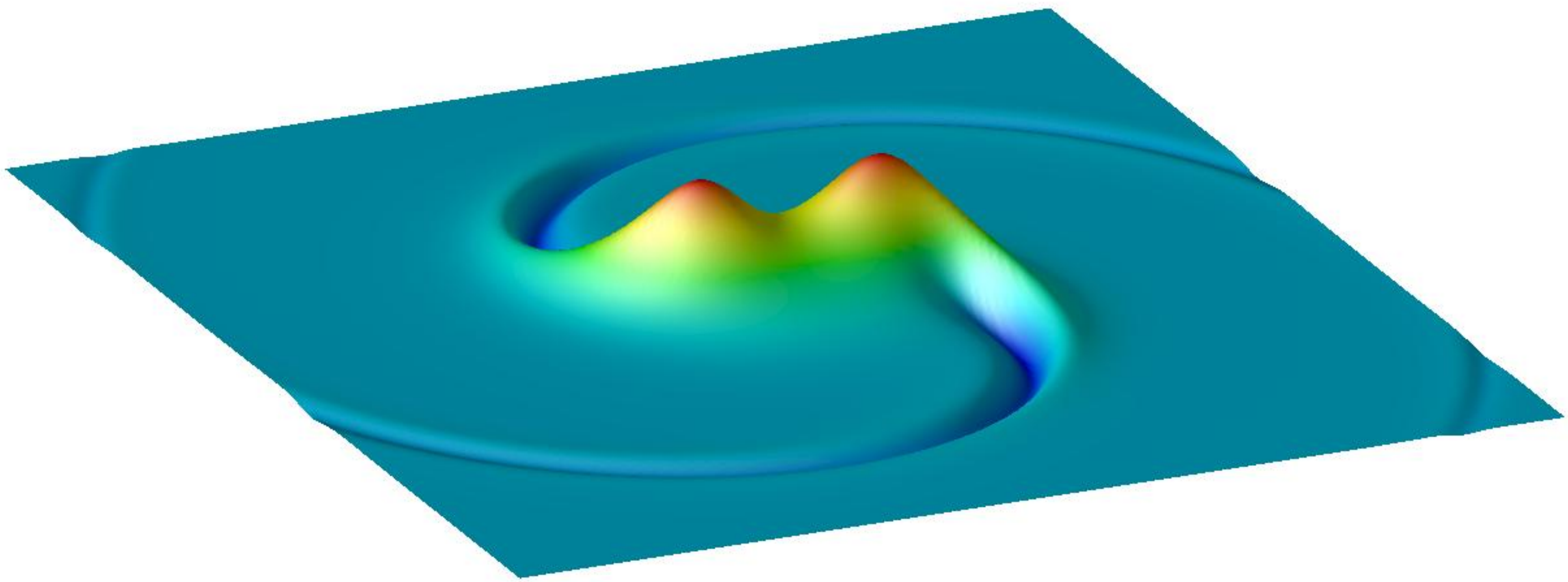
PERIMETER INSTITUTE
FOR THEORETICAL PHYSICS

Holographic Theories with Fundamental Matter



with: Rowan Thomson & David Mateos
(Kruczenski, Hovdebo, Starinets, ...)

Outline:

1. Gauge/Gravity from Dp-branes:
   add probe branes ~ fundamental matter
   → "universal" meson spectra

RCM + Thomson (hep-th/0605017)
Arean + Ramallo (hep-th/0602174)

2. Probe branes in thermal backgrounds
   → self-similar embeddings
   → first-order phase transition

Mateos, RCM + Thomson (hep-th/0605046)

Aharony, Sonnenschein & Yankielowicz (hep-th/0604161)
Albash, Filev, Johnson & Kundu (hep-th/0605088; hep-th/0605175)
Karch & O'Bannon (hep-th/0605120)
Peeters, Sonnenschein & Zamaklar (hep-th/0606195)

3. Conclusions/Future Directions

Itzhaki, Maldacena, Sonnenschein & Yankielowcz

Gauge/Gravity Duality from Dp-branes: (p<5)

d=10 Type II superstrings in Dp-brane throat with N_c units of RR flux

equivalent to

d=p+1 $\mathcal{N}$ =4 U(N_c) super-Yang-Mills

dimensionless effective coupling:

$$g_{YM}^2 \sim g_s \, \ell_s^{p-3}$$
$$g_{YM}^2 N_c \sim (R_p/\ell_s)^{7-p} \, \ell_s^{p-3}$$

YM coupling dimensionful !

Itzhaki, Maldacena, Sonnenschein & Yankielowcz

Gauge/Gravity Duality from Dp-branes: (p<5)

d=10 Type II superstrings in Dp-brane throat with N_c units of RR flux

dilaton and curvature run!

equivalent to

d=p+1 $\mathcal{N}$ =4 U(N_c) super-Yang-Mills

dimensionless effective coupling:

$$g_{eff}^2 = g_{YM}^2 N_c U^{p-3}$$
$$U = r/\ell_s^2$$

Supergravity only in intermediate region: $1 \ll g_{eff} \ll N_c^{\frac{4}{7-p}}$

Itzhaki, Maldacena, Sonnenschein & Yankielowcz

Gauge/Gravity Duality from Dp-branes: (p<5)

d=10 Type II superstrings in Dp-brane throat with N_c units of RR flux

equivalent to

d=p+1 $\mathcal{N}$ =4 U(N_c) super-Yang-Mills

all adjoint fields!

dimensionless effective coupling:

$$
\begin{aligned}
g_{eff}^2 &= g_{YM}^2 N_c U^{p-3} \\
U &= r/\ell_s^2
\end{aligned}
$$

Supergravity only in intermediate region: $1 \ll g_{eff} \ll N_c^{\frac{4}{7-p}}$

(Karch and Katz)

Fundamental fields:

Decoupling limit of N_c Dp-branes with N_f Dq-branes

Low-energy limit with $\alpha' E^2, L^2/\alpha' \to 0$

Field theory:

U(Nc) super-Yang-Mills ← $U(N_c)$ adjoint

coupled to N_f massive hypermultiplets ← fund. in $U(N_c)$ & global $U(N_f)$

(SUSY: $\mathcal{N}=4 \to \mathcal{N}=2$)

Gravity theory:

Dp-throat containing Dq probe brane

Gauge/gravity dictionary:

supergravity modes: $h_{\mu\nu} \quad \leftrightarrow \quad T_{\mu\nu}$

Dq-brane modes: $A_\mu^{ij} \quad \leftrightarrow \quad J_\mu^{ij} \simeq \mathrm{Tr}\left[\bar{\psi}^i \gamma_\mu \psi^j + \Phi^i D_\mu \Phi^j\right]$

SUSY embeddings: stable

Dp/D(p+4): generalizes D3/D7

(p+1)-dim. gauge theory with fundamental matter

Dp/D(p+2): with fundamental matter **on codim. 1 defect**

Dp/Dp: with fundamental matter **on codim. 2 defect**

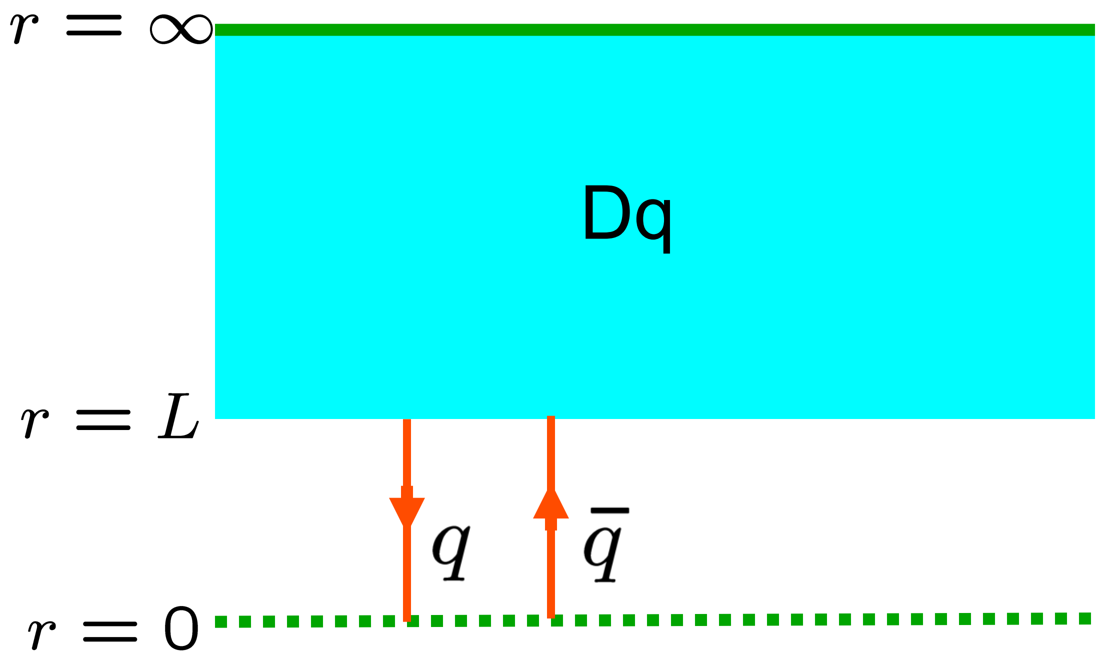
Probe approximation: $N_f / N_c \rightarrow 0$

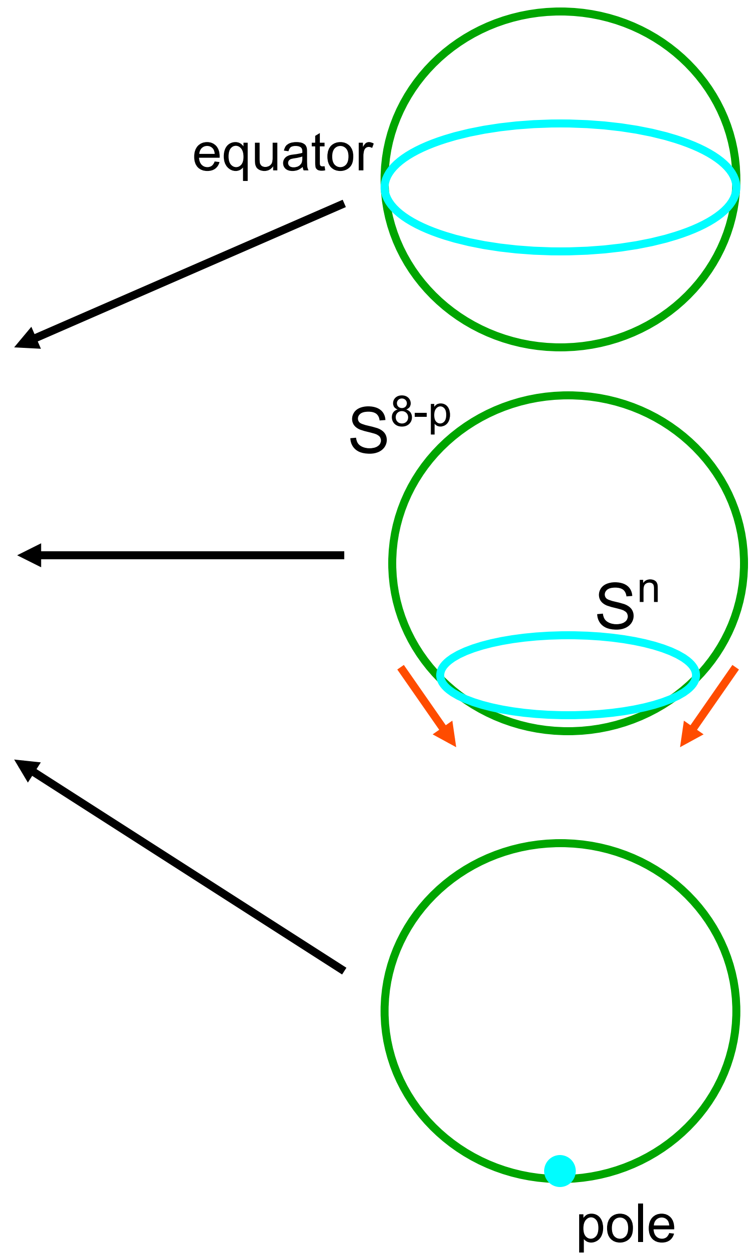
The above construction does not take into account the "gravitational" back-reaction of the D7-branes!

$\rightarrow$ considering large-N_c limit with N_f fixed

(see, however: Burrington et al; Kirsch & Vaman; Casero, Nunez & Paredes)

Geometry:

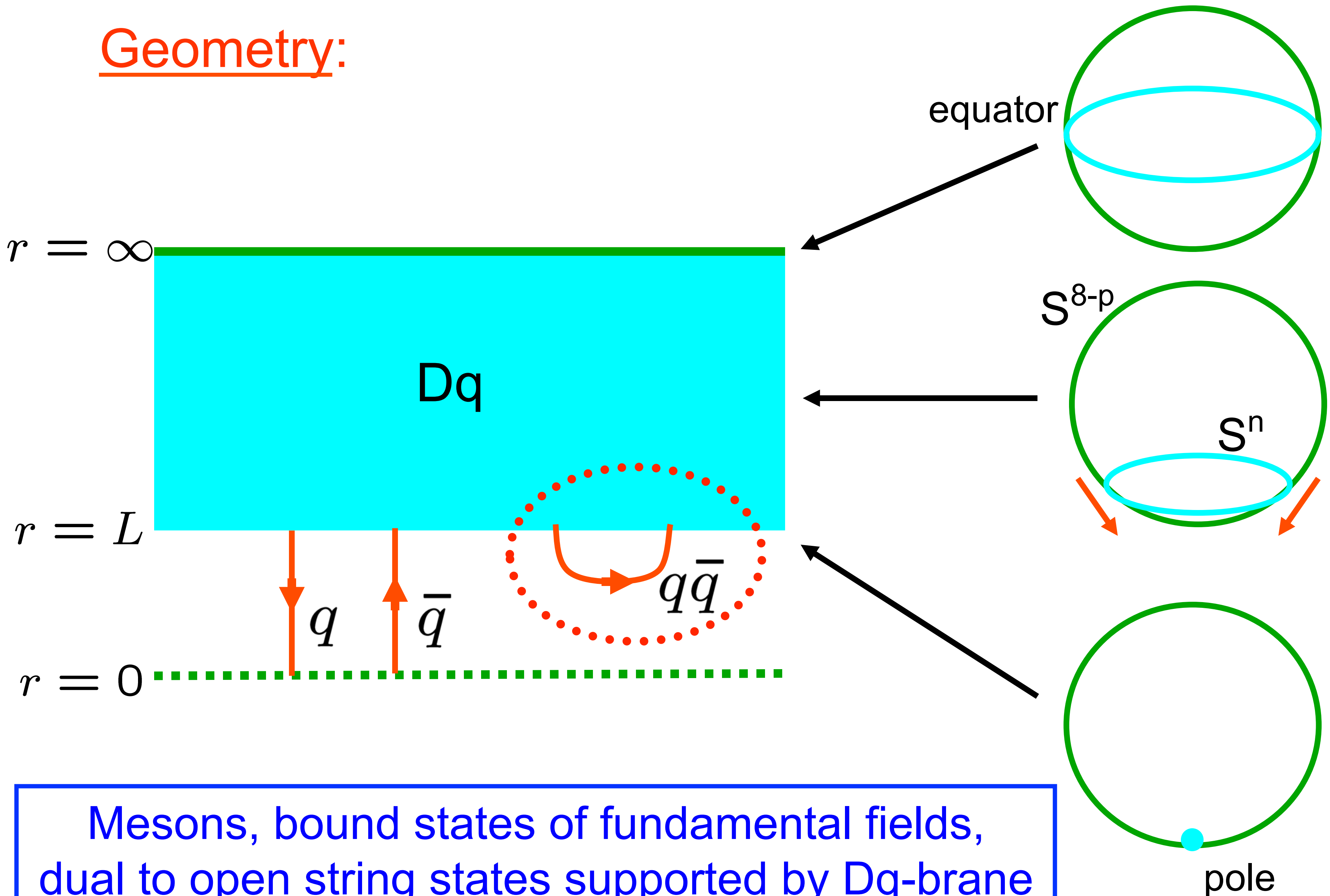




Free quarks appear with mass:

$$m_q = \frac{L}{2\pi\alpha'}$$

Geometry:



Mesons, bound states of fundamental fields, dual to open string states supported by Dq-brane

Kruczenski, Mateos, RCM and Winters
(hep-th/0304032)

Mesons:

lowest lying open string states are excitations of the massless modes on probe brane: vector, scalars (& spinors)

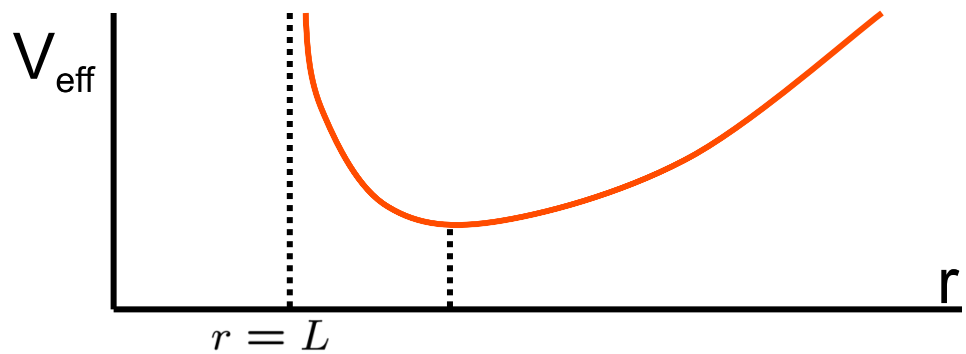
→ their dynamics is governed by usual worldvolume action:

$$I_q = -T_q \int \mathrm{d}^{q+1}\xi \sqrt{-\det\left(P[G]_{ab} + 2\pi\alpha' F_{ab}\right)} + T_q \int \sum P[C^{(n)}]\ \exp[2\pi\alpha' F]$$

free spectrum:

- expand action to second order in fluctuations around bkgd.
- solve linearized eq's of motion by separation of variables

→ discrete spectrum



(RCM & Thomson; Arean & Ramallo)

Mesons:

Detailed results rely on numerical solution of radial ODE

Spectra have universal features:

discrete, deeply bound, mass gap

$$M \sim \frac{m_q}{g_{eff}(m_q)}$$ (with $g^2_{eff}(m_q) = g^2_{YM} N_c \, m_q^{p-3}$)

mass scale dictated by Holography!

Holography:

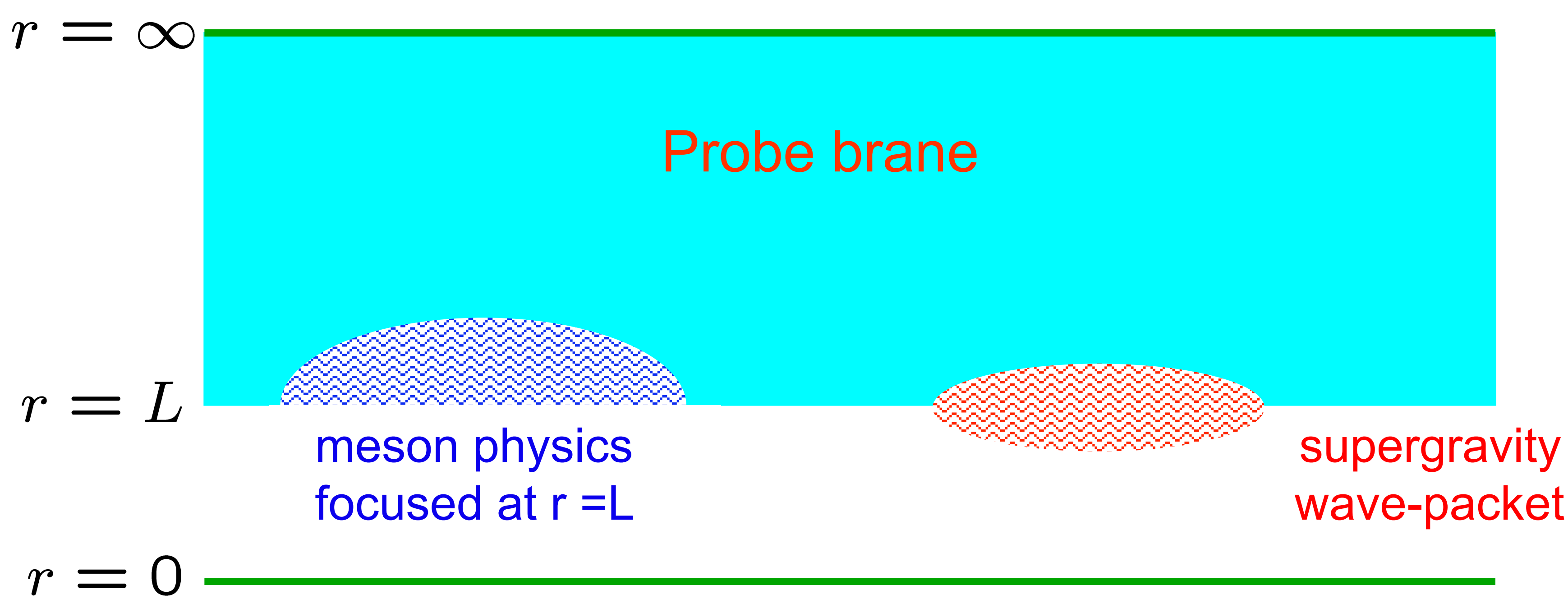
(p+2)-dim. gravity ⟷ (p+1)-dim. gauge theory

radius → $r/\ell_s^2 = U$ ← energy

- gravity physics is local in radius

(Horowitz & Polchinski)

→ gauge physics must be local in energy!



Holography:

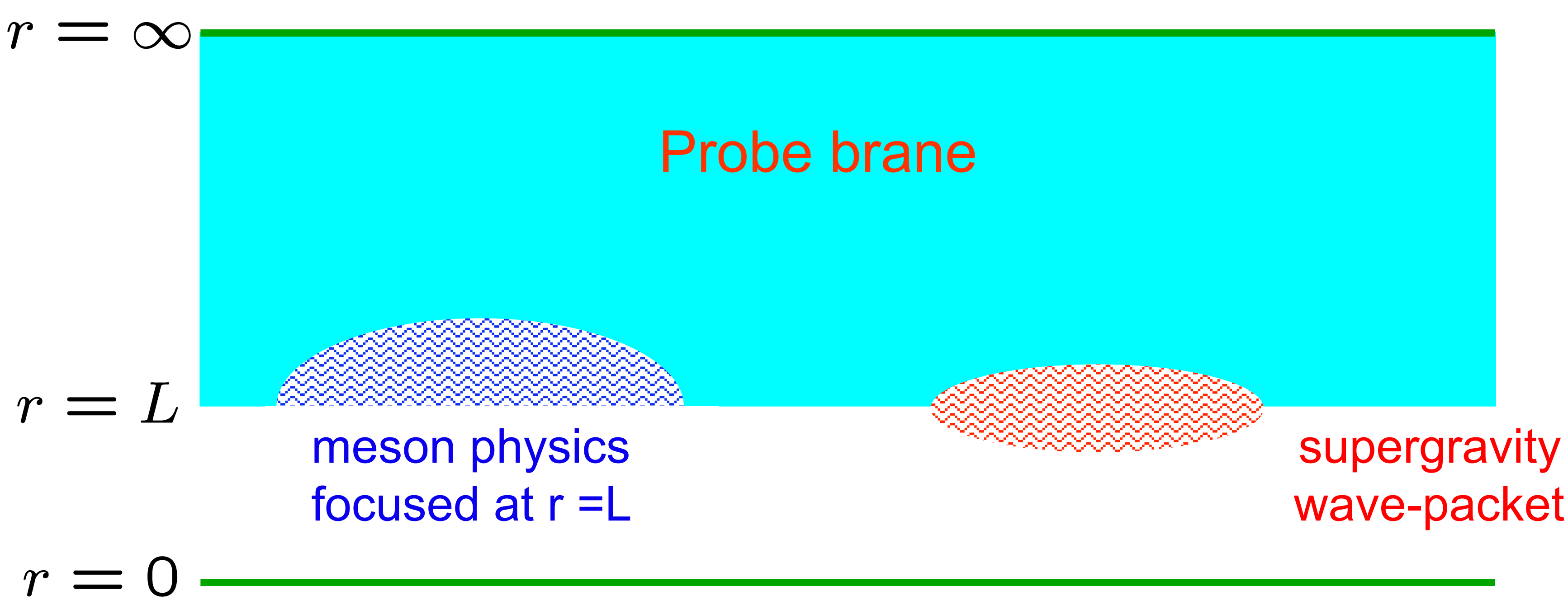
(Peet & Polchinski)

String energy: $E = U = r/\ell_s^2$

Supergravity energy: $$E = \frac{U}{\sqrt{g_{YM}^2 N_c U^{\frac{p-3}{2}}}} = \frac{U}{g_{eff}(U)}$$

Precisely matches meson energy scale with $U = m_q$!

Locality in energy preserved between open and closed strings!



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mass scale dictated by Holography!

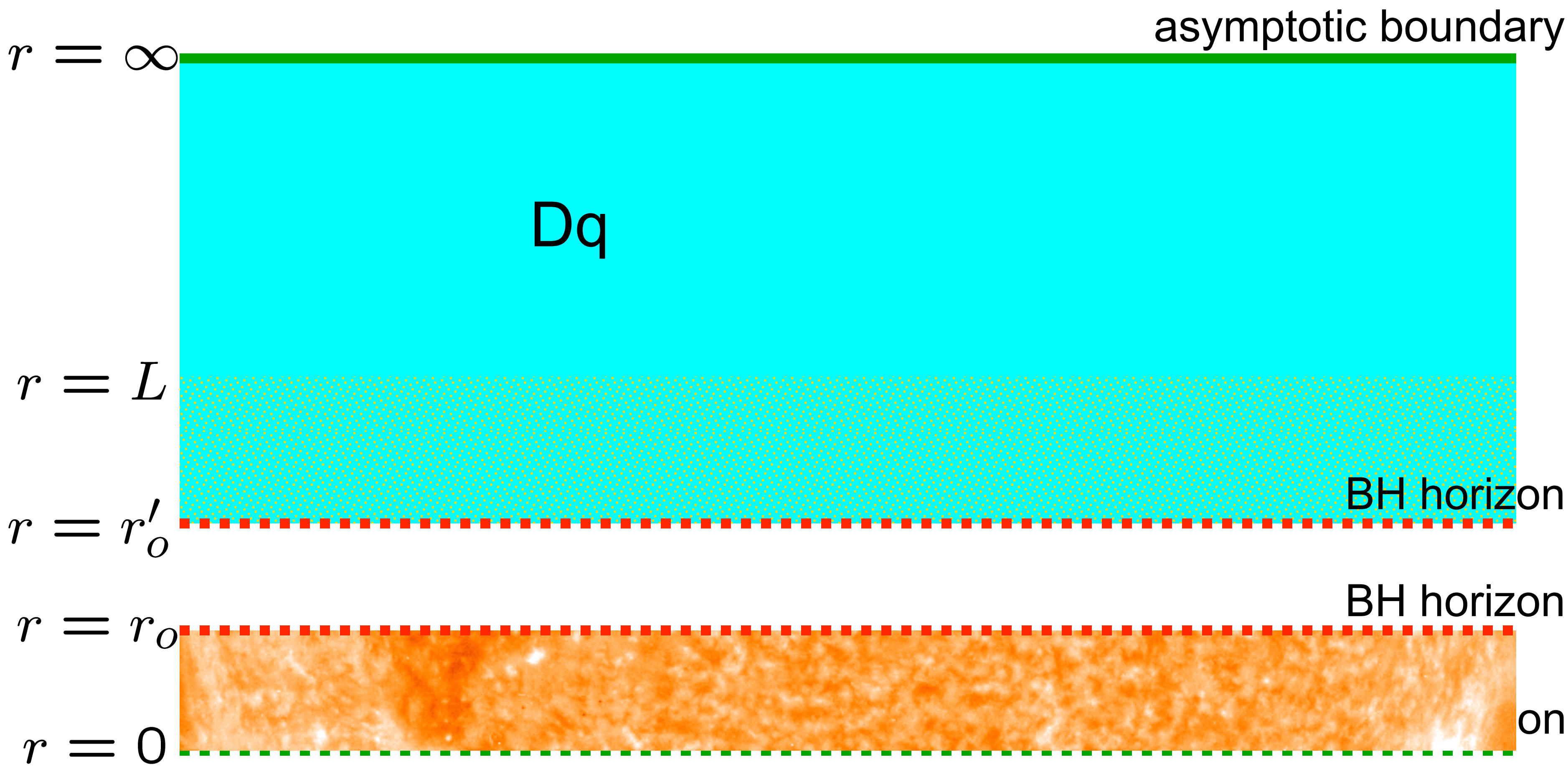
- SUSY ⟶ nonSUSY: bare mass ⟶ constituent mass
- Goldstone modes

Gauge/Gravity thermodynamics with probe branes:

Witten

Gauge theory thermodynamics = Black hole thermodynamics

put Dq-probe in throat geometry of black Dp-brane



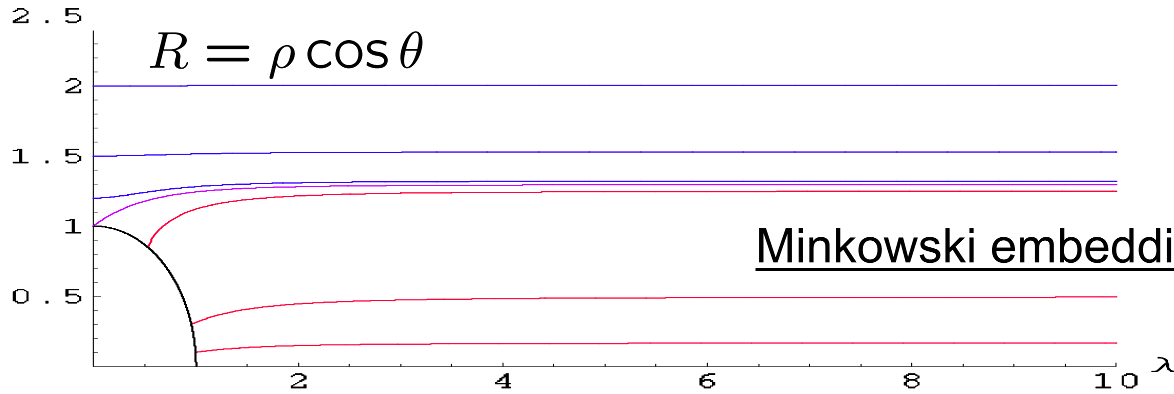
for L ≤ r_o, … r

Expect to see a phase transition!

D3/D7: Babington, Erdmenger, Evans, Guralnik and Kirsch (hep-th/0306018)

Dq-brane embedding in black Dp-background:

Numerical solutions:



Minkowski embeddings: close off smoothly

Black hole embeddings: fall through horizon

BH horizon: $\rho = r_o$

$\lambda = \rho \sin\theta$

D3/D7: Babington, Erdmenger, Evans, Guralnik and Kirsch (hep-th/0306018)

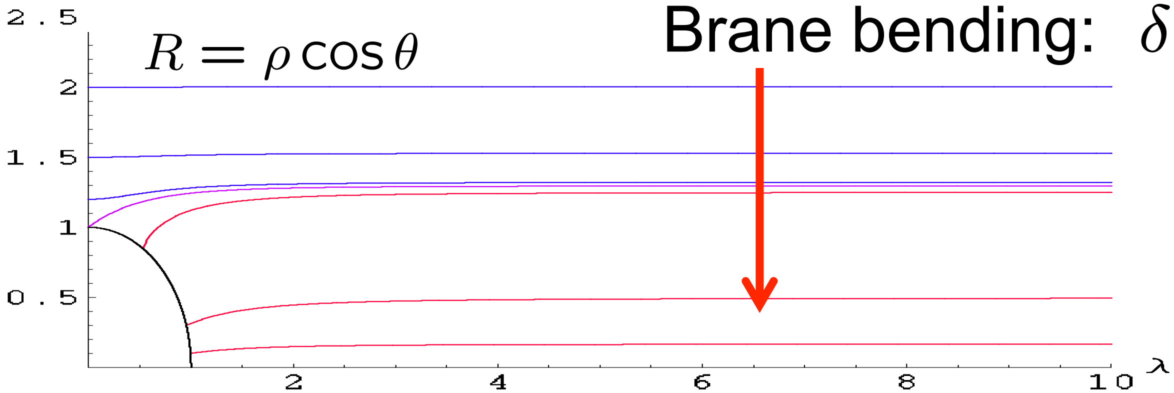
Dq-brane embedding in black Dp-background:

$$m_q = \frac{L}{2\pi\ell_s^2}\ ; \qquad \langle\bar\psi\,\psi\rangle = -\frac{1}{2\sqrt{2}\pi}\sqrt{\lambda}N_cT^3\,\tilde c$$

Numerical solutions:

$R = \rho\cos\theta$

Brane bending: $\delta R(\rho) \simeq \frac{r_o^3}{\rho^2}\,\tilde c$



$R_\infty = L$

BH horizon: $\rho = r_o$

$\lambda = \rho\sin\theta$

Frolov, Larsen and Christensen (hep-th/9811148)
Frolov (gr-qc/0604114)

Critical Self-Similar Embeddings:

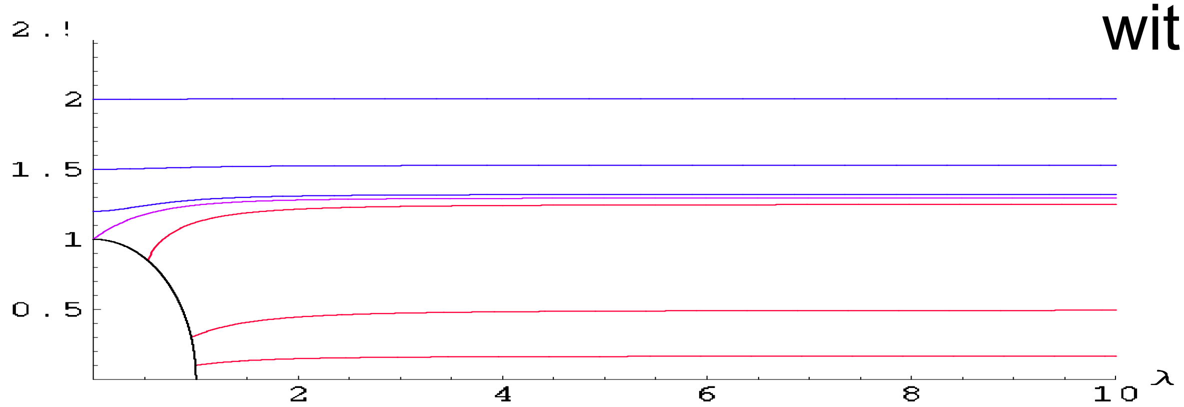
Thermal D3 background:

D7 embedding: θ(ρ)

$$ds^2 = \frac{1}{2}\frac{\rho^2}{R^2}\left(-\frac{f(\rho)^2}{\tilde{f}(\rho)}dt^2 + \tilde{f}(\rho)d\vec{x}^{\,2}\right) + \frac{R^2}{\rho^2}\left[d\rho^2 + \rho^2\left(d\theta^2 + \cos^2\theta\, d\phi^2 + \sin^2\theta\, d\Omega_3^2\right)\right]$$

with $f(\rho) = 1 - \frac{r_0^4}{\rho^4}$

$\tilde{f}(\rho) = 1 + \frac{r_0^4}{\rho^4}$



Frolov, Larsen and Christensen

Critical Self-Similar Embeddings:

$$r = r_0 + \tfrac{1}{2}\kappa Z^2 \ , \quad \theta = \frac{R}{L} \ , \quad \vec{x} = \frac{L}{r_0}\vec{y} \qquad \text{with } \kappa = 2\frac{{r_0}^2}{R^4}$$

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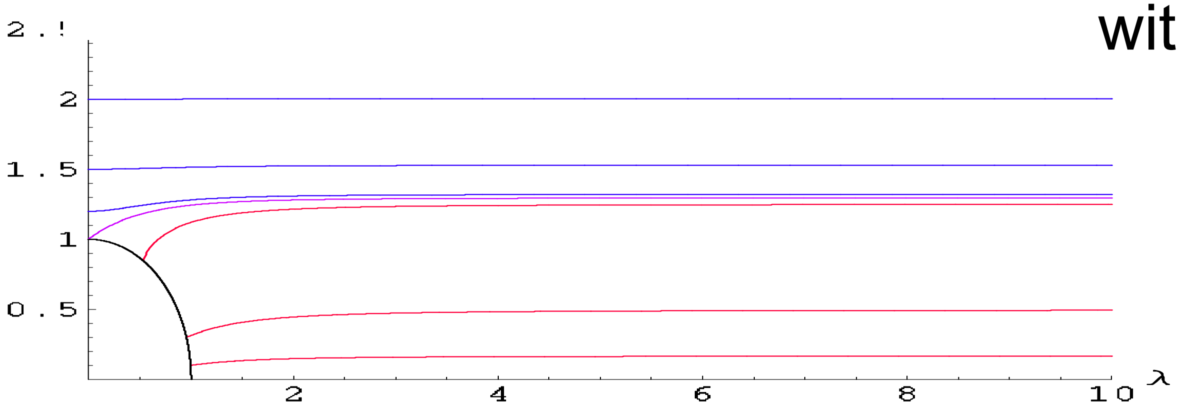

Zoom in on horizon at axis

Frolov, Larsen and Christensen

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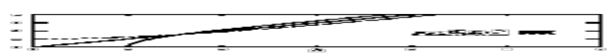
with $\kappa = 2\frac{{r_0}^2}{L^4}$

$$ds^2 = -\kappa^2 Z^2\, dt^2 + dZ^2 + dR^2 + R^2\, d\Omega_3^2 + d\vec{y}^2 + L^2\, d\phi^2$$

D7 embedding: R(Z)



← axis

horizon ↓

Frolov, Larsen and Christensen

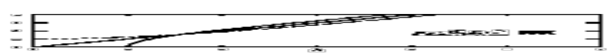
Critical Self-Similar Embeddings:

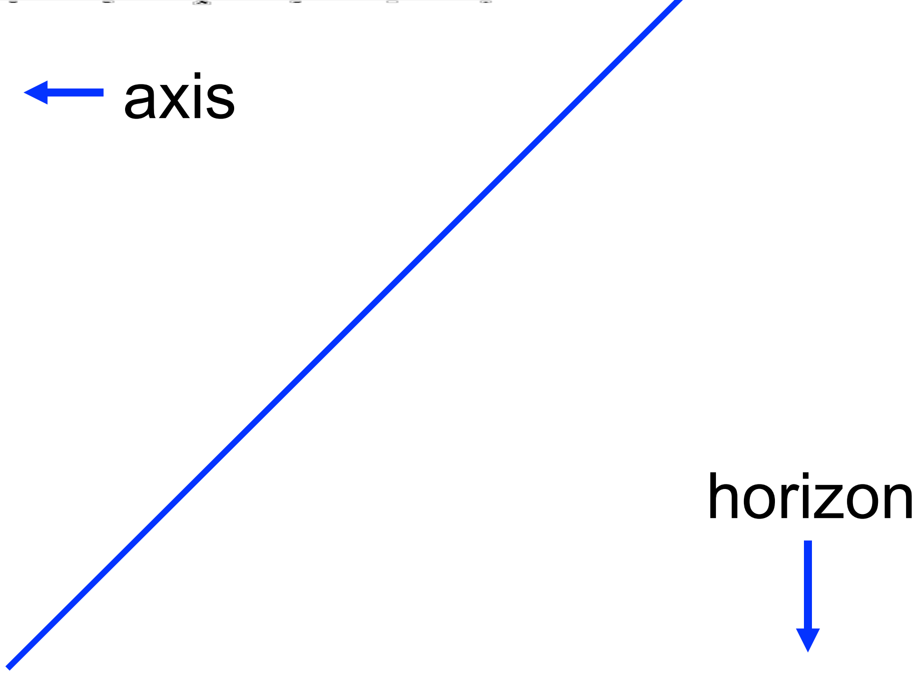
embedding eqn: $Z\,R\,R'' + (R\,R' - 3\,Z)\left(1 + R'^2\right) = 0$

critical soln: $R = \sqrt{3}\,Z$ → (m_q^*, c^*)

scaling: $R = f(Z)$ → $R = f(\lambda Z)/\lambda$

$R(Z=0) = R_0$ → $R(Z=0) = R_0/\lambda$





Frolov, Larsen and Christensen

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scaling: $R = f(Z)$ $\longrightarrow$ $R = f(\lambda Z)/\lambda$

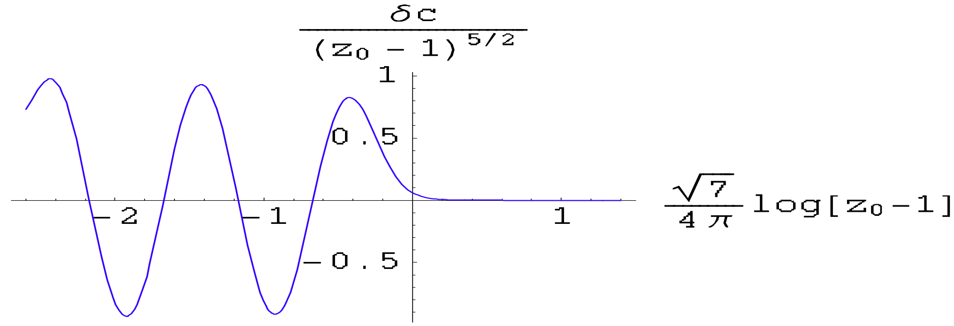
$R(Z=0) = R_0$ $\longrightarrow$ $R(Z=0) = R_0/\lambda$

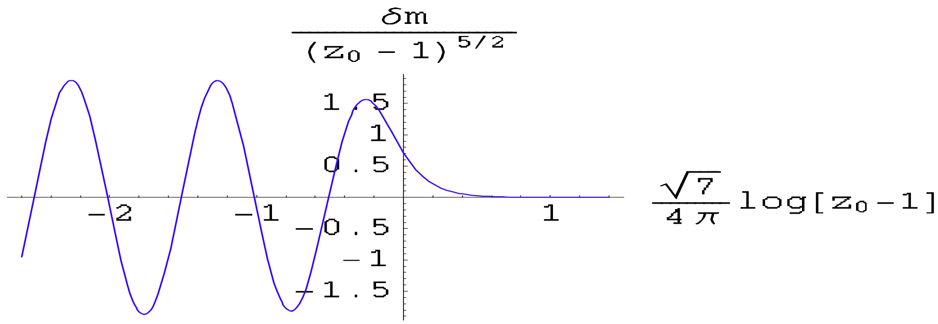
oscillations: linearize $R = \sqrt{3}Z + p(Z)$ (far from *Z=0*)

$$R = \sqrt{3}Z + Z^{-3/2}\left[a\,\sin\left(\frac{\sqrt{7}}{2}\log Z\right) + b\,\cos\left(\frac{\sqrt{7}}{2}\log Z\right)\right]$$

$\Rightarrow$ find oscillations in asymptotic boundary cond's

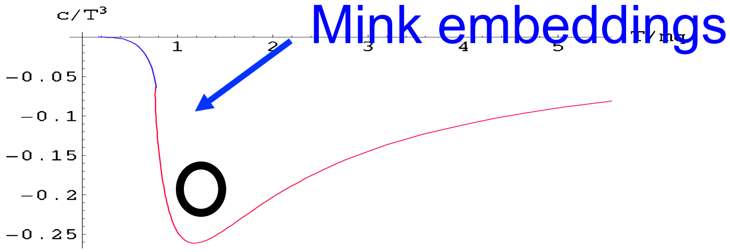
$R_0^{-5/2}\,\delta m_q$, $R_0^{-5/2}\,\delta c$ are periodic functions of $\sqrt{7}/4\pi\,\log R_0$



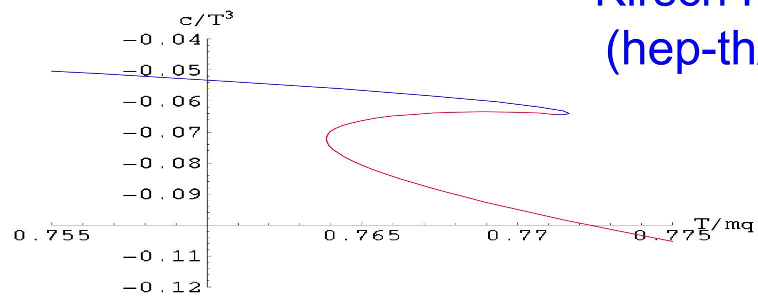


$R_0^{-5/2}\,\delta m_q$, $R_0^{-5/2}\,\delta c$ are periodic functions of $\sqrt{7}/4\pi \log R_0$

Phases do not join “smoothly” rather spiral in on critical solution

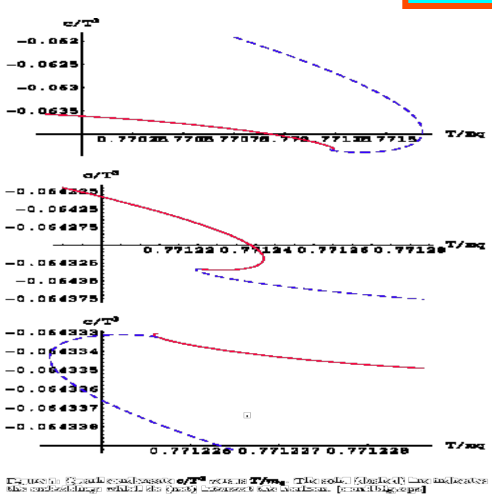


Kirsch PhD thesis
(hep-th/0406274)

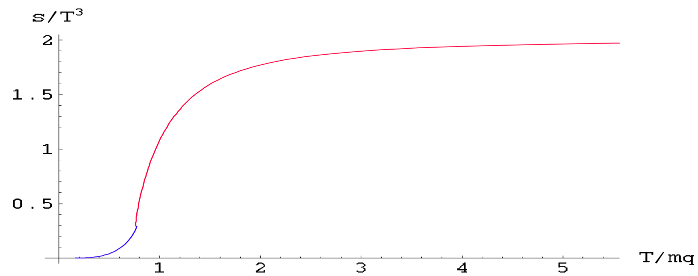


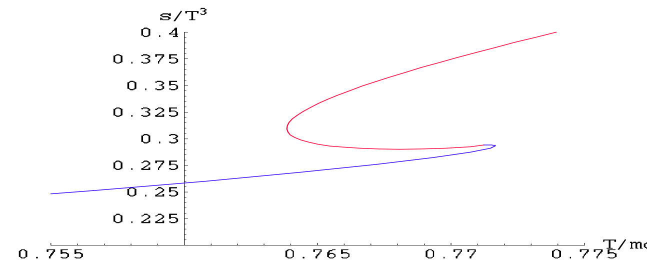
BH embeddings

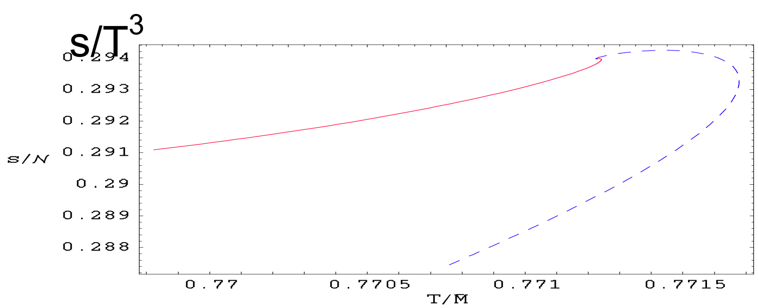
Similarly for other physical properties!
e.g., energy density, entropy

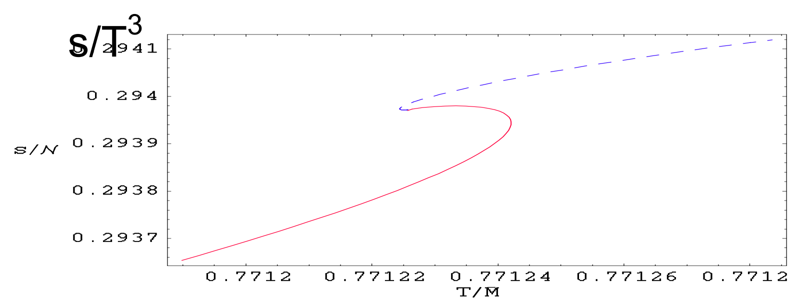


Brane entropy: apply Euclidean action techniques to brane







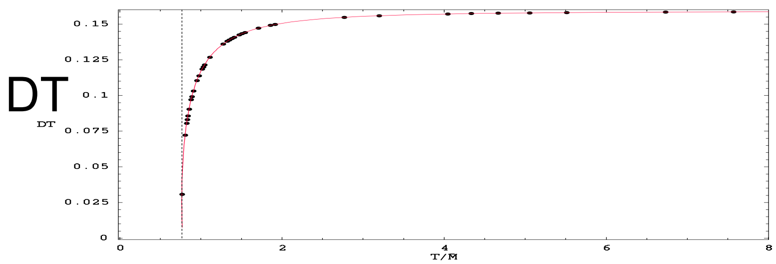


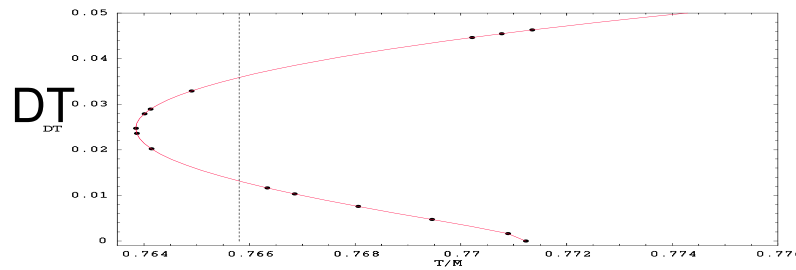
Kovtun, Son & Starinets (hep-th/0309213)

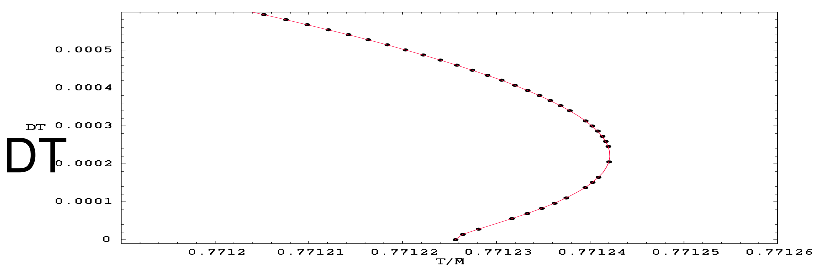
Diffusion constant: diffusion of flavor current in BH phase

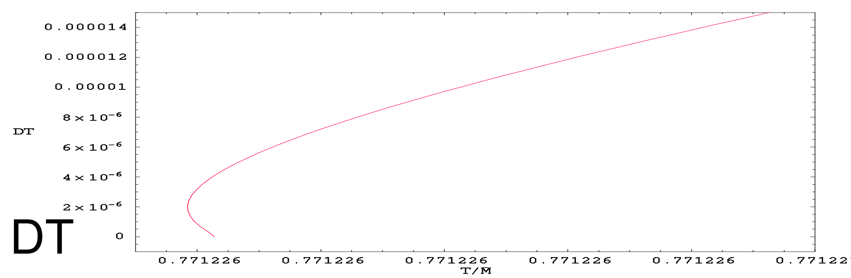
$$J^x = -\mathbf{D}\,\partial_x J^0$$

D3/D7: $1/2\pi$



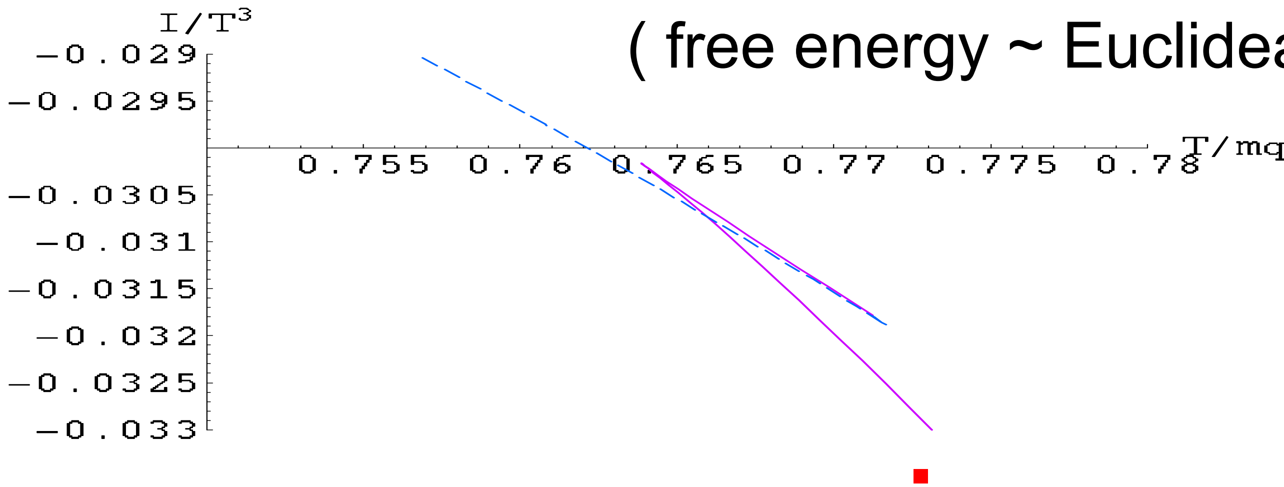






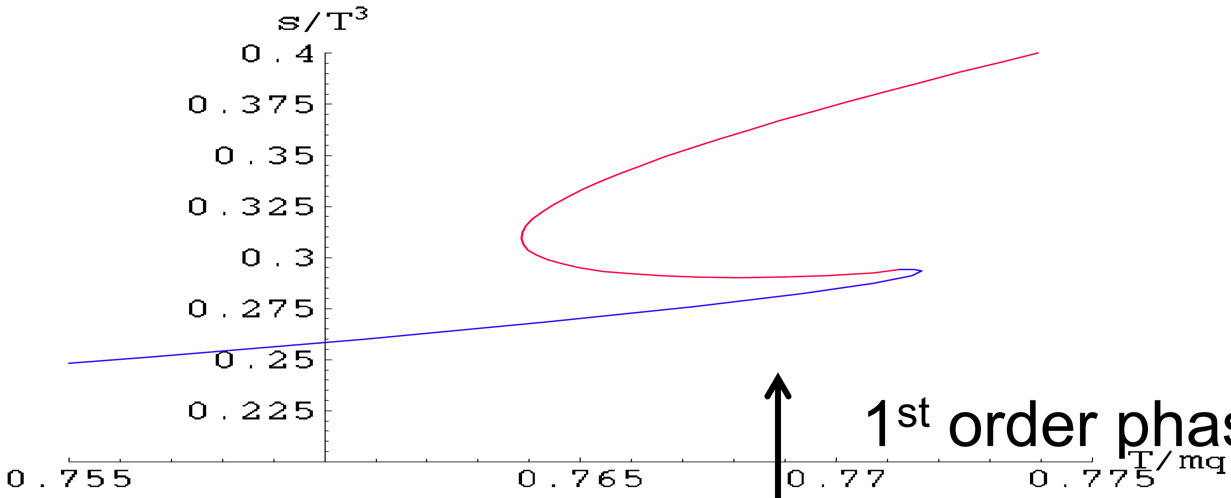
thermal properties multi-valued → free energy determines physical configuration

(free energy ~ Euclidean brane action)



Phase transition ← Critical Behavior

Brane entropy:



1st order phase transition

Transition temperature:

$$T_{fun} \sim m_q / g_{eff}(m_q)$$

Brane Entropy:

Dq-probe introducing (d+1)-dim. defect in Dp-branes

$$s \sim N_{\mathrm{f}}\, N_{\mathrm{c}}\, T^{d}\, g_{\mathrm{eff}}(T)^{2\frac{d-1}{5-p}}$$

effective coupling: $g^2_{\mathrm{eff}}(T) = g^2_{\mathrm{YM}} N_{\mathrm{c}}\, T^{p-3}$

Recall gluons and adjoint matter contribute:

$$s \sim N_{\mathrm{c}}^2\, T^{p}\, g_{\mathrm{eff}}(T)^{2\frac{p-3}{5-p}}$$

Conclusions/future directions:

- holographic meson spectra have universal features:
  discrete, deeply bound, mass gap
- mass scale dictated by holography
- Self-similar embeddings are "universal" for probe branes in black hole bkgd's (brane topology R^{q+1})
  → universal feature for many gauge/gravity dualities with fundamental matter
- first order phase transition also seems universal
- how robust are these features?
  → finite $N_f/N_c \sim O(1)$? Hawking radiation $O(1/N_c^2)$?
- (Brane) Entropy = (Horizon) Area ?? (okay)
- hydrodynamic transport properties:
  e.g., shear viscosity: $\eta/s = 1/4\pi$?? (in progress)