Exactly Solvable Dynamics for Rolling D-brane

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Based on :hep-th/0605013 with S.J. Rey, Y. Sugawara,
hep-th/0507040 with S.J. R, Y. S,
hep-th/0412038 with K. Panigrahi, S.J. R, and H. Takayanagi
hep-th/0406173 with Y. S, H.T

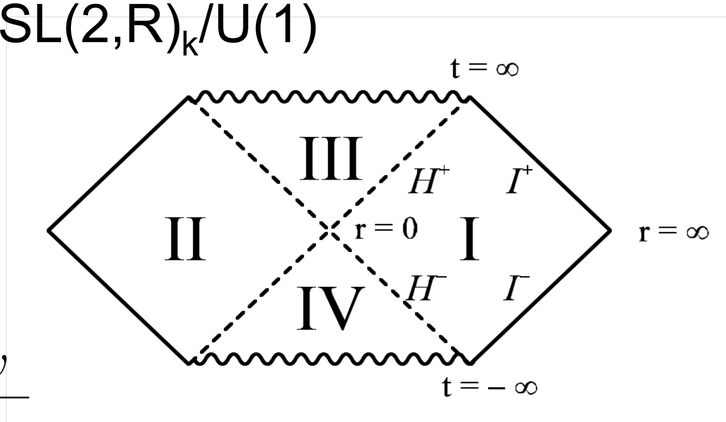
Purpose of the talk

- Large charge (+BPS) vs. Small Charge (+non-BPS)
  - Black hole / String phase transition
  - Hawking temperature vs. Hagedorn temperature
  - Is 2D (pure) Black hole really black?
- Analyticity vs. Non-analyticity
  - Universality of Tachyon-Radion correspondence
  - Wick rotation in curved space
- Unitarity vs. Open/Closed duality
  - Optical theorem
  - Lorentzian world-sheet vs. Euclidean world-sheet

2D (fermionic) Black Hole

- 2D black hole is the simplest black hole geometry as an exact string background (Witten; Mandal, Senguputa, Wadia) $SL(2,R)_k/U(1)$

$$ds^2 = 2k(d\rho^2 - \tanh^2\rho\, dt^2) \qquad e^{\Phi} = \frac{e^{\Phi_0}}{\cosh\rho}$$

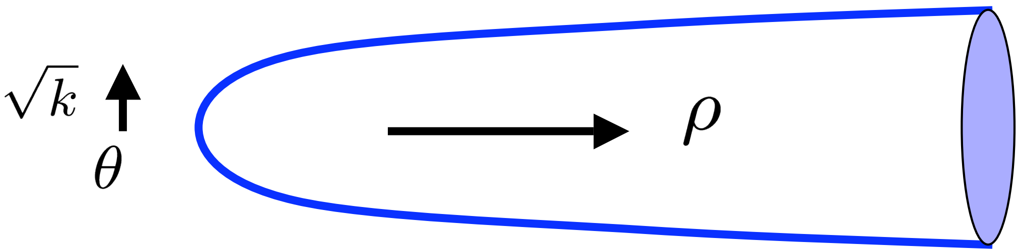


- Global metric looks Schwarzshild-like

$$u = \sinh\rho\, e^{t} \quad v = -\sinh\rho\, e^{-t} \quad ds^2 = -2k\frac{du dv}{1-uv}$$

- In Euclidean geometry, 2D black hole is cigar geometry:

$$ds^2 = 2k(d\rho^2 + \tanh^2\rho\, d\theta^2) \qquad e^{\Phi} = \frac{e^{\Phi_0}}{\cosh\rho}$$



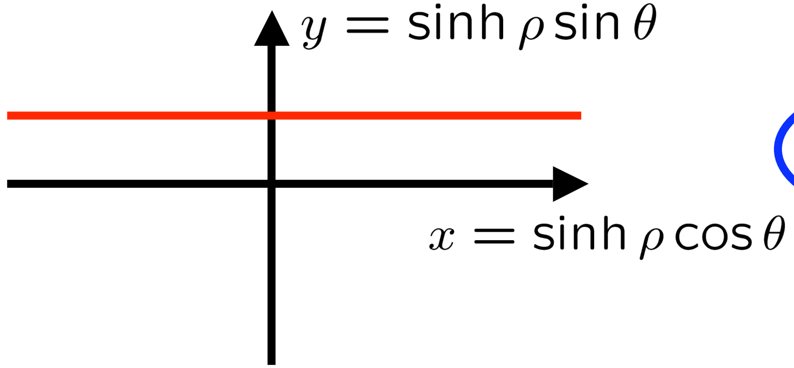
$$T_{Hw} = \frac{1}{\beta_{Hw}} = \frac{1}{2\pi\sqrt{2k}}$$

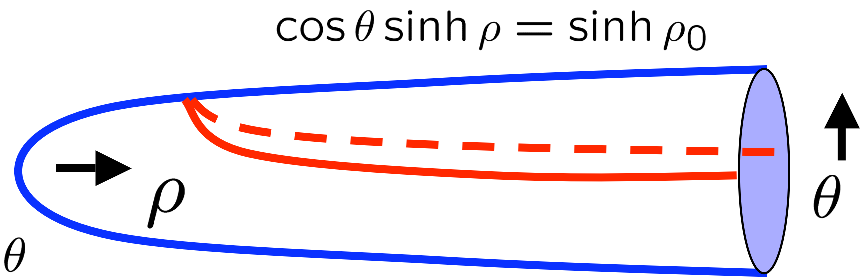
$$T_{Hg} = \frac{1}{\beta_{Hg}} = \frac{1}{4\pi\sqrt{1-\frac{1}{2k}}}$$

Branes in 2D Black Hole geometries

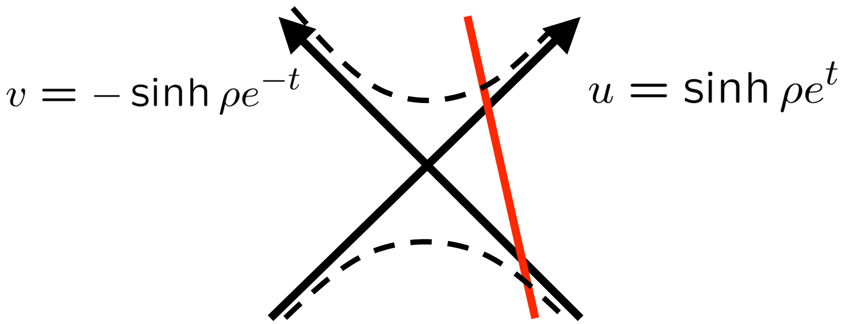
Classical D-branes are classified by solutions of DBI action.

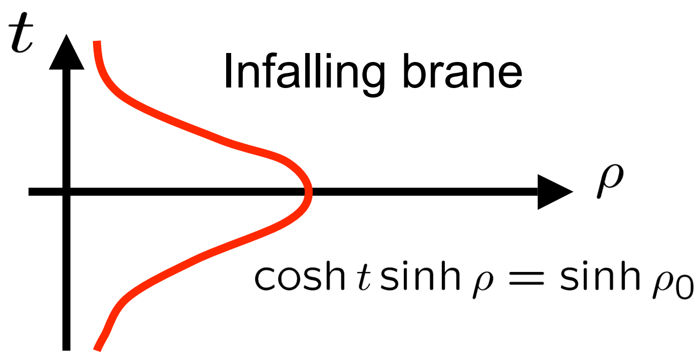
- Euclidean case (Class 2' brane "hairpin brane": Ribault-Schomerus. see also Eguchi-Sugawara, Ahn-Stanishkov-Yamamoto, Lukyanov-Vitcev-Zamolodchikov)





- Lorentzian case ("Rolling D-brane": Kutasov, NST, Yogendran, NRS)





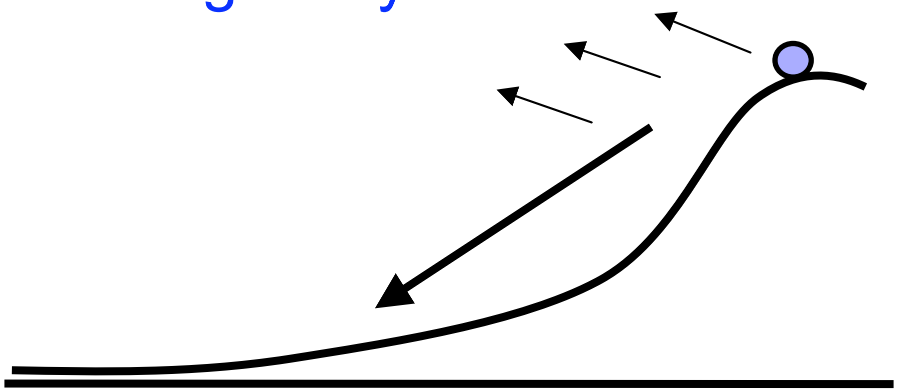
Tachyon-Radion correspondence

- D-brane near NS5-brane shows resemblance to rolling tachyon (Kutasov): Rolling D-brane

$$L_{\mathrm{D0}} = -e^{-\Phi}\sqrt{\left(\frac{ds}{dt}\right)^2} = -V(X)\sqrt{1-\dot{X}^2}$$

$$\text{where} \quad V(X) = M_0\, e^{\sqrt{\frac{2}{k}}X}.$$

- Rolling tachyon has similar form (Sen …).



$$L_T = -V(T)\sqrt{1-\dot{T}^2}$$

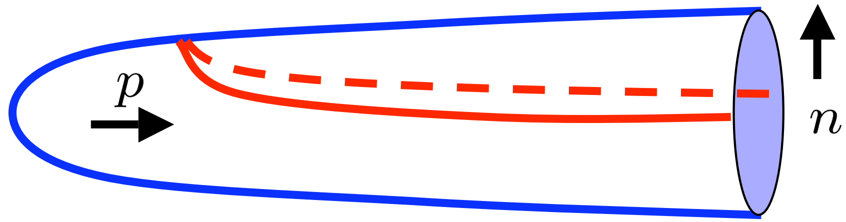
$$V(T) \sim \frac{1}{\cosh T}$$

Is tachyon-radion correspondence universal?
Artifact at the level of effective action?

Euclidean boundary states

- Class 2' boundary states in Euclidean BH

$$\cos\theta \sinh\rho = \sinh\rho_0$$



$$|B;\rho_0\rangle = \int_0^\infty \frac{dp}{2\pi} \sum_{n\in Z} \Psi_{D1}(\rho_0;p,n)\,|p,n\rangle,$$

$$\Psi_{D1}(\rho_0;p,n) = \mathcal{N}(k)\frac{\Gamma(ip)\Gamma\left(1+\frac{ip}{k}\right)}{\Gamma\left(\frac{1}{2}+\frac{ip+n}{2}\right)\Gamma\left(\frac{1}{2}+\frac{ip-n}{2}\right)} [e^{-ip\rho_0}+(-1)^n e^{ip\rho_0}]\ .$$

- Effect of 1/k correction
  - Delta function localized trajectory → smeared wavefunction

    Poisson distribution: $\delta(\phi) \sim \sqrt{\frac{1}{k-1}}$
  - The steeper the hairpin, the wider the trajectory (NRPT).

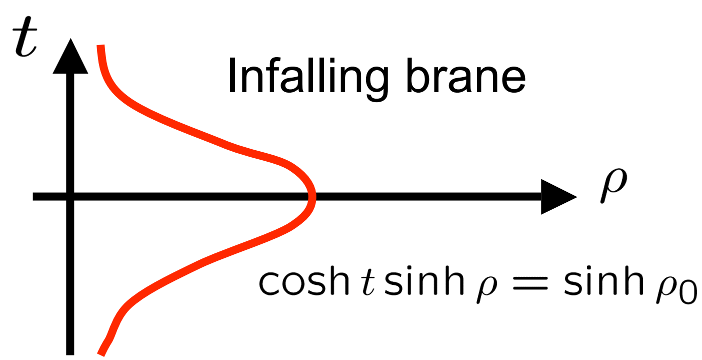
Wick rotation: rolling D-brane boundary states

Naïve momentum space Wick rotation $n \to i\omega$ does not work.

- Performing Wick rotation in coordinate space, or choosing the contour integral properly,

$$\Psi(\rho_0;p,\omega) = \frac{\Gamma(\frac{1}{2} - i\frac{p+\omega}{2})\Gamma(\frac{1}{2} - i\frac{p-\omega}{2})\Gamma(1+\frac{ip}{k})}{\Gamma(1-ip)}\left[e^{-ip\rho_0} - \frac{\cosh\left(\pi\frac{p-\omega}{2}\right)}{\cosh\left(\pi\frac{p+\omega}{2}\right)}e^{ip\rho_0}\right]$$

$$|falling\rangle = \int \frac{d\omega}{2\pi}\frac{dp}{2\pi}\Psi(\rho_0;p,\omega)|U;p,\omega\rangle$$

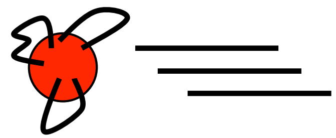


- Finite k correction:
  - Trajectory is smeared (NPRT)

$$\delta(\phi) \sim \sqrt{\frac{1}{k-1}}$$

  - Rolling D-brane gathers moss (Kutasov)

→ analytic continuation of winding tachyon?



D-brane radiation

From boundary states, we can compute closed string emission from falling D-branes (NST,NRS see also Sahakyan).

- Let us assume k>1. For fixed mass level, $\omega^2(p,M) = p^2 + 2kM^2$

$$
\begin{aligned}
N(M) &\sim \int \frac{d^{5-p}\mathbf{k}_\perp}{(2\pi)^{5-p}} \int_0^\infty \frac{dp}{2\pi} \frac{1}{2\omega(p,M)} |\Psi(\rho_0; p, \omega(p,M))|^2 \\
&\sim \frac{1}{M} \int \frac{d^{5-p}\mathbf{k}_\perp}{(2\pi)^{5-p}} \int_0^\infty dp\, e^{\pi(1-\frac{1}{k})p - \pi\sqrt{p^2+2k(M^2+\mathbf{k}_\perp^2)}} \\
&\sim M^{2-\frac{p}{2}}\, e^{-2\pi M\sqrt{1-\frac{1}{2k}}} = M^{2-\frac{p}{2}} e^{-2\pi M \frac{\beta_H g}{2}}
\end{aligned}
$$

- Saddle point approximation is used as M → ∞
- Hagedorn temperature (with α' correction) appeared in infalling mode!
- We are adding extra directions so that the theory is critical.

Tachyon-radion correspondence

- We can sum over all the final states

$$N = \sum_M N(M) \sim \int dM \rho(M) N(M) \sim \int \frac{dM}{M} M^{-p/2}$$

$$\rho(M) \sim \frac{1}{M^3} e^{2\pi M \sqrt{1-\frac{1}{2k}}} \qquad N(M) \sim M^{2-\frac{p}{2}} e^{-2\pi M \sqrt{1-\frac{1}{2k}}}$$

- Density of states exactly cancels with the radiation density → shows the same behavior in rolling tachyon (Lambert-Liu-Maldacena)

Tachyon-radion correspondence is true at the stringy level.

- Remarkable cancellation of stringy corrections → universal property of rolling (falling) D-brane?

Black hole/ String transition at k = 1

Evaluation changes drastically at k=1 (BH/String transition)

- There is no nontrivial saddle point for k<1

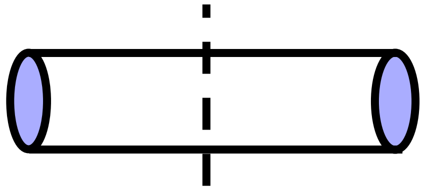
$$N(M) \sim \int_0^\infty dp\, e^{\pi\left(1-\frac{1}{k}\right)p-\pi\sqrt{p^2+2kM^2}}$$
$$\sim e^{-2\pi M\sqrt{\frac{k}{2}}}$$

- Emission rate is UV convergent (BH/String transition)
- Summary:
  - Tachyon-radion correspondence is universal (k>1)
  - BH/String transition is observed in physical quantities (radiation rate)
  - At k = 1, $T_{Hw} = T_{Hg}$ and k<1, BH interpretation is questionable.
  - 1/k corrections are crucial.

Unitarity and Open/Closed duality (NRS: hep-th/0605013)

- Is unitarity consistent with open/closed duality?

$$\mathrm{Im} Z_{cylinder} = \quad \langle B|$$

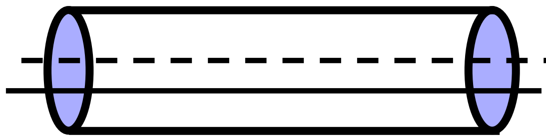


$$|B\rangle$$

$$= \sum_M \int \frac{d^{5-p}\mathbf{k}_\perp}{(2\pi)^{5-p}} \int_0^\infty \frac{dp}{2\pi} \frac{1}{2\omega(p,M)} |\Psi(\rho_0; p, \omega(p,M))|^2$$

- Open string channel?

$$\mathrm{Im} Z_{cylinder} \stackrel{?}{=} \langle B|$$



$$|B\rangle$$

- Euclidean vs. Lorentzian worldsheet
  - Gives the same answer in rolling tachyon (Karzcmarek-Liu-Maldacena-Strominger), but… (Okuyama-Rozalli, NRS)

Open string computation

- Modular transform is (only) well-defined in Lorentzian signature world sheet.

$$\int ds_c \int_0^\infty dp \int_{-\infty}^\infty d\omega \frac{\sinh(\pi p)}{(\cosh(\pi\omega)+\cosh(\pi p))\sinh(\pi p/k)} q^{\frac{1}{2k}(p^2-\omega^2)} Z_{osc} \quad \text{closed}$$

$$= \int \frac{ds_o}{s_o} \int_0^\infty dp \int_{-\infty}^\infty d\omega \frac{\sinh(\pi\omega)}{(\cosh(\pi p)+\cosh(\pi\omega))\sinh(\pi\omega/k)} \tilde{q}^{\frac{1}{2k}(p^2-\omega^2)} Z_{osc} \quad \text{open}$$

- Imaginary part consists of two parts

$$\mathrm{Im} Z_{cyl} = \mathrm{Im} Z_{naive} + \mathrm{Im} Z_{pole}$$

- Naïve part corresponds to contribution easily guessed in the Euclidean approach (but not enough)

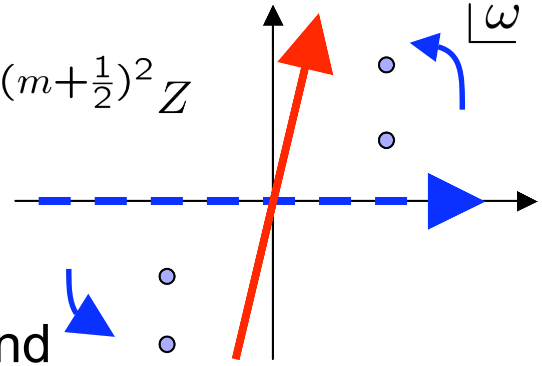
$$Z_{naive} = \int_0^\infty \frac{dt_o}{t_o} \int_0^\infty dp \int_{-\infty}^\infty d\omega \frac{\sin(\pi\omega)\tilde{q}^{\frac{1}{2k}(\omega^2+p^2)}}{(\cos(\pi\omega)+\cosh(\pi p))\sin(\pi\omega/k)} Z_{osc}$$

$$\mathrm{Im} Z_{naive} = \sum_{n=1}^\infty \int \frac{dt_o}{t_o} \int_{-\infty}^\infty dp \frac{(-1)^{n+1}\sin(\pi nk) e^{-2\pi t_o(\frac{p^2}{2k}+\frac{kn^2}{2})}}{\cos(\pi nk)+\cosh(\pi p)} Z_{osc}$$

Unitarity meets open/closed duality

- Pole part comes from poles in Euclidean (Wick) rotation

$$\mathrm{Im} Z_{pole} = 2\int_0^\infty \frac{dt_o}{t_o} \sum_{n=1}^{\infty} \sum_{m=0}^{[\frac{kn}{2}-\frac{1}{2}]} (-1)^{n+1} e^{-\pi t_o n(2m+1)+\frac{2\pi t_o}{k}(m+\frac{1}{2})^2} Z$$



- Both contributions are imperative to understand
  - Unitarity
  - Tachyon-Radion correspondence
- Summary
  - In Euclidean approach, no apparent reason to include/exclude pole contributions.
  - Unitarity demands its existence, and Lorentzian theory automatically knows it.
  - Fortuitously no pole contribution in rolling tachyon (in 2D ZZ-brane decay as well).

Summary and Outlook

- Exact boundary states for rolling D-brane is constructed.
- Tachyon-Radion correspondence is proved in α' exact way.

  → Full proof in string field theory?
- BH/String transition is observed at k=1.

  → Is 2D pure BH really black? Matrix model?
- Consistency between unitarity and open/closed duality requires careful analytic continuation (Wick rotation).

The shortest path between two truths in the real domain passes through the complex domain. ---- J. Hadamard