New Results from Entropy Function:

Rotating (non-supersymmetric) Attractors

References:

Astefanesei, Goldstein, Jena, A.S., Trivedi, to appear

Earlier related work:

A.S. , hep-th/0506177, $\cdots$

Goldstein, Iizuka, Jena, Trivedi, hep-th/0507095, $\cdots$

Work on rotating SUSY attractors in D=5:

BMPV, hep-th/9602065
Kallosh, Rajaraman, Wong, hep-th/9611094
Kraus, Larsen, hep-th/0503219

Li, Strominger, hep-th/0605139

Entropy function analysis provides a good understanding of the attractor mechanism for **spherically symmetric** extremal black holes if

• we consider a theory of gravity coupled to abelian (p-form) gauge fields and neutral scalar fields

• the Lagrangian density $\mathcal{L}$ is gauge and general coordinate invariant

• define an extremal black hole to be one whose near horizon geometry is $AdS_2 \times S^2$ (in $D = 4$).

The theory **need not be supersymmetric** and $\mathcal{L}$ could contain **higher derivative terms**.

For such black holes one can define an ‘entropy function’ $\mathcal{E}$ as follows:

$$\mathcal{E} = 2\pi \left(q_i e_i - \int_{Horizon} \sqrt{-\det g}\, \mathcal{L} \right)$$

q_i: electric charges

e_i: near horizon radial electric field

$\mathcal{E}$ is a function of the q_i’s and various parameters labelling the SO(2,1)$\times$SO(3) symmetric near horizon background (*e.g.* sizes of AdS_2 and S^2, vev of scalars, radial electric fields e_i, radial magnetic fields p_i)

Results:

• For a black hole with given electric charges $\vec{q}$ and magnetic charges $\vec{p}$, all other near horizon parameters are obtained by extremizing $\mathcal{E}$ with respect to these parameters.

• The entropy is given by the value of $\mathcal{E}$ at its extremum. A.S.

Consequences

If $\mathcal{E}$ has a unique extremum with no flat directions then its extremization determines the near horizon background completely.

Hence there cannot be any dependence of the near horizon background and entropy on the asymptotic data on the moduli fields.

If $\mathcal{E}$ has flat directions then the near horizon field configuration is not completely determined by the extremization principle.

It can depend on the asymptotic data on the moduli fields.

Nevertheless the entropy, being independent of the flat directions, does not depend on the asymptotic data on the moduli fields.

Question: Can we generalize these results to rotating extremal black holes?

Answer: Yes

For simplicity we shall focus on black holes in D=4 with event horizon of spherical topology, but generalizations to other cases are possible.

- Supersymmetry plays no role in this analysis.

- Our analysis does not show the existence of such black holes, but derives its properties given its existence.

Question: How do we define extremal rotating black holes in a general higher derivative theory of gravity?

Clue: For extremal Kerr and Kerr-Newman black holes the near horizon geometry has $SO(2,1)\times U(1)$ isometry Bardeen, Horowitz

$\to$ symmetries of $\mathrm{AdS}_2\times\mathrm{S}^1$

We shall take this as the definition of extremal rotating black holes in a general theory of gravity coupled to matter fields.

(Tested in several two derivative theories)

Consider an arbitrary general coordinate invariant theory of gravity coupled to a set of abelian gauge fields $A_\mu^{(i)}$ and neutral scalar fields $\{\phi_s\}$.

Action:

$$\int d^4x \sqrt{-\det g}\, \mathcal{L}$$

$\mathcal{L}$: gauge and general coordinate invariant Lagrangian density

$\rightarrow$ function of metric, gauge field strengths, Riemann tensor, scalar fields and their covariant derivatives.

General form of the near horizon geometry with $SO(2,1)\times U(1)$ symmetry:

$$\begin{aligned} ds^2 &= v_1(\theta)\left(-r^2dt^2+\frac{dr^2}{r^2}\right)+\beta^2\,d\theta^2 \\ &\quad +\beta^2\,v_2(\theta)(d\phi-\alpha\,rdt)^2 \end{aligned}$$

$$\begin{aligned} \frac{1}{2}\,F^{(i)}_{\mu\nu}dx^\mu\wedge dx^\nu &= (e_i-\alpha\,b_i(\theta))dr\wedge dt \\ &\quad +\partial_\theta b_i(\theta)d\theta\wedge(d\phi-\alpha\,rdt)\,, \\ \Phi_s \;&=\; u_s(\theta) \end{aligned}$$

$$0\le\theta\le\pi, \qquad \phi\equiv\phi+2\pi$$

β, α, e_i: constant parameters

$v_1(\theta)$, $v_2(\theta)$, $u_s(\theta)$, $b_i(\theta)$: functions

$$ds^2 = v_1(\theta)\left(-r^2 dt^2 + \frac{dr^2}{r^2}\right) + \beta^2\, d\theta^2$$
$$+\beta^2\, v_2(\theta)(d\phi - \alpha r dt)^2$$

Smoothness at $\theta = 0, \pi$ requires:

$$v_2(\theta) \simeq \sin^2\theta + \mathcal{O}(\sin^4\theta)$$

$$\frac{1}{2} F^{(i)}_{\mu\nu} dx^\mu \wedge dx^\nu = (e_i - \alpha b_i(\theta)) dr \wedge dt$$
$$+ \partial_\theta b_i(\theta) d\theta \wedge (d\phi - \alpha r dt)\,,$$

Magnetic charge:

$$p_i = \int d\theta d\phi F_{\theta\phi} = 2\pi \{ b_i(\pi) - b_i(0) \}$$

By adjusting e_i we can choose boundary conditions:

$$b_i(0) = -\frac{p_i}{4\pi}, \qquad b_i(\pi) = \frac{p_i}{4\pi}$$

Define:

$$
\begin{aligned}
& f[\alpha, \beta, \vec{e}, v_1(\theta), v_2(\theta), \vec{u}(\theta), \vec{b}(\theta)] \\
= & \int d\theta d\phi \sqrt{-\det g}\, \mathcal{L}\Big|_{horizon} .
\end{aligned}
$$

f is a function of α, β, e_i and a functional of $v_1(\theta)$, $v_2(\theta)$, $u_s(\theta)$ and $b_i(\theta)$.

Define:

$$
\begin{aligned}
& \mathcal{E}[J, \vec{q}, \alpha, \beta, \vec{e}, v_1(\theta), v_2(\theta), \vec{u}(\theta), \vec{b}(\theta)] \\
= & 2\pi\Big(J\alpha + \vec{q} \cdot \vec{e} \\
& -f[\alpha, \beta, \vec{e}, v_1(\theta), v_2(\theta), \vec{u}(\theta), \vec{b}(\theta)]\Big) .
\end{aligned}
$$

Results:

For an extremal black hole of electric charge $\vec{q}$, magnetic charge $\vec{p}$ and angular momentum J:

1. The near horizon background is obtained by extremizing the entropy function $\mathcal{E}$ with respect to the parameters characterizing the near horizon background:

$$\frac{\partial \mathcal{E}}{\partial \alpha} = 0, \quad \frac{\partial \mathcal{E}}{\partial \beta} = 0, \quad \frac{\partial \mathcal{E}}{\partial e_i} = 0\,, \quad \frac{\delta \mathcal{E}}{\delta v_1(\theta)} = 0\,,$$
$$\frac{\delta \mathcal{E}}{\delta v_2(\theta)} = 0, \quad \frac{\delta \mathcal{E}}{\delta u_s(\theta)} = 0, \quad \frac{\delta \mathcal{E}}{\delta b_i(\theta)} = 0\,.$$

2. The entropy $S_{BH}(\vec{q},\vec{p},J)$ is given by

$$S_{BH} = \mathcal{E}[J,\vec{q},\alpha,\beta,\vec{e},v_1(\theta),v_2(\theta),\vec{u}(\theta),\vec{b}(\theta)]$$

at the extremum of $\mathcal{E}$ with respect to

$$\alpha,\beta,\vec{e},v_1(\theta),v_2(\theta),\vec{u}(\theta),\vec{b}(\theta)$$

The dependence on the magnetic charge $\vec{p}$ enters through the boundary condition:

$$b_i(0) = -\frac{p_i}{4\pi}, \qquad b_i(\pi) = \frac{p_i}{4\pi}$$

These results follow from straightforward use of equations of motion and Wald's formula for black hole entropy.

These results lead to a generalized attractor mechanism.

1. Extremization of the single 'entropy function' $\mathcal{E}$ determines the near horizon values of

– the scalar fields,

– the metric

– the gauge field strengths

in terms of the charges $\vec{q}$, $\vec{p}$ and angular momentum J.

2. If $\mathcal{E}$ has no flat directions then the extremization of $\mathcal{E}$ determines the near horizon background completely in terms of $\vec{q}$, $\vec{p}$ and J.

$\rightarrow S_{BH} = \mathcal{E}$ is independent of the asymptotic values of the scalar fields.

If $\mathcal{E}$ has flat directions, then extremization of $\mathcal{E}$ does not determine the near horizon background completely.

But since $\mathcal{E}$ does not depend on the flat directions, $S_{BH} = \mathcal{E}$ is still independent of the asymptotic values of the scalar fields.

Test of this formalism:

1. We have computed the entropy function for extremal Kerr and Kerr-Newman black holes in ordinary two derivative theories of gravity coupled to Maxwell field.

Extremization of the entropy function reproduces:

- correct form of the near horizon geometry
- correct value of the entropy

2. We have applied this formalism to study extremal rotating black holes in

– five dimensional ordinary gravity compactified on a circle (Kaluza-Klein theory)

– more general toroidally compactified heterotic string theory

at the level of two derivative action, and compared with known answers.

Rasheed; Matos and Mora; Larsen

Cvetic, Youm; Jatkar, Mukherji, Panda

Since the two theories have very similar properties and are related by duality, we shall focus on the black holes in the Kaluza-Klein theory.

4-dimensional fields:

Metric $g_{\mu\nu}$, a gauge field A_μ, a scalar field Φ

Lagrangian density:

$$\mathcal{L} = R - 2g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi - e^{2\sqrt{3}\Phi} g^{\mu\rho} g^{\nu\sigma} F_{\mu\nu}F_{\rho\sigma} \, .$$

Extremal black hole solutions in this theory are characterized by electric charge Q, magnetic charge P and angular momentum J.

Rasheed; Matos and Mora; Larsen

There are two distinct types of rotating extremal black holes arising from two different limits.

But both types of solutions have $SO(2,1) \times U(1)$ symmetric near horizon background and hence can be described using entropy function formalism.

None of these solutions preserve supersymmetry.

1. ergo-branch: This branch of solutions exist for

$$|J| > |P\,Q|$$

and have ergo-sphere.

Entropy: $2\pi\sqrt{J^2 - P^2Q^2}$

– independent of the asymptotic values of moduli in accordance with our general arguments.

However the near horizon background, including the metric, depends on the asymptotic values of the moduli fields.

$\to \mathcal{E}$ has flat directions

2. ergo-free branch: This branch of solutions exist for

$$|J| < |P\,Q|$$

and have no ergo-sphere.

Entropy: $2\pi\sqrt{P^2Q^2 - J^2}$

– independent of the asymptotic moduli in accordance with our general arguments.

On this branch the near horizon background is independent of the asymptotic values of the moduli fields.

$\rightarrow$ rotating attractor.

Radial evolution of Φ at fixed θ for $J = 16\pi/3$, $P = Q = 4\sqrt{\pi}$

Conclusion

The entropy function method can be used effectively to prove and study generalized attractor behaviour for rotating black holes.