Exactly Solvable Dynamics for Rolling D-brane

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Purpose of the talk

- **Large charge** (+BPS) vs. **Small Charge** (+non-BPS)
  - Black hole / String phase transition
  - Hawking temperature vs. Hagedorn temperature
  - Is 2D (pure) Black hole really black?
- **Analyticity** vs. **Non-analyticity**
  - Universality of Tachyon-Radion correspondence
  - Wick rotation in curved space
- **Unitarity** vs. **Open/Closed duality**
  - Optical theorem
  - Lorentzian world-sheet vs. Euclidean world-sheet
2D (fermionic) Black Hole

- **2D black hole** is the simplest black hole geometry as an exact string background (Witten; Mandal, Senguputa, Wadia) $SL(2,R)_k/U(1)$

$$ds^2 = 2k(d\rho^2 - \tanh^2 \rho dt^2) \quad e^\Phi = \frac{e^{\Phi_0}}{\cosh \rho}$$

- Global metric looks **Schwarzshild-like**

$$u = \sinh \rho e^t \quad v = -\sinh \rho e^{-t} \quad ds^2 = -2k \frac{du dv}{1 - uv}$$

- In **Euclidean** geometry, 2D black hole is **cigar geometry**:

$$ds^2 = 2k(d\rho^2 + \tanh^2 \rho d\theta^2) \quad e^\Phi = \frac{e^{\Phi_0}}{\cosh \rho}$$

$$T_{Hw} = \frac{1}{\beta_{Hw}} = \frac{1}{2\pi \sqrt{2k}}$$

$$T_{Hg} = \frac{1}{\beta_{Hg}} = \frac{1}{4\pi \sqrt{1 - \frac{1}{2k}}}$$
Branes in 2D Black Hole geometries

Classical D-branes are classified by solutions of DBI action.

- Euclidean case (Class 2’ brane “hairpin brane”: Ribault-Schomerus. see also Eguchi-Sugawara, Ahn-Stanishkov-Yamamoto, Lukyanov-Vitcev-Zamolodchikov)

\[
y = \sinh \rho \sin \theta
\]
\[
x = \sinh \rho \cos \theta
\]
\[
\cos \theta \sinh \rho = \sinh \rho_0
\]

- Lorentzian case (“Rolling D-brane”: Kutasov, NST, Yogendran, NRS)

\[
v = -\sinh \rho e^{-t}
\]
\[
u = \sinh \rho e^{t}
\]
\[
cosh t \sinh \rho = \sinh \rho_0
\]
Tachyon-Radion correspondence

- D-brane near NS5-brane shows resemblance to rolling tachyon (Kutasov): Rolling D-brane
  \[
  L_{D0} = -e^{-\Phi} \sqrt{\left( \frac{ds}{dt} \right)^2} = -V(X) \sqrt{1 - \dot{X}^2}
  \]
  where \( V(X) = M_0 e^{\sqrt{\frac{2}{k}} X} \).

- Rolling tachyon has similar form (Sen ...).

\[
L_T = -V(T) \sqrt{1 - \dot{T}^2}
\]

\[
V(T) \sim \frac{1}{\cosh T}
\]

Is tachyon-radion correspondence universal? Artifact at the level of effective action?
Euclidean boundary states

- Class 2’ boundary states in Euclidean BH

\[ \cos \theta \sinh \rho = \sinh \rho_0 \]

\[ |B; \rho_0 \rangle = \int_0^\infty \frac{dp}{2\pi} \sum_{n \in \mathbb{Z}} \Psi_{D1}(\rho_0; p, n) |p, n\rangle, \]

\[ \Psi_{D1}(\rho_0; p, n) = \mathcal{N}(k) \frac{\Gamma(ip)\Gamma\left(1 + \frac{ip}{k}\right)}{\Gamma\left(\frac{1}{2} + \frac{ip+n}{2}\right) \Gamma\left(\frac{1}{2} + \frac{ip-n}{2}\right)} \left[ e^{-ip\rho_0} + (-1)^n e^{ip\rho_0} \right]. \]

- Effect of 1/k correction
  - Delta function localized trajectory \( \rightarrow \) smeared wavefunction
    Poisson distribution:
    \[ \delta(\phi) \sim \sqrt{\frac{1}{k-1}} \]
  - The steeper the hairpin, the wider the trajectory (NRPT).
Wick rotation: rolling D-brane boundary states

Naïve momentum space Wick rotation $n \rightarrow i \omega$ does not work.

- Performing Wick rotation in **coordinate space**, or choosing the **contour integral** properly,

$$
\Psi(\rho_0; p, \omega) = \frac{\Gamma\left(\frac{1}{2} - \frac{i p + \omega}{2}\right) \Gamma\left(\frac{1}{2} - \frac{i p - \omega}{2}\right) \Gamma(1 + i p)}{\Gamma(1 - i p)} \left[ e^{-i p \rho_0} - \frac{\cosh\left(\frac{\pi (p - \omega)}{2}\right)}{\cosh\left(\frac{\pi (p + \omega)}{2}\right)} e^{i p \rho_0} \right]
$$

$$
|\text{falling}\rangle = \int \frac{d\omega}{2\pi} \frac{dp}{2\pi} \Psi(\rho_0; p, \omega) |U; p, \omega\rangle
$$

- **Finite k correction:**
  - Trajectory is **smeared** (NPRT)
    $$
    \delta(\phi) \sim \sqrt{\frac{1}{k - 1}}
    $$
  - Rolling D-brane gathers moss (Kutasov)
    $\Rightarrow$ analytic continuation of **winding tachyon**?

Infalling brane

$$
\cosh t \sinh \rho = \sinh \rho_0
$$
D-brane radiation

- Let us assume $k > 1$. For fixed mass level, $\omega^2(p, M) = p^2 + 2kM^2$
  
  \[
  N(M) \sim \int \frac{d^{5-p}k_\perp}{(2\pi)^{5-p}} \int_0^\infty \frac{dp}{2\pi} \frac{1}{2\omega(p, M)} |\Psi(\rho_0; p, \omega(p, M))|^2 \\
  \sim \frac{1}{M} \int \frac{d^{5-p}k_\perp}{(2\pi)^{5-p}} \int_0^\infty \frac{dp}{2\pi} e^{(1-\frac{1}{k})p-\pi \sqrt{p^2 + 2k(M^2 + k_\perp^2)}} \\
  \sim M^{2-p/2} e^{-2\pi M \sqrt{1-1/2k}} = M^{2-p/2} e^{-2\pi M \frac{\beta H_g}{2}}
  \]

- Saddle point approximation is used as $M \to \infty$
- Hagedorn temperature (with $\alpha'$ correction) appeared in infalling mode!
- We are adding extra directions so that the theory is critical.

From boundary states, we can compute closed string emission from falling D-branes (NST, NRS see also Sahakyan).
Tachyon-radion correspondence

- We can sum over all the final states

\[ N = \sum_M N(M) \sim \int dM \rho(M) N(M) \sim \int \frac{dM}{M} M^{-p/2} \]

\[ \rho(M) \sim \frac{1}{M^3} e^{2\pi M \sqrt{1 - \frac{1}{2k}}} \quad N(M) \sim M^{2-p/2} e^{-2\pi M \sqrt{1 - \frac{1}{2k}}} \]

- Density of states exactly cancels with the radiation density \( \rightarrow \) shows the same behavior in rolling tachyon
  (Lambert-Liu-Maldacena)

  Tachyon-radion correspondence is true at the stringy level.

- Remarkable cancellation of stringy corrections \( \rightarrow \) universal property of rolling (falling) D-brane?
Black hole/ String transition at $k = 1$

- There is no nontrivial saddle point for $k<1$

$$N(M) \sim \int_0^\infty dp \, e^{\pi \left(1 - \frac{1}{k}\right)p - \pi \sqrt{p^2 + 2kM^2}}$$
$$\sim e^{-2\pi M \sqrt{\frac{k}{2}}}$$

- Emission rate is UV convergent (BH/String transition)

Summary:

- Tachyon-radion correspondence is universal ($k>1$)
- BH/String transition is observed in physical quantities (radiation rate)
- At $k = 1$, $T_{HW} = T_{Hg}$ and $k<1$, BH interpretation is questionable.
- $1/k$ corrections are crucial.
Unitarity and Open/Closed duality (NRS: hep-th/0605013)

- Is unitarity consistent with open/closed duality?

\[ \text{Im} Z_{cylinder} = \langle B \mid \text{cylinder} \mid B \rangle \]

\[ = \sum_M \int \frac{d^{5-p}k_{\perp}}{(2\pi)^{5-p}} \int_0^\infty dp \frac{1}{2\pi 2\omega(p, M)} |\psi(\rho_0; p, \omega(p, M))|^2 \]

- Open string channel?

\[ \text{Im} Z_{cylinder} \overset{?}{=} \langle B \mid \text{open string channel} \mid B \rangle \]

- Euclidean vs. Lorentzian worldsheet
  - Gives the same answer in rolling tachyon (Karzcmarek-Liu-Maldacena-Strominger), but... (Okuyama-Rozalli, NRS)
Open string computation

- Modular transform is (only) well-defined in Lorentzian signature world sheet.

\[
\int ds_c \int_0^\infty dp \int_{-\infty}^\infty d\omega \frac{\sinh(\pi p)}{(\cosh(\pi \omega) + \cosh(\pi p)) \sinh(\pi p/k)} q^{\frac{1}{2k}}(p^2 - \omega^2) Z_{osc} \quad \text{closed}
\]

\[
= \int \frac{ds_o}{s_o} \int_0^\infty dp \int_{-\infty}^\infty d\omega \frac{\sinh(\pi \omega)}{(\cosh(\pi p) + \cosh(\pi \omega)) \sinh(\pi \omega/k)} q^{\frac{1}{2k}}(p^2 - \omega^2) Z_{osc} \quad \text{open}
\]

- Imaginary part consists of \textbf{two} parts

\[
\text{Im} Z_{cyl} = \text{Im} Z_{naive} + \text{Im} Z_{pole}
\]

- Naïve part corresponds to contribution easily guessed in the Euclidean approach (but not enough)

\[
Z_{naive} = \int_0^\infty \frac{dt_o}{t_o} \int_0^\infty dp \int_{-\infty}^\infty d\omega \frac{\sin(\pi \omega) q^{\frac{1}{2k}}(\omega^2 + p^2)}{(\cos(\pi \omega) + \cosh(\pi p)) \sin(\pi \omega/k)} Z_{osc}
\]

\[
\text{Im} Z_{naive} = \sum_{n=1}^{\infty} \int \frac{dt_o}{t_o} \int_{-\infty}^\infty dp (-1)^{n+1} \frac{\sin(\pi nk) e^{-2\pi t_o (\frac{p^2}{2k} + \frac{kn^2}{2})}}{\cos(\pi nk) + \cosh(\pi p)} Z_{osc}
\]
Unitarity meets open/closed duality

- Pole part comes from poles in Euclidean (Wick) rotation

\[ \text{Im} Z_{\text{pole}} = 2 \int_0^\infty \frac{dt_0}{t_0} \sum_{n=1}^{\infty} \sum_{m=0}^{[\frac{kn}{2} - \frac{1}{2}]} (-1)^{n+1} e^{-\pi t_0 n(2m+1)} + \frac{2\pi t_0}{k} (m+\frac{1}{2})^2 Z \]

- Both contributions are imperative to understand
  - Unitarity
  - Tachyon-Radion correspondence

Summary

- In Euclidean approach, no apparent reason to include/exclude pole contributions.
- Unitarity demands its existence, and Lorentzian theory automatically knows it.
- Fortuitously no pole contribution in rolling tachyon (in 2D ZZ-brane decay as well).
Summary and Outlook

- **Exact boundary states** for rolling D-brane is constructed.
- **Tachyon-Radion correspondence** is proved in $\alpha'$ exact way.
  → Full proof in string field theory?
- **BH/String transition** is observed at $k=1$.
  → Is 2D pure BH really black? Matrix model?
- Consistency between **unitarity** and **open/closed duality** requires careful analytic continuation (Wick rotation).

*The shortest path between two truths in the real domain passes through the complex domain.* ---- J. Hadamard