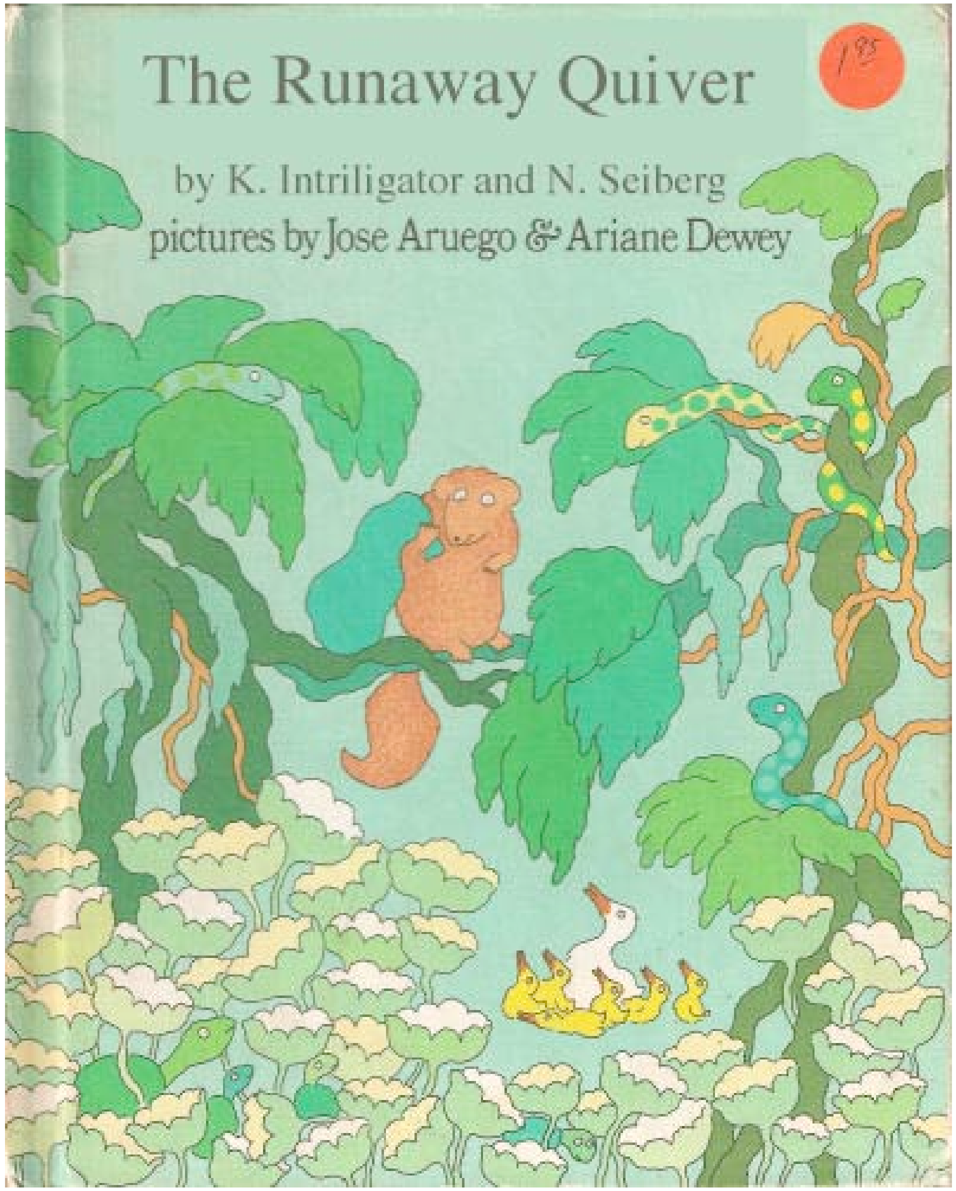
# On the capture of runaway quivers

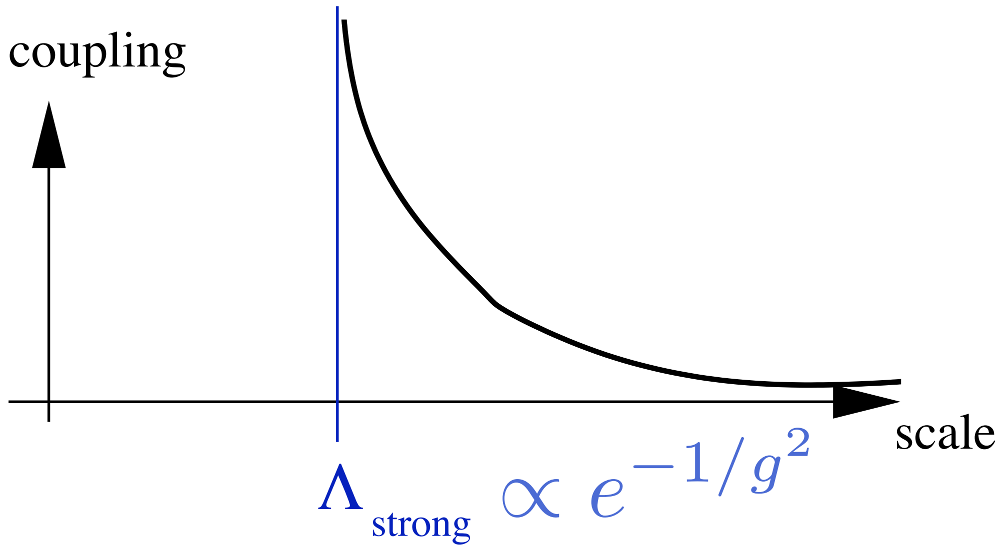
with

B. Florea, S. Kachru, N. Saulina

hep-th/060????


The Runaway Quiver
by K. Intriligator and N. Seiberg
pictures by Jose Aruego & Ariane Dewey

Dimensional transmutation
is a mechanism which
generates large ratios of scales.




Dynamical supersymmetry breaking (DSB)

$$\langle F \rangle \propto \Lambda^2_{\text{strong}}$$

uses it to explain the ratio $\dfrac{M_W}{M_{Pl}}$ .

# How does DSB happen in string theory?

Can we find vacua of string theory
which have their SUSY broken dynamically?

$$\langle F \rangle \propto e^{-1/g}$$

This is not an idle question to ask of
our UV completion of gravity

for at least two reasons:

1

# Moduli stabilization

In string theory, there are no coupling constants.
When gauge theory couplings become fields,
the vacuum structure gets rearranged.

2

# UV sensitivity from SUSY

RG hides microphysics
but the superpotential is not renormalized.
**Technical naturalness of**
**an arbitrarily chosen superpotential**
**is a source of UV sensitivity.**

# Outline

0. Motivation

1. ‘SUSY breaking by obstructed deformation’

and its discontents

2. Stringy nonperturbative effects

in the presence of space-filling branes

3. D3 instantons in a CY with $dP_1$ singularity

4. Vacuum structure

# DSB by D-branes

D-branes carry gauge theories.

Interesting ones live on branes at singularities.

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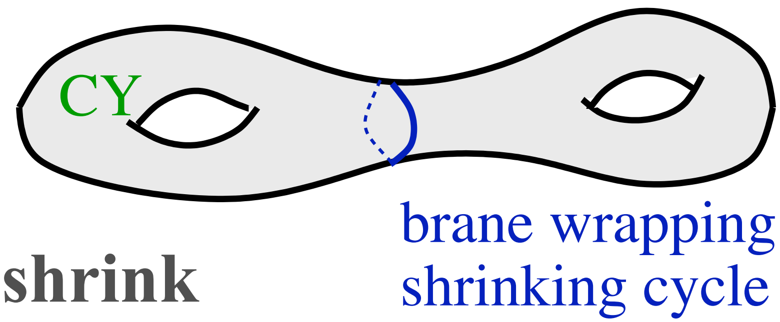
**Singularities arise from shrinking things.**




**What can shrink supersymmetrically?**

shrinking a curve in CY → conifold.

**Next case: surfaces**

A surface in a CY which can be shrunk **is a del Pezzo surface.**

# Branes stuck to shrinking dPs

Berenstein Herzog Ouyang Pinansky, hep-th/0505029
Franco Hanany Saad Uranga, hep-th/0505040
Bertolini Bigazzi Coltrone, hep-th/0505055

gauge-string duality

gaugino condensates ⟷ complex structure deformations

(Klebanov-Strassler, Vafa)

del Pezzo cones are not complete intersections

(unlike conifold)

⇒ hard to deform (Altmann)

Looks like gravity dual of Konishi anomaly:

$$\mathrm{tr} W_\alpha W^\alpha \propto \frac{\partial W}{\partial \phi} = F_\phi$$

# the DSB representation of $dP_1$

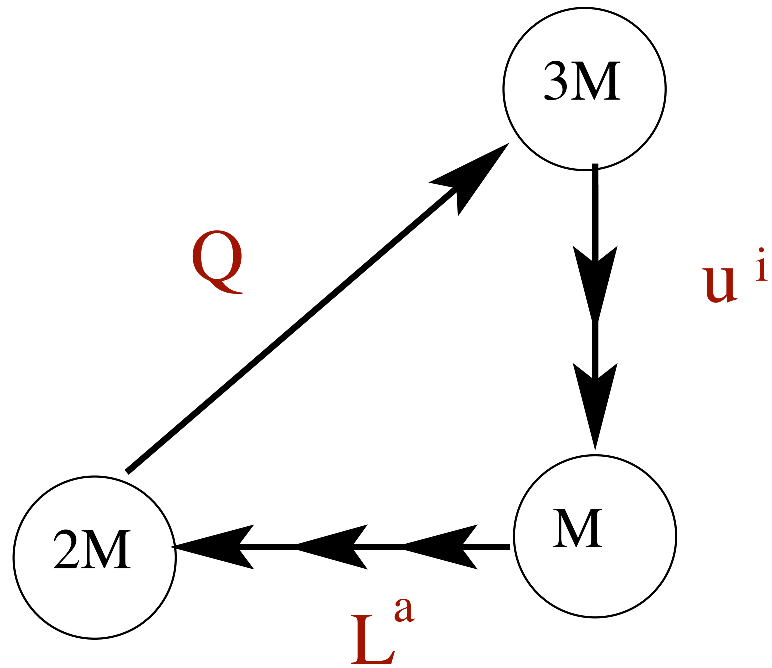
Lots of work was done
to figure out what quiver
corresponds to what geometry.

very similar to 3-2 model

Affleck Dine Seiberg 1984

$$W_{\text{tree}} = \lambda_{ia}\ Q u^i L^a$$

$$a = 1, 2, 3 \quad i = 1, 2$$



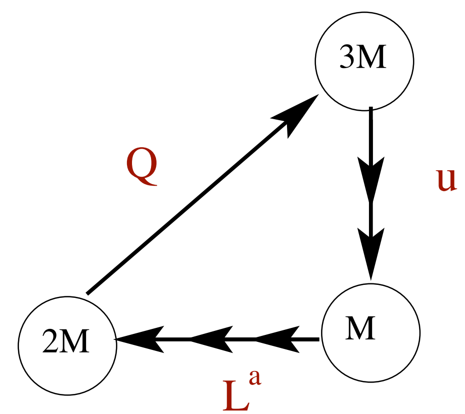

breaks flavor symmetry $SU(3) \times SU(2) \longrightarrow SU(2)_{\text{diag}}$

$SU(M)$ and $U(1)$ factors are IR free.

# Symmetries of the quiver

| | gauge symmetries | | | global symmetries | | |
|---|---|---|---|---|---|---|
| | $SU(3M)$ | $SU(2M)$ | $SU(M)$ | $[SU(2)$ | $U(1)_F$ | $U(1)_R]$ |
| $Q$ | $\mathbf{3M}$ | $\overline{\mathbf{2M}}$ | $\mathbf{1}$ | $\mathbf{1}$ | $1$ | $-1$ |
| $\overline{u}$ | $\overline{\mathbf{3M}}$ | $\mathbf{1}$ | $\mathbf{M}$ | $\mathbf{2}$ | $-1$ | $0$ |
| $L$ | $\mathbf{1}$ | $\mathbf{2M}$ | $\overline{\mathbf{M}}$ | $\mathbf{2}$ | $0$ | $3$ |
| $L_3$ | $\mathbf{1}$ | $\mathbf{2M}$ | $\overline{\mathbf{M}}$ | $\mathbf{1}$ | $-3$ | $-1,$ |

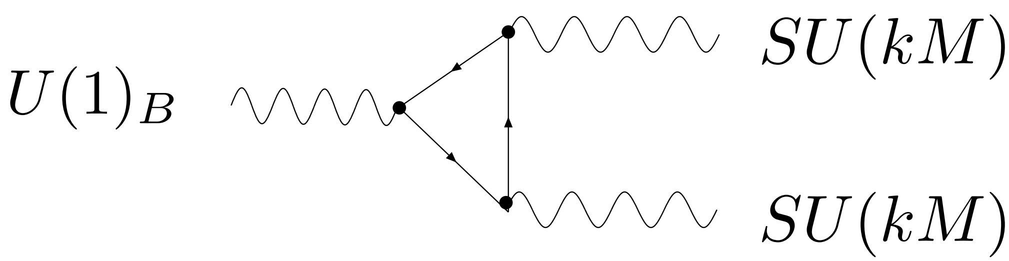
$$W_{\text{tree}} = \lambda Q \epsilon_{ij} u^i L^j$$

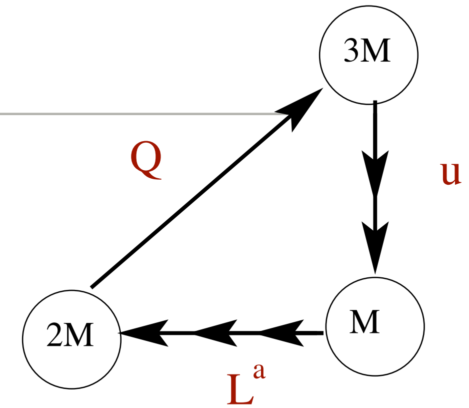



For M=1, SU(3) has $N_f = N_c - 1$

$$\Rightarrow \quad W_{ADS} = \frac{\Lambda_3^7}{\det Q \cdot u}$$

# anomalies




Mixed anomalies give mass to the baryonic U(1)s by the GS mechanism.

Dine Seiberg Witten 1985

$$L = ... + \phi \,\mathrm{tr} F \wedge F + m^2 (\partial\phi + A)^2$$

$\phi$ is a RR axion.

$$A \rightarrow A + d\lambda$$
$$\phi \rightarrow \phi - \lambda$$

Massless closed strings are inextricably involved in the problem.

# Runaway

Intriligator Seiberg, hep-th/0512347 :

The theory with gauge group $SU(3) \times SU(2)$ (M=1)

has no vacuum at finite distance in field space.

$L$ s run away: $\mathcal{V}(V) \propto (V^\dagger V)^{-1/6}$

$$V^a \equiv \det(L^a, L^b)\epsilon_{abc}$$

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'SUSY-BOG' crucially used D-term conditions from $U(1)_B$ s:

$\sum |L|^2 = \xi$ $\Longrightarrow$ $L$'s are bounded.

# This isn't the end of the story:

This is the theory in a certain decoupling limit of "local $dP_1$" where

$$m(U(1)_B) \to \infty.$$

In a compact CY, with $m_s < \infty$, $U(1)_B$s matter.

( It can be embedded in a compact CY. )

Diaconescu Florea Kachru Svrcek, hep-th/0512170

# Including the baryonic U(1)s

There are two independent anomalies.

$dP_1$ has two 2-cycles, $c, f$ .

$$\phi_S \equiv \int_{dP_1} C^{(4)}_{RR} \qquad \phi_c \equiv \int_{dP_1} C^{(2)}_{RR} \wedge c \qquad \phi_f \equiv \int_{dP_1} C^{(2)}_{RR} \wedge f$$

We find their charges by demanding that

$$\delta\Gamma_{\text{eff}} = -\delta\left(\sum_{\alpha=1}^{3}\int_{\text{branes, }\alpha}\sum_{p} C^{(p)}_{RR}\wedge \sqrt{\text{Td}}\wedge \text{ch}V_\alpha \wedge \text{tr}_\alpha F\wedge F\right)$$

| | $U(1)_1$ | $U(1)_2$ | $U(1)_3$ |
|---|---|---|---|
| $e^{i\phi_S}$ | -4 | -14 | -18 |
| $e^{i\phi_c}$ | 1 | 2 | -3 |
| $e^{i\phi_f}$ | 2 | 4 | -6 |

$\exists$ Neutral combination: $2\phi_c - \phi_f$

# “Kahler moduli are charged”

**important question:**

kahler moduli in IIB are stabilized by

euclidean D3-branes $\Delta W \sim e^{-\rho}$

Witten, hep-th/9604030
KKLT, hep-th/0301240

$$\rho \equiv \int_D (J^2 + iC^{(4)}_{RR}) = \sigma + i\phi_S$$

but now this isn’t gauge invariant!

$$\rho \mapsto \rho + i\lambda, \;\; A_B \mapsto A_B + d\lambda$$

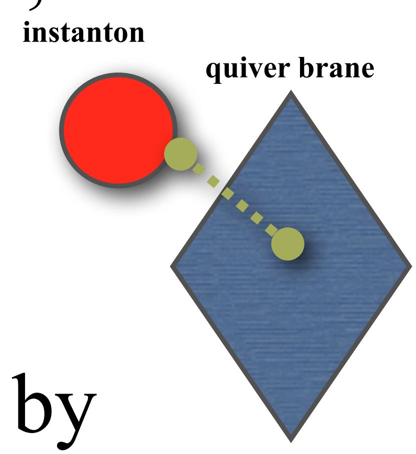
How to make a gauge-inv’t potential
for kahler moduli?

# A hint

**A Note on zeros of superpotentials in F theory.**
Ori J. Ganor (Princeton U.) . PUPT-1672, Dec 1996. 12pp.
Published in **Nucl.Phys.B499:55-66,1997**
e-Print Archive: **hep-th/9612077**

Massless strings stretching between the instanton and spacefilling branes

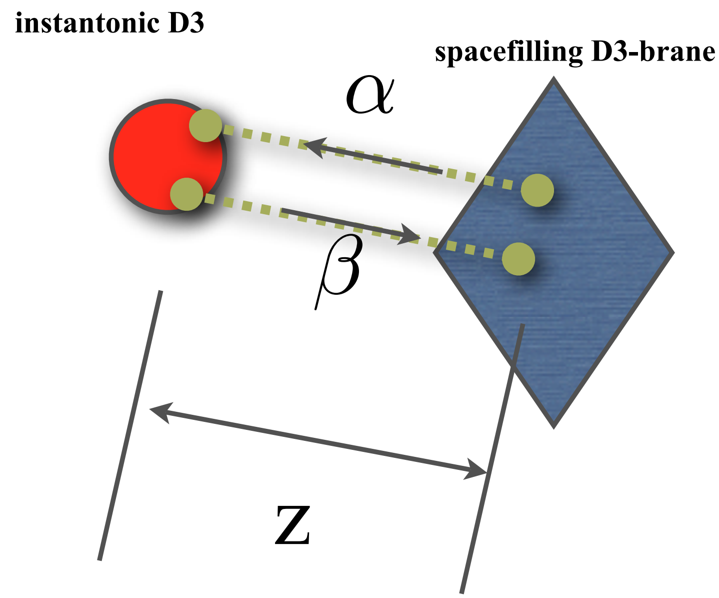
act like collective coords of the instanton, and couple to quiver fields.




Integrating out these modes multiplies the instanton contribution by a function of the quiver fields.

# The instanton prefactor is a field theory operator

Ganor, hep-th/9612077




$$L_{\rm disc} = \alpha \cdot Z \cdot \beta$$

an ordinary
Grassmann integral

$$\Delta W(\rho, Z) \sim e^{-\rho} \int d\alpha d\beta \; e^{\alpha \cdot Z \cdot \beta} \sim Z \; e^{-\rho}$$

# Which D-branes contribute?

# del Pezzo D-geometry

Wijnholt Herzog Walcher Aspinwall Karp Melnikov Nogin...

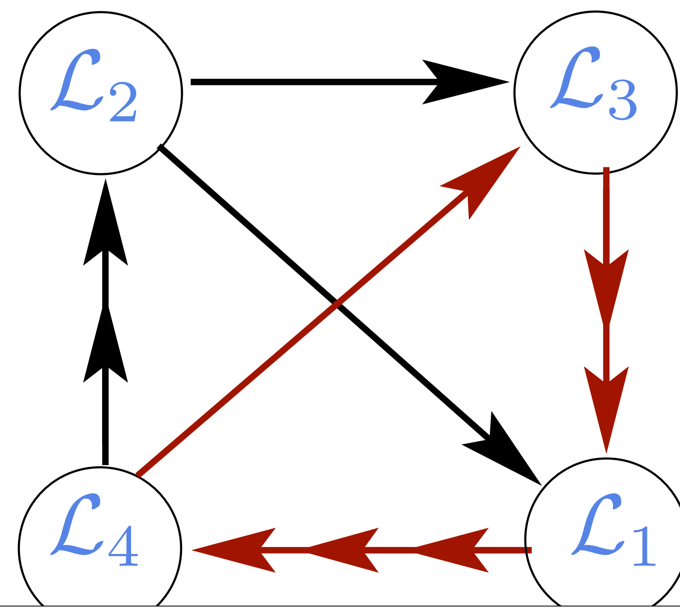
an “exceptional collection” of branes on $dP_1$ is:

$$\{\mathcal{L}_1, \ldots, \mathcal{L}_4\} \equiv$$

$$\{\mathcal{O}_{dP_1}, \mathcal{O}_{dP_1}(c+f), \overline{\mathcal{O}_{dP_1}(f)}, \overline{\mathcal{O}_{dP_1}(c)}\}$$

(the DSB representation above is $\mathcal{L}_1 \oplus 2\mathcal{L}_4 \oplus 3\mathcal{L}_3$ )

**we need to know this because we are going to study euclidean branes and their interactions with these D7s**



# Counting Ganor strings

Twisting of 3-7 strings:

reduction of hypermultiplet on dP

- net number of 3-7 bosons is counted by

$$h^0(dP, \mathcal{L}_A \otimes \mathcal{L}_B^\star) - h^0(dP, \mathcal{L}_B \otimes \mathcal{L}_A^\star)$$

- net number of 3-7 fermions are counted by

$$\chi(\mathcal{L}_A \otimes \mathcal{L}_B^\star) \equiv \sum_{p=0}^{3} (-1)^p h^p(dP, \mathcal{L}_A \otimes \mathcal{L}_B^\star)$$

- fermions from $h^0(...)$ are paired with bosons by two supercharges

# The ADS instanton is a D3 brane

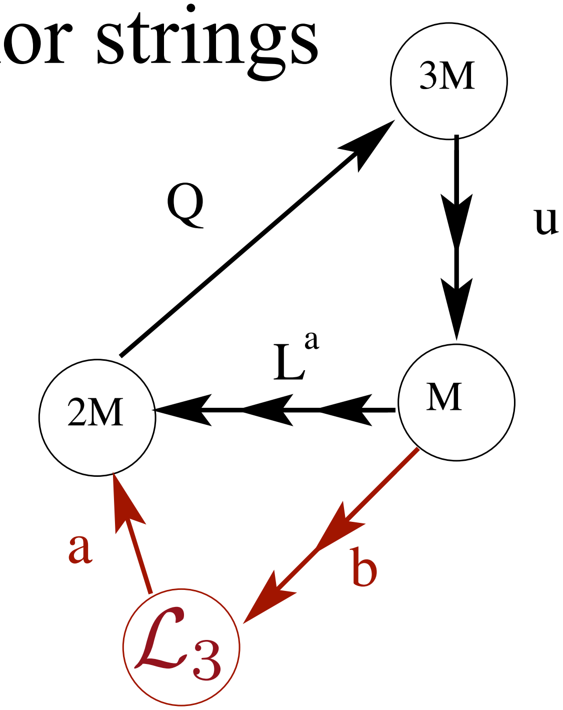
(for M=1)

D3 on SU(3) node = field theory instanton

There is a net number of *bosonic* Ganor strings

$$L_{\rm disc} \sim a(Q \cdot u^i) b_i$$

$$\Delta W \propto \int dadb \; e^{a(Q\cdot u)b} = \frac{1}{\det Q \cdot u}$$




see: Bershadsky et al, hep-th/9612052

# What about Witten's criterion?

Witten, hep-th/9604030

In the M-theory lift, an M5-brane wrapping a divisor D contributes $\exp\left(-\int_D \left(J^3 + iC^{(6)}\right)\right)$.

This carries R-charge: $2\chi(D) = 2\sum_{p=0}^{3}(-1)^p h^{0,p}(D)$

If this is to be a term in W: $\chi = 1$

Our D3-branes lift to M5-branes with $\chi = 0$.

But, the Ganor strings produce an operator $\mathcal{O}$ with R-charge $q_R = 2$.

**Generalized criterion:** $2\chi + q_R(\mathcal{O}) = 2$

# Other instantons

For a certain class of line bundles

$$X_n \equiv \overline{\mathcal{O}(2(1-n)c + nf)}$$

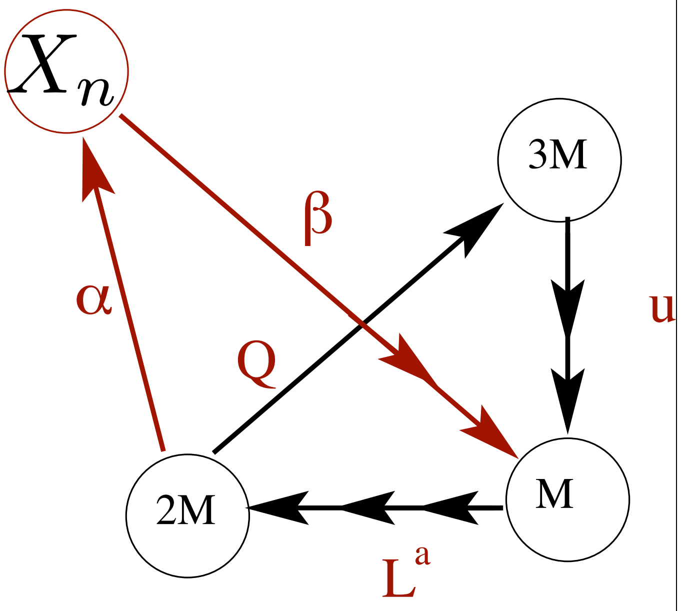
there is a net number of *fermionic* Ganor strings

$$L_{\text{disc}} \sim \alpha(L^a d_a^i)\beta_i$$

$d_a^i$ are some numbers

$$\Delta W \propto \int d\alpha d\beta \; e^{\alpha(L^a)\beta_i d_a^i}$$

$$= \det(L^2, L^3) = V^1$$



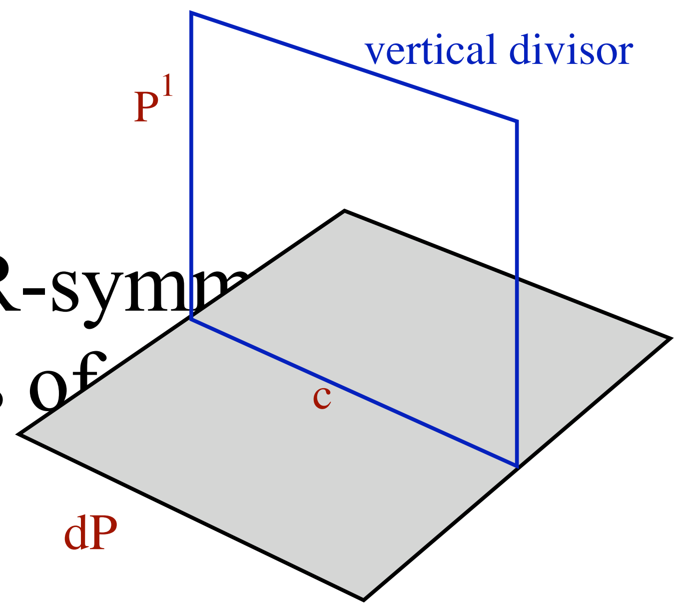

This cancels the charges of the instanton action factor.

# Other instantons

1 Many other candidate instantons vanish because of unpaired fermion zeromodes:

All euclidean D-strings,

and all ‘vertical’ branes: $\mathbb{P}^1 \rightarrow$ curve in $dP_1$




2 The non-anomalous R-symm
forbids multicovers of

# cartoon of result

$$W = QuL + \frac{e^{-\rho_1}}{\det Qu} + e^{-\rho_2} \det\left(L^2, L^3\right)$$

anomalous

Note that the baryon breaks the flavor symmetry.
It must do this to preserve $U(1)_R$

non-anomalous

# more accurate version of result

$$W = \lambda Q u^i L^j \epsilon_{ij} + \frac{e^{-\rho_1}}{\det Qu}$$

$$+e^{-\rho_1}\left(\sum_n c_n e^{-n\rho'}\right)\det(L^2, L^3)$$

contribution of D3 on $X_n$

# Vacuum structure

# Effect of baryon term

A very similar field theory was studied in

Poppitz Shadmi Trivedi, hep-th/9606184

(w/o anomalous U(1)s and inflow).

ALSO: comment in Intriligator Seiberg.

$$W_{\text{tree}} = \lambda_{ia} Q u^i L^a + \epsilon_{a_1 \ldots a_N} \alpha^{a_1} \det(L^{a_2}, \ldots, L^{a_N})$$

$$i = 1..N-1, \; a = 1..N$$

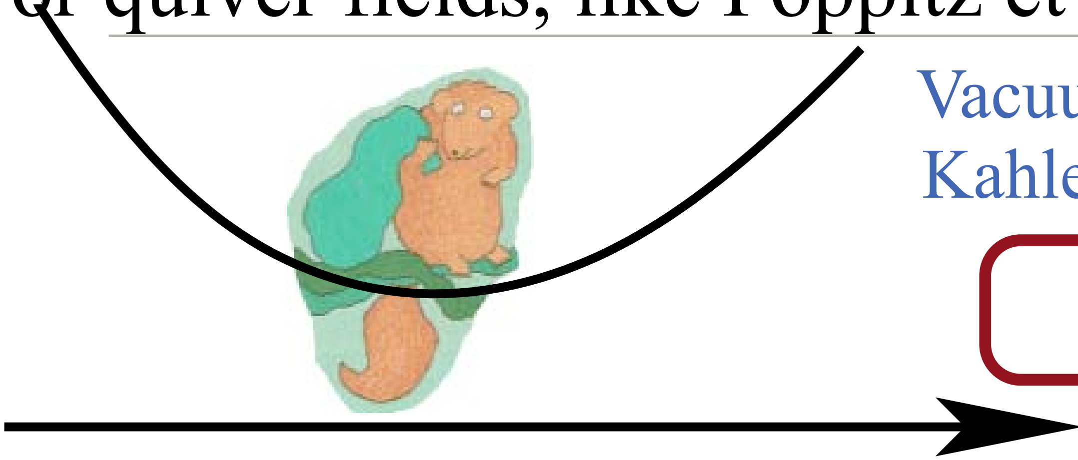
DSB if: $\lambda_{ia}\alpha^a \neq 0$

This is the condition that the flavor symmetry is completely broken.

*V* lifts the classical flat directions.

# Summary of vacuum structure

For quiver fields, like Poppitz et al.



Vacuum occurs at vevs where Kahler potential is canonical.

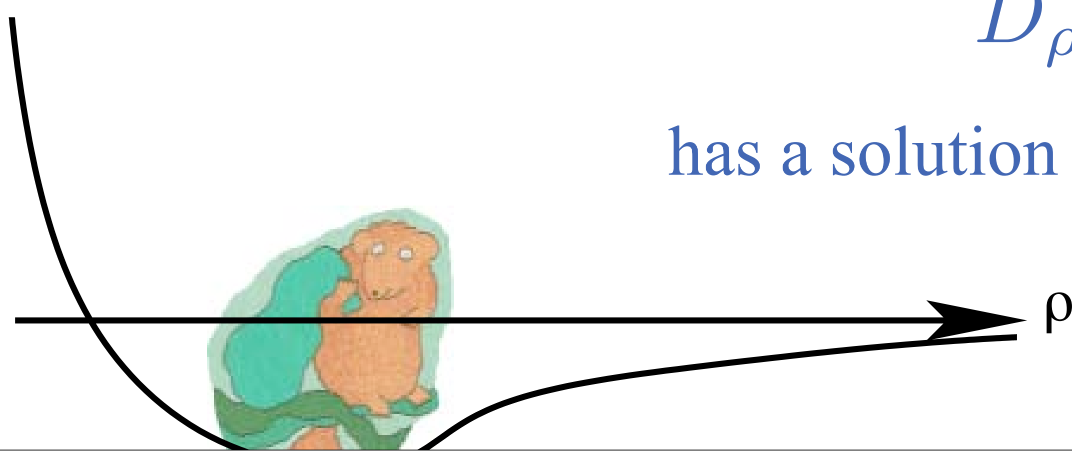
$$\langle F \rangle \propto \Lambda_3^{\#}$$

quiver fields

For Kahler moduli, like KKLT. $W = W_0 + \langle \mathcal{O} \rangle e^{-\alpha\rho}$

$$D_\rho W = 0$$

has a solution for generic $K(\rho, \bar{\rho})$.



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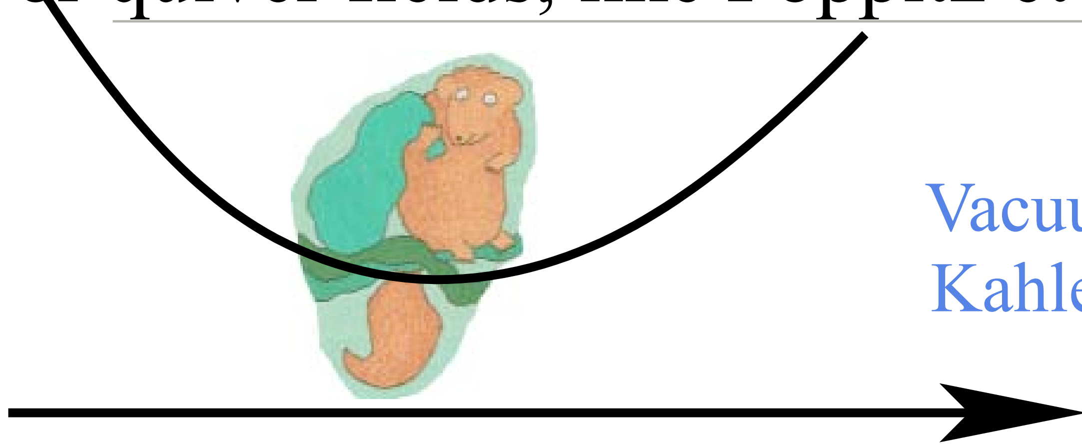


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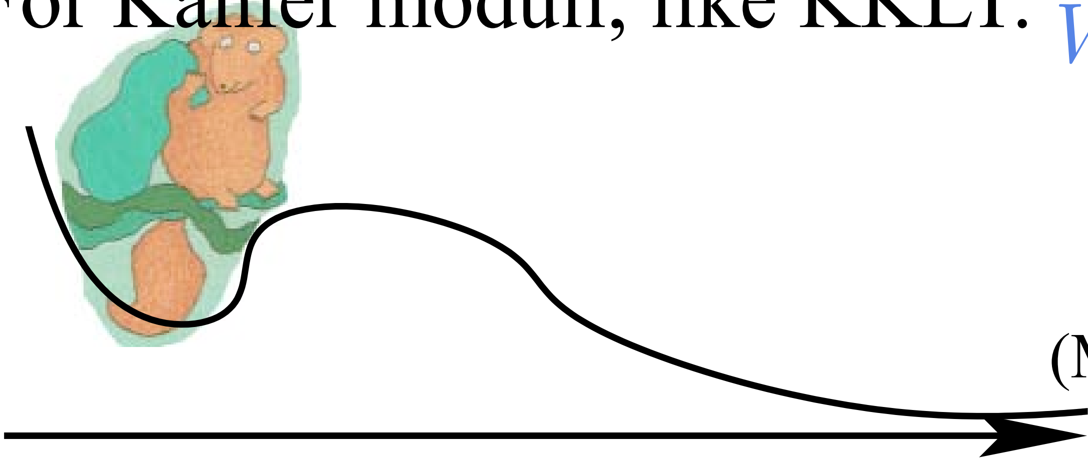



(Metastable in rho direction.)

Lifetime determined by constant in W.

# Some final words

# Final comments

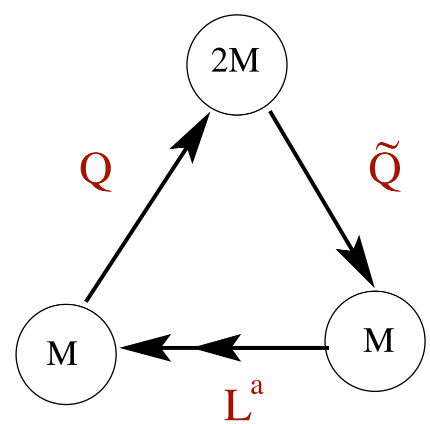
**1** $V$ can be thought of as position of D3 dissolved in quiver.

$\Delta W \propto V$ reduces to Ganor's result.

**2** $\Delta W \propto V$ is not a field theory instanton here, but perhaps it is in another UV completion.

**3** Sensitivity to embedding in compact model?

**4** This technology generalizes to other DSB representations:



the end