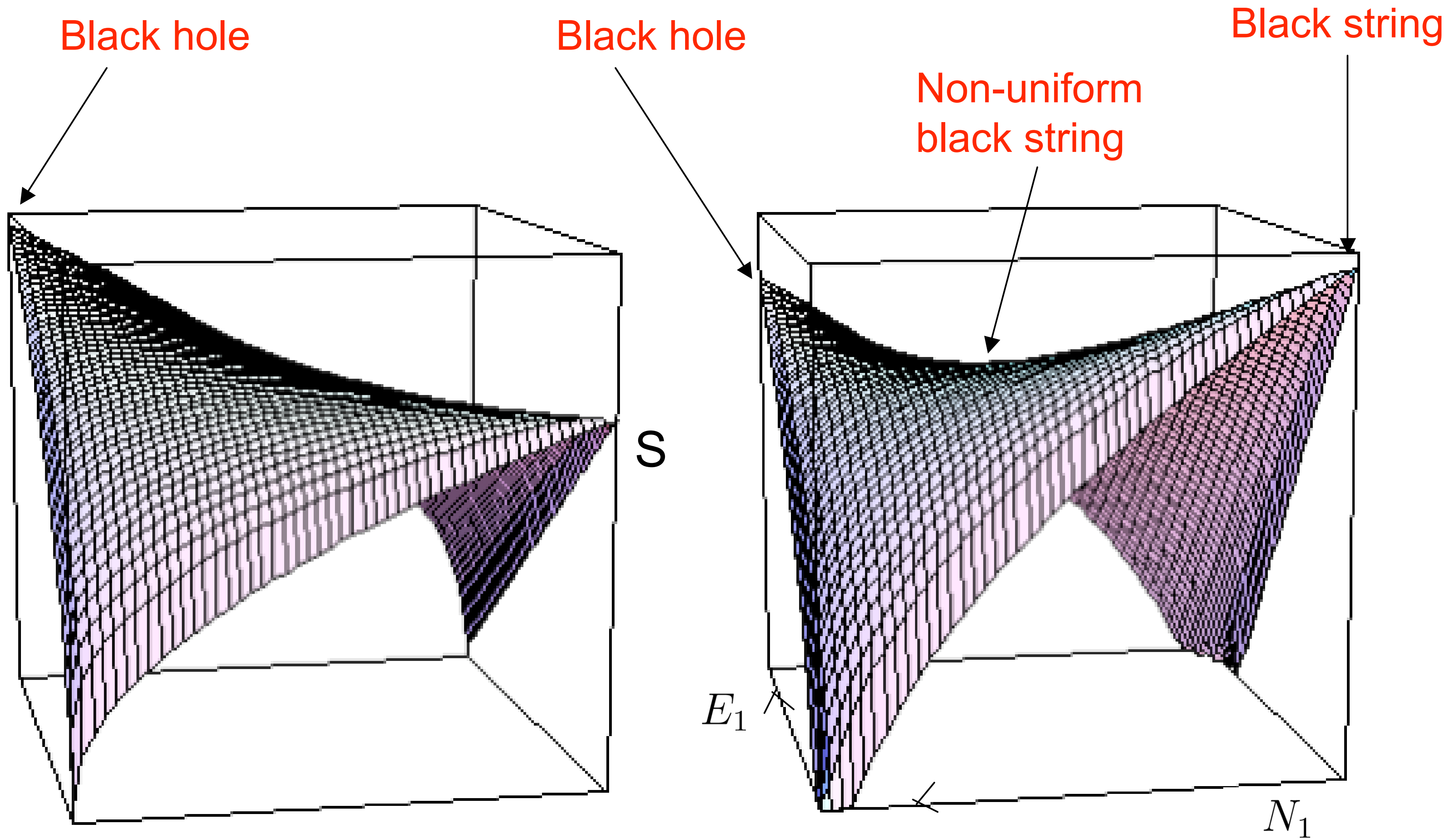
D1-D5
+ P $\overline{\text{P}}$





D1-D5
+ P $\overline{\text{P}}$
+ KK $\overline{\text{KK}}$

$$S = 2\pi\sqrt{N_1}\,\left(2\sqrt{\frac{E_1}{2m_p}}\right) \quad + \quad 2\pi\sqrt{(N-N_1)}\,\frac{(E-E_1)}{m_p m_{kk}}$$



$E/(2m_{kk}) = 0.5$ $\qquad$ $E/(2m_{kk}) = 1.2$

Computing the tension

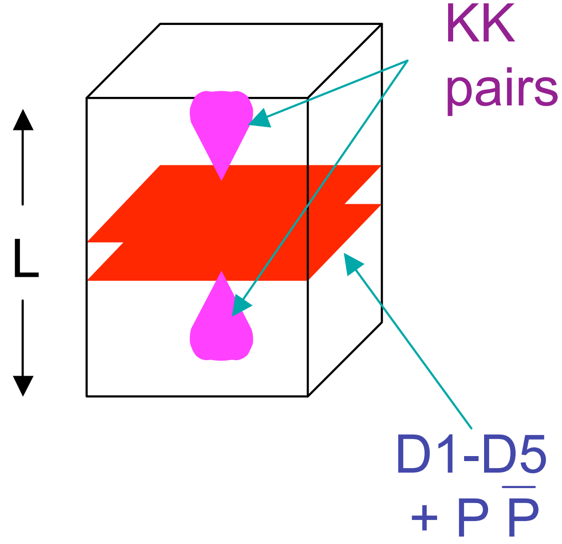
The microscopic model treats the branes as non-interacting

For the black hole solution Tensions (pressures) are found to agree with gravity

(*Horowitz+Maldacena+Strominger '96*)

$$\mathcal{T} = \mathcal{T}_{D1} + \mathcal{T}_{D5} + \mathcal{T}_{P\bar{P}} + \mathcal{T}_{KK\bar{K}K}$$

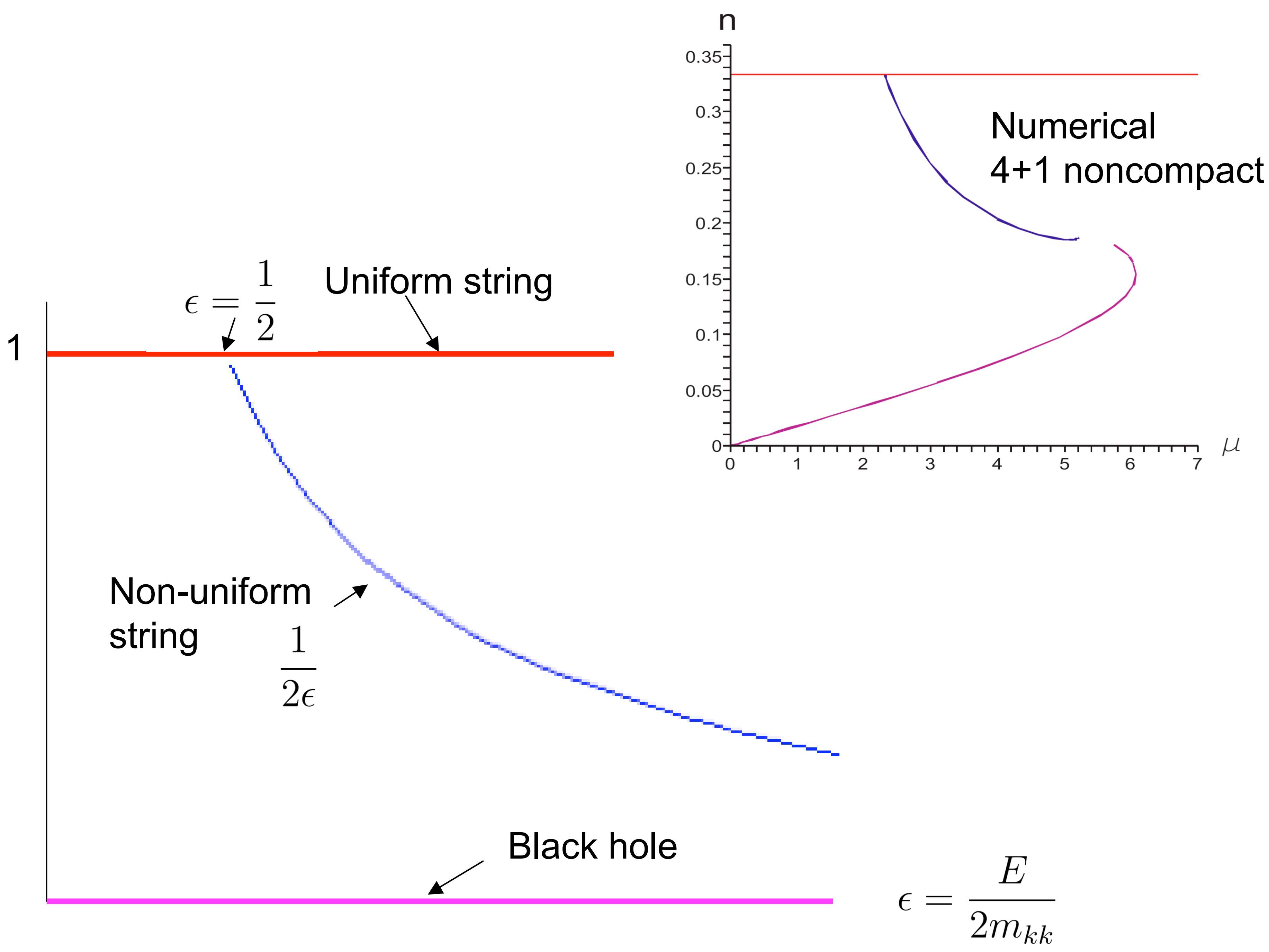
$$\mathcal{T}_{D1} = 0, \quad \mathcal{T}_{D5} = 0, \quad \mathcal{T}_{P\bar{P}} = 0, \quad \mathcal{T}_{KK\bar{K}K} = 2n_{kk}\frac{dm_{kk}}{dL}$$



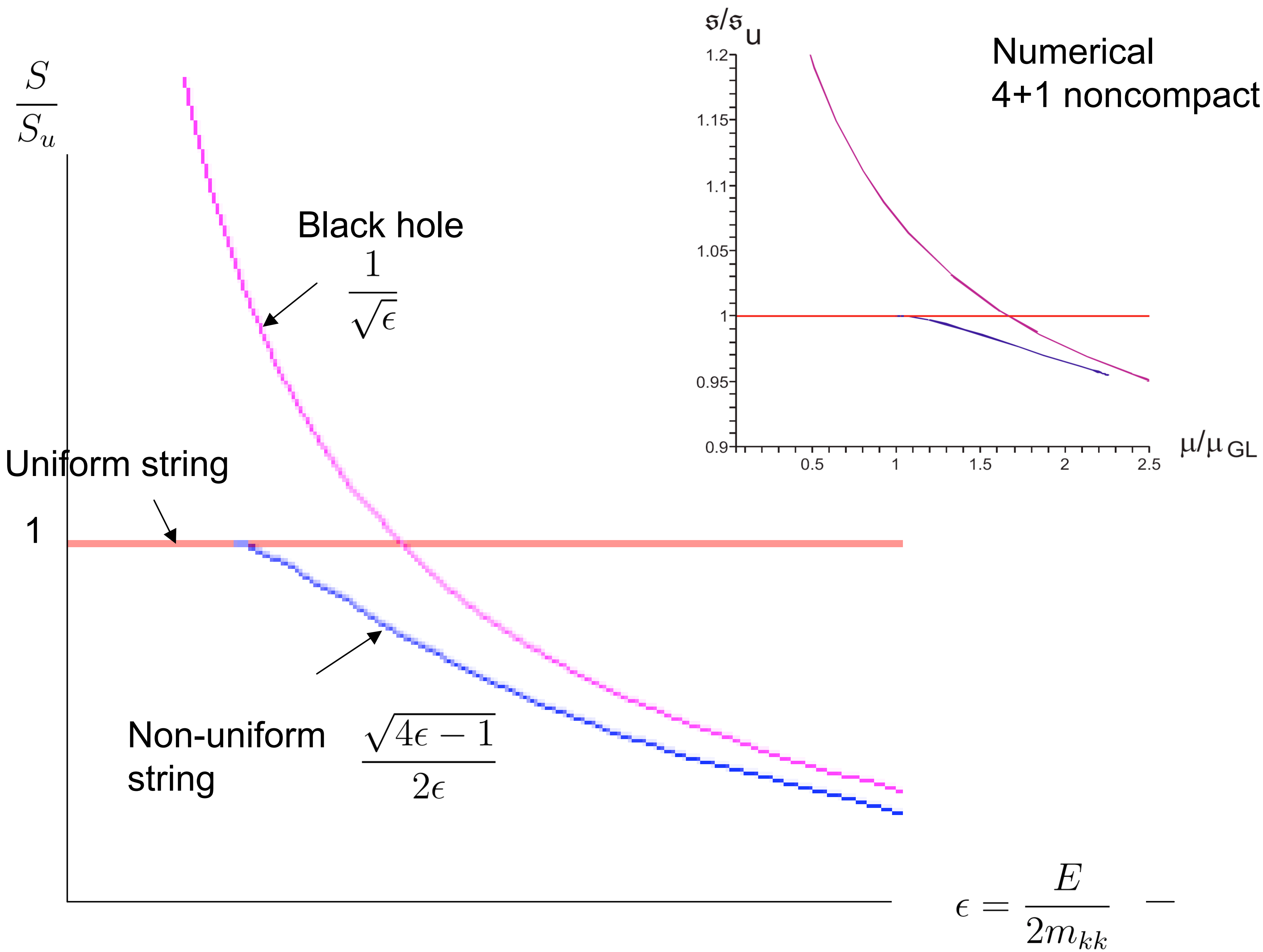
$$n_{kk} = \frac{1}{2m_{kk}}\left(\frac{E-E_1}{2}\right)$$

$$m_{kk} \propto L^2, \qquad \frac{dm_{kk}}{dL} = \frac{2m_{kk}}{L}$$

$$\mathcal{T} = \frac{E-E_1}{L}$$



n
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0 1 2 3 4 5 6 7
μ
Numerical
4+1 noncompact
1
$\epsilon = \frac{1}{2}$
Uniform string
Non-uniform
string
$\frac{1}{2\epsilon}$
Black hole
$\epsilon = \frac{E}{2m_{kk}}$



$\frac{S}{S_u}$
Black hole
$\frac{1}{\sqrt{\epsilon}}$
Uniform string
1
Non-uniform string
$\frac{\sqrt{4\epsilon-1}}{2\epsilon}$
$\epsilon = \frac{E}{2m_{kk}}$
Numerical
4+1 noncompact
$\mathfrak{s}/\mathfrak{s}_u$
1.2
1.15
1.1
1.05
1
0.95
0.9
0.5
1
1.5
2
2.5
μ/μ_{GL}

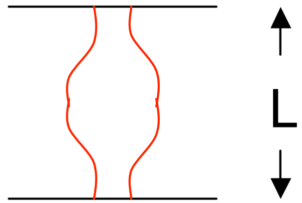
Smarr relations

Black hole and black string solutions satisfy Smarr relations

For 3+1 noncompact dimensions, near extremal, this relation is

$$TS = (2-r)E$$

$$r = \left(\frac{\partial E}{\partial L}\right)_S \frac{L}{E} \quad \text{(tension)}$$



Non-uniform string: *Conjectured gravity ansatz satisfies Smarr*

Microscopic computation:

$$S = A\sqrt{4\epsilon - 1}, \quad E = 2m_{kk}\epsilon$$

$$\frac{1}{T} = \frac{dS}{dE} = \frac{A}{m_{kk}} \frac{1}{\sqrt{4\epsilon - 1}}$$

$$r = \frac{dE}{dL}\frac{L}{E} = \frac{1}{2\epsilon} \Longrightarrow TS = (2-r)E$$

This supports the identification of the saddle point solution with the non-uniform string

Summary of results:

We have made a simple assumption about the nature of fractionation (energy and charges split between two different kinds of excitations)

With this assumption we get a phase diagram that has most of the features of the numerically obtained phase diagram

Black holes and the Uniform string satisfy Smarr relations.

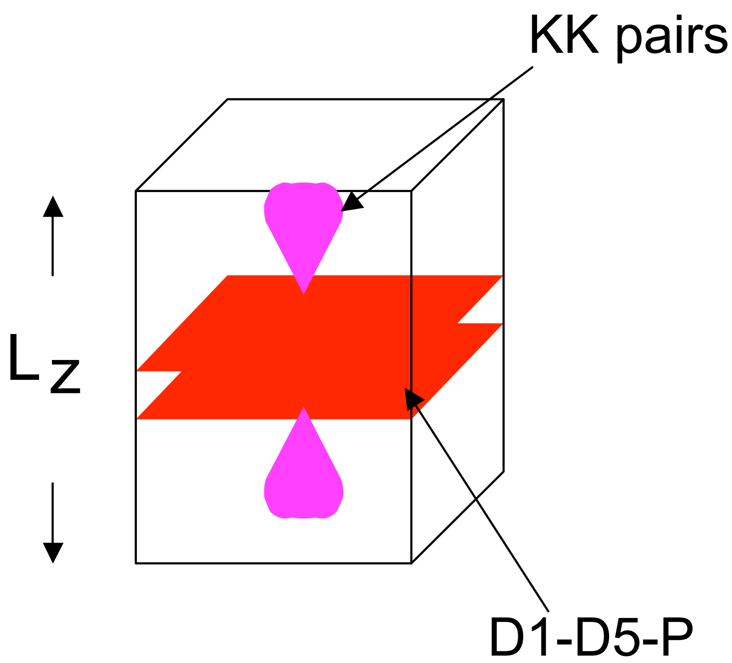
A non-interacting superposition of two systems satisfying Smarr will always satisfy the Smarr relation.

The fact that the numerical gravity solution satisfies Smarr suggests that using a non-interacting model is a good assumption …

The computations we have used are similar to those used to get a crude estimate of the D1-D5-P bound state

(SDM '97)

Put a D1-D5-P extremal bound state in a box of length L_z

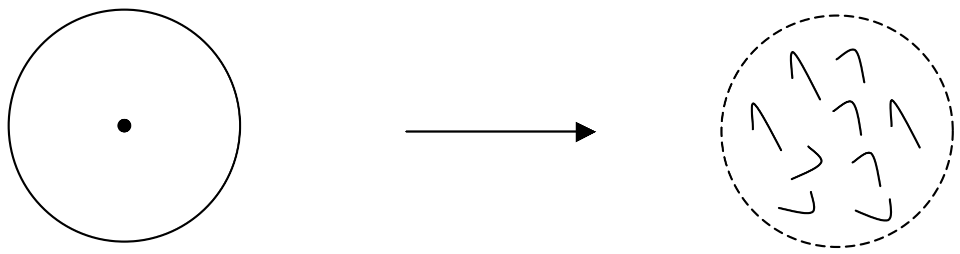


First fractional KK pairs appear when

$$L_z \sim [\frac{g^2 \alpha'^4 \sqrt{n_1 n_5 n_p}}{VR}]^{\frac{1}{3}} \sim R_s$$

(i.e., box size is if order horizon radius)

Construction of microstate geometries



'Fuzzball'

??

2-charge in 4+1 non-compact dimensions: Lunin+SDM

3 charge in 4+1, U(1) X U(1) symmetry: Giusto+SDM+Saxena, Lunin

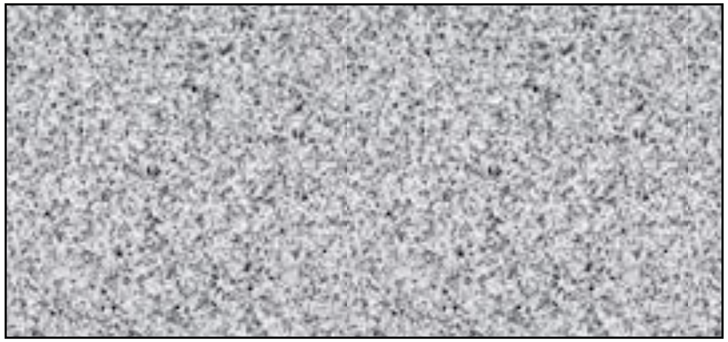
3 charges in 3+1, U(1) X U(1) symmetry: Bena+Kraus

3 charges in 4+1, U(1) symmetry: Bena+Warner, Berglund+Gimon+Levy

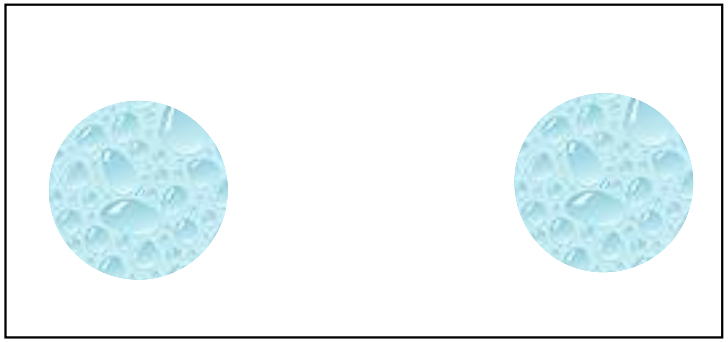
4 charges in 3+1, U(1) X U(1) symmetry: Saxena+Potvin+Giusto+Peet

4 charges in 3+1, U(1) symmetry: Balasubramanian+Gimon+Levi

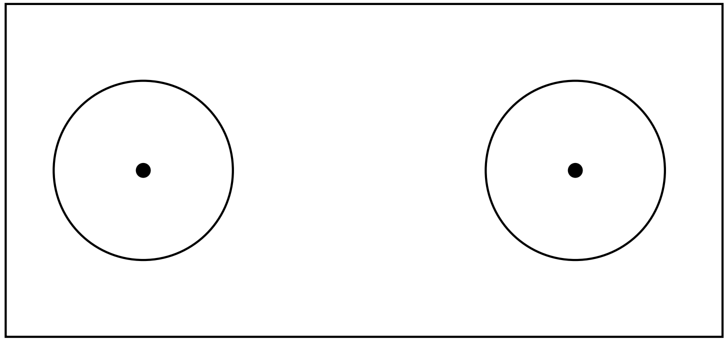
Possible implications for Cosmology … the maximal entropy states are very quantum fuzzy fluids ….



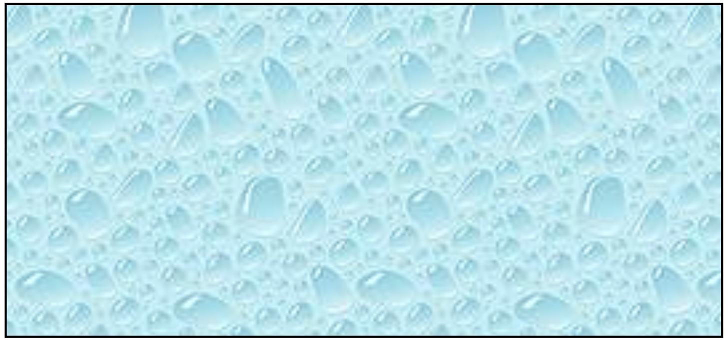
Dust filled Universe



Black holes → Fuzzballs



All matter in Black Holes



Fuzzball matter:
Quantum correlations across
Horizon scales ??