Tiny Graviton Matrix Theory

DLCQ of tpye IIB strings on
the $AdS_5 \times S^5$ or the plane-wave background

By: M.M. Sheikh-Jabbari

Based on:

M.M.Sh-J, [hep-th/0406214]

M.M.Sh-J, M. Torabian,[hep-th/0501001]
M. Ali-Akbari, M.M.Sh-J, M. Torabian,[
hep-th/0512037 & 0606117]

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Plan of the Talk

- Inspirations from BFSS Matrix Model
- DLCQ of string/M- theory on $AdS_p \times S^q$ bg's.
- Some facts about the ten dim. plane-wave,
- The proposal for DLCQ of strings on $AdS_5 \times S^5$ or plane-wave background, the Tiny Graviton Matrix Theory (TGMT).
- Analysis of and Evidence for the Model.
- Summary, works in progress and a to-do-list.

- According to the BFSS conjecture

The Discrete Light-Cone Quantization (DLCQ) of M-theory on the flat $11d$ space in the sector with J units of the light-cone momentum is described by a

$U(J)$ SUSic Quantum Mechanics, i.e. a $U(J)$ $0+1$ dim. SYM theory with 16 SUSY.

Remarks:

- This theory is describing or described by a dynamics of J $D0$-branes.

- $D0$-branes are 1/2 BPS objects.

SUSY is a crucial ingredient for the consistency of the conjecture.

- There is a $U(N)$ gauge symmetry structure corresponding to N D0-branes.

- On the other hand $D0$-branes are nothing but gravity waves form the $11d$ viewpoint.

Quantum Gravity (M-theory) is a theory of quantized gravitons

• The BFSS matrix Model has been extended to describe DLCQ of M-theory on weakly curved backgrounds. It is done by adding proper deformations to the $0+1$ SYM action. There is a one-to-one relation between the background and the deformations [W. Taylor & M. van Raamsdonk '98, '99].

▶ Q1: What about the strongly curved backgrounds, namely, the $AdS_{4,7} \times S^{7,4}$ or the max. SUSic $11d$ plane-wave, bg's?

▶ Q2 Does DLCQ of M-theory on the above bg's admit a Matrix theory formulation?

Short Answers:

⋆ Yes,there is a Matrix Theory formulation.
⋆ DLCQ of M/String theory on the AdS bg. is the same as the corresponding plane-wave.
⋆ The DLCQ of M-theory on the $11d$ plane-wave is the BMN Matrix Model [hep-th/0202021].

■ Ingredients of DLCQ:

- (Globally defined) Light-like Killing vector.
- Compactification along light-like direction.

Consider $AdS_{p+2} \times S^{q+2}$ geometry:

$$ds^2 = R_A^2\Big(-\cosh^2\rho d\tau^2 + d\rho^2 + \sinh^2\rho d\Omega_p^2\Big) + R_S^2\Big(\cos^2\theta d\psi^2 + d\theta^2 + \sin^2\theta d\Omega_q^2\Big)$$

light-like geodesics

i) Inside AdS along the radial direction ρ.
ii) Inside sphere and along the ψ direction, at

$$\rho = \theta = 0, \;\; R_A \tau = \pm R_S \psi.$$

Only ii) is appropriate for the purpose of DLCQ and the light-like compactification.

• Next, let us follow the light-like observer and elaborate on the geometry seen by this observer....

Systematically this geometry is known as the Penrose limit of the original background.

For the $AdS_{p+2} \times S^{q+2}$ background, that is

$$ds^2 = -2dx^+dx^- - \mu^2(\vec{x}_p^2 + \kappa^2\vec{x}_q^2)(dx^+)^2 + d\vec{x}_p d\vec{x}_p + d\vec{x}_q d\vec{x}_q$$

$\kappa = \frac{R_s}{R_A}$ and μ is an arbitrary parameter of dimension of energy (length^{-1}).

This is a PLANE-WAVE geometry.

It has a globally defined light-like Killing vector:

$$p^+ = \frac{\partial}{\partial x^-}.$$

- For $(p,q) = (3,3)$, the $AdS_5 \times S^5$ case:

$$ds^2 = -2dx^+dx^- - \mu^2(X_i^2 + X_a^2)(dx^+)^2$$
$$+ dX_i dX_i + dX_a dX_a$$

$i = 1,2,3,4,\ a = 5,6,7,8$. This metric supplemented with

$$F_{+ijkl} = \frac{\mu}{4! g_s}\epsilon_{ijkl}\ ,\ F_{+abcd} = \frac{\mu}{4! g_s}\epsilon_{abcd}$$
$$e^{\phi} = g_s = const.$$

is the IIB $10d$ plane-wave bg.

DLCQ of type IIB strings on the $AdS_5 \times S^5$ geometry is the same as DLCQ of strings on the $10d$ plane-wave background.

Bosonic Isometries of the 10d plane-wave

- Translation along x^- and x^+:

$$H = P_- = i\frac{\partial}{\partial x^+}$$
$$p^+ = -i\frac{\partial}{\partial x^-}$$

- $SO(4)_i \times SO(4)_a$ rotations, generated by:

$$J_{ij}, \; J_{ab}.$$

- There are 16 other isometries not manifest in the above coordinate system, (K_i, L_i) and (K_a, L_a):

$$[K_i, L_j] = \mu p^+ \delta ij \;\; ; \;\; [K_a, L_b] = \mu p^+ \delta ab$$
$$[K_i, K_a] = [L_a, L_b] = [K_i, L_a] = [K_a, L_i] = 0$$

Altogether, #isometries=$2 + 12 + 16 = 30$.

Note:

$$dim\,(so(4,2) \times so(6)) = 30$$

$$dim\,(Iso(9,1)) = 55$$

$$dim\,(Iso(8)) = 36.$$

Fermionic Isometries of the 10d plane-wave

$SO(8)$ fermions can be decomposed into the $SO(4) \times SO(4)$ spinors as:

- (Complexified) $\mathbf{8_s} \to (\psi_{\alpha\beta},\ \psi_{\dot\alpha\dot\beta})$
- (Complexified) $\mathbf{8_c} \to (\psi_{\alpha\dot\beta},\ \psi_{\dot\alpha\beta})$

$\alpha, \dot\alpha = 1, 2$ are the $SO(4)$ Weyl indices.

■ Supercharges:

▶ Kinematical supercharges: $q_{\alpha\beta}, q_{\dot\alpha\dot\beta}$, and their complex conjugates, $\# = 16$.

▶ Dynamical supercharges: $Q_{\alpha\dot\beta}, Q_{\dot\alpha\beta}$, and their complex conjugates, $\# = 16$.

■ Kinematical SUSY:

$$\{q_{\alpha\beta}, q^{\dagger\rho\lambda}\} = 2\delta_\alpha^\rho \delta_\beta^\lambda P^+$$

$$\{q_{\dot\alpha\dot\beta}, q^{\dagger\dot\rho\dot\lambda}\} = 2\delta_{\dot\alpha}^{\dot\rho} \delta_{\dot\beta}^{\dot\lambda} P^+$$

$$[q_{\alpha\beta}, H] = \mu q_{\alpha\beta}\ ,\ [q_{\dot\alpha\dot\beta}, H] = -\mu q_{\dot\alpha\dot\beta}$$

$$[q_{\alpha\beta}, P^+] = 0$$

■ Dynamical SUSY:

$$\{Q_{\alpha\dot{\beta}}, Q^{\dagger\rho\dot{\lambda}}\} = 2\delta^{\rho}_{\alpha}\delta^{\dot{\lambda}}_{\dot{\beta}} H + 2\mu\delta^{\dot{\lambda}}_{\dot{\beta}}(\sigma^{ij})^{\rho}_{\alpha} J_{ij} + 2\mu\delta^{\rho}_{\alpha}(\sigma^{ab})^{\dot{\lambda}}_{\dot{\beta}} J_{ab}$$

$$\{Q_{\dot{\alpha}\beta}, Q^{\dagger\rho\dot{\lambda}}\} = 0$$

$$\{Q_{\dot{\alpha}\beta}, Q^{\dagger\dot{\rho}\lambda}\} = \delta^{\dot{\rho}}_{\dot{\alpha}}\delta^{\lambda}_{\beta} H + 2\mu\delta^{\lambda}_{\beta}(\sigma^{ij})^{\dot{\rho}}_{\dot{\alpha}} J_{ij} + 2\mu\delta^{\dot{\rho}}_{\dot{\alpha}}(\sigma^{ab})^{\lambda}_{\beta} J_{ab}$$

$$[Q_{\alpha\dot{\beta}}, H] = [Q_{\dot{\alpha}\beta}, H] = 0$$

$$[Q_{\alpha\dot{\beta}}, P^{+}] = [Q_{\alpha\dot{\beta}}, P^{+}] = 0$$

For the full SUSY algebra see
[D. Sadri, M.M. Sh-J, hep-th/0310119]
[M. Ali-Akbari, M.M. Sh-J, M. Torabian, hep-th/0512037].

• The plane-wave SUSY algebra can be obtained as the Penrose contract of $PSU(2,2|4)$.

• The dynamical part of the SUSY algebra is

$$PSU(2|2) \times PSU(2|2) \times U(1)_H \times U(1)_{p^+}$$

which is a subalgebra of $psu(2,2|4)$ w/ 16 SUSY.

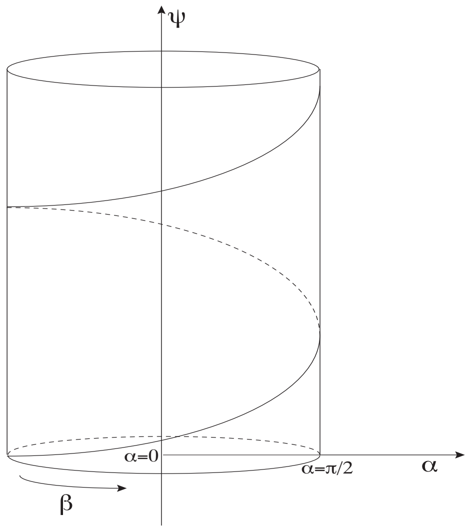
• p^+ is at the center of the whole plane-wave SUSY, i.e. it commutes with all supercharges. This should be contrasted with the flat space.

Penrose diagram of the plane-wave

By a series of coordinate transformations and analytic extension on the range of coordinates, the plane-wave metric can be brought to a form conformal to the Einstein static universe (conformal to $R \times S^9$): [Berenstein, Nastase, hep-th/0205048]

$$ds^2 = \frac{1}{\mu^2}\frac{1}{|e^{i\psi} - \sin\alpha e^{i\beta}|^2}\Big(- d\psi^2 + \sin^2\alpha d\beta^2 + d\alpha^2 + \cos^2\alpha d\Omega_7^2\Big)$$

$\alpha \in [0, \pi/2]$, $\beta \in [0, 2\pi]$, $\psi \in \mathbb{R}$.



The $\psi = \beta$, $\alpha = \pi/2$ is the casual boundary of the plane-wave, which is one dimensional light-like.

▶ Key Question:
What is playing the role of D0-branes for the $AdS_5 \times S^5$ or the plane-wave backgrounds?

Answer:
1/2 BPS Spherical branes, the giant gravitons.

■ Giant Gravitons, A quick review

• Dp-branes are objects carrying $p+1$-form RR-charges proportional to their volume form.

• (Topologically) spherical D-brane can't carry the corresponding RR charge. It can, however, carry electric dipole moment of the RR form.

• In the absence of any other force a spherical brane would collapse under its own tension.

• The electric dipole can be used to stabilize the brane, iff we have a moving brane in the corresponding background RR flux.

• Such a magnetic form-field flux exists in $AdS_{p+2} \times S^{q+2}$, $(p,q) = (2,5), (3,3), (5,2)$ sol'ns.

• It turns out that it is possible to stabilize spherical p or q branes in $AdS_{p+2} \times S^{q+2}$ spaces, that is,
spherical D3-branes in $AdS_5 \times S^5$ geometry and spherical M2 and M5 branes in $AdS_{4,7} \times S^{7,4}$.

• This is possible only when the branes are moving with the speed of light, i.e. when they are following a light-like geodesic and hence they are like graviton, the Giant Gravitons.

• Giant Gravitons are 1/2 BPS objects in the above backgrounds.

• Giant 3-brane Gravitons are D-branes and hence there is a $U(J)$ gauge symmetry structure associated to J number giant gravitons.

• Their size is then fixed by their angular momentum J as:

$$\left(\frac{R_{giant}}{R_{AdS}}\right)^{p-1} = \frac{J}{N}$$

where

$$(R_{AdS})^{p+1} = (l_p)^{p+1} N.$$

Therefore,

$$\left(\frac{R_{giant}}{l_p}\right)^{p-1} = \left(\frac{l_p}{R_{AdS}}\right)^2 J = \frac{J}{N^{\frac{2}{p+1}}}$$

What if J takes it minimal value $J = 1$?

In this case we call them TINY Gravitons. The scale associated with the tiny gravitons, ℓ, is much smaller than the Planck units.

▶ Tiny 3-brane Gravitons:

$$\boxed{\ell^2 = \frac{l_p^2}{R_{AdS}} \quad OR \quad \ell^4 = l_p^4 \times \frac{1}{N}}$$

Size of a generic giant three brane graviton is

$$\boxed{R_{giant}^2 = \ell^2 J}$$

Q: Can we use the same observation, but now with tiny three branes, for DLCQ formulation of type IIB strings on the $10d$ plane-wave bg?

Note:

Tiny three-brane gravitons, by definition, are D3-branes carry one unit of the angular (or light-cone) momentum, and they are 1/2 BPS objects, exactly like D0-branes. Hence they are perfect objects for obtaining the DLCQ formulation of type IIB string theory on the $AdS_5 \times S^5$ or the plane-wave background, the

Tiny Graviton Matrix Theory

OR

the TGMT, in short.

The TGMT Conjecture

DLCQ of type IIB string theory on the $AdS_5 \times S^5$ or the ten dim. max. SUSic plane-wave background in the sector with J units of the the light-cone momentum is described by a $U(J)$ $0+1$ dim. supersymmetric gauge theory; i.e. a $U(J)$ SUSY QM, with the Hamiltonian

$$\mathcal{H} = R_- \; Tr \Bigg[\frac{1}{2} \Pi_I^2 + \frac{1}{2} \left(\frac{\mu}{R_-} \right)^2 X_I^2 + \frac{1}{2 \cdot 3! g_s^2} [X^I, X^J, X^K, \mathcal{L}_5][X^I, X^J, X^K, \mathcal{L}_5] - \frac{\mu}{3! R_- g_s} \Big(\epsilon_{ijkl} X^i [X^j, X^k, X^l, \mathcal{L}_5] + \epsilon_{abcd} X^a [X^b, X^c, X^d, \mathcal{L}_5] \Big) + Fermions \Bigg]$$

where $I, J, K = 1, 2 \cdots, 8$, $I = \{i, a\}$ and $i, j = 1, 2, 3, 4$ and $a, b, c = 5, 6, 7, 8$.

- $\mathcal{L}_5$ is a $J \times J$ hermitian matrix where

$$\mathcal{L}_5{}^2 = \mathbf{1}_{J\times J} \ , \quad Tr\mathcal{L}_5 = 0$$

- $$[F_1, F_2, F_3, F_4] \equiv \epsilon^{ijkl} F_i F_j F_k F_l$$

is the quantized Nambu 4-bracket.

Nambu brackets are a direct generalization of Poisson brackets. They define an algebra, though not the symplectic one.

This algebra is closely related to the Quantum version of the Area Preserving Diffeomorphisms of a four (or three) dimensional surface.

- The TGMT Hamiltonian is obtained from the Discretized or Quantized version of the action of a D3-brane in the ten dimensional plane-wave background.

This is similarly to the BFSS or BMN matrix models which are obtained from discretized (regularized) membrane action in the light-cone gauge.

• The TGMT enjoys the $U(J)$ gauge symmetry

$$\begin{aligned} \Phi &\rightarrow U\Phi U^{-1}, \quad U \in \ U(J) \\ \mathcal{L}_5 &\rightarrow U\mathcal{L}_5 U^{-1} \end{aligned}$$

where $\Phi \in \{X^I, \Pi^I, \ Fermions\}$.
Although a gauge theory, TGMT is not a Yang-Mills theory.

• It has global $SO(4)_i \times SO(4)_a$ as well as $i \leftrightarrow a$ $\mathbb{Z}_2$ symmetries, under which the $\mathcal{L}_5$ is invariant.

• The TGMT is invariant under superalgebra

$$PSU(2|2) \times PSU(2|2) \times U(1)_H$$

which is the dynamical part the plane-wave SUSY algebra.

• The Physical states of the TGMT are subject to the Gauss Law constraint:

$$\left(i[X^I, \Pi^I] + 2\Psi^{\dagger\alpha\beta}\Psi_{\alpha\beta} + 2\Psi^{\dagger\dot{\alpha}\dot{\beta}}\Psi_{\dot{\alpha}\dot{\beta}}\right) |\phi\rangle_{phys} = 0$$

• TGMT, besides J has two dimensionless parameters,
R_-/μ which is the radius of the light-like compactification (in string units) and g_s.

• When used for the plane-wave bg, R_- is an arbitrary parameter.
When used for DLCQ of $AdS_5 \times S^5$ bg,

$$\frac{R_-}{\mu} = \frac{R^2_{AdS}}{l_s^2} = (g_s N)^{1/2}.$$

The tiny graviton scale ℓ, in the Language of the TGMT Hamiltonian is then

$$\ell^2 = \frac{\mu g_s}{R_-} l_s^2$$

• The plane-wave string theory is recovered in the (continuum) limit:

$$J, R_- \to \infty, \;\; p^+ = \frac{J}{R_-}, \mu, g_s = fixed$$

OR in other words, $\ell \to 0, \;\; \ell^2 J, \;\; l_p = fixed,$
that is the BMN limit

$$J, N \to \infty \;\; , \;\; g_s, l_p, J^2/N = fixed.$$

▶ For the TINY Membrane Gravitons:

$$\ell = \frac{l_p^3}{R^2_{Ads_4}} = l_p \; N^{-2/3}$$

Remarks:

M5branes do not become tiny in $AdS_4 \times S^7$ bg, while they do become tiny in $AdS_7 \times S^4$ bg.

Similarly, tiny membrane gravitons, may be used to give a DLCQ description of M-theory on the $AdS_4 \times S^7$ bg, or the $11d$ plane-wave. That is,

tiny membrane gravitons play the role of $D0$-branes of BFSS in this DLCQ.

In fact the BMN matrix model is nothing but the theory of J tiny gravitons, i.e.

$11d$ plane-wave matrix theory

$\equiv$

tiny (membrane) graviton matrix theory.

Evidence for the TGMT conjecture

⊛ ⊛ ⊛ ⊛ ⊛ ⊛ ⊛ ⊛ ⊛ ⊛ ⊛⊛

⊛ The fact that the TGMT Hamiltonian is invariant under the expected SUSY and that the superalgebra is a big one (with 16 SUSY), puts severe restrictions on the form of the Hamiltonian. (To my knowledge, however, there is no no-go theorem on this.)

To be precise the TGMT naturally contains some of the (central) extensions of the

$$PSU(2|2) \times PSU(2|2) \times U(1)$$

and in particular an R_{ijab} 4-form, corresponding to the dipole moment of the self-dual RR five-form.

For more details see [M. Ali-Akbari, M.M. Sh-J, M. Torabian, hep-th/0512037].

• 1/2 BPS (zero energy) solutions.

$$V_B = R_- Tr\left[\frac{1}{2}\left(\frac{\mu}{R_-}X^i - \frac{1}{3!g_s}\epsilon_{ijkl}[X^j, X^k, X^l, \mathcal{L}_5]\right)^2 + \frac{1}{2}\left(\frac{\mu}{R_-}X^a - \frac{1}{3!g_s}\epsilon_{abcd}[X^b, X^c, X^d, \mathcal{L}_5]\right)^2 + \frac{1}{4g_s^2}\left([X^a, X^b, X_i, \mathcal{L}_5]^2 + [X^i, X^j, X_a, \mathcal{L}_5]^2\right)\right].$$

All four terms are +ve definite and hence to have zero energy, they should all vanish:

$$[X^j, X^k, X^l, \mathcal{L}_5] = \ell^2 \epsilon_{ijkl} X^i$$
$$[X^b, X^c, X^d, \mathcal{L}_5] = \ell^2 \epsilon_{abcd} X^a$$
$$[X^a, X^b, X^i, \mathcal{L}_5] = [X^a, X^i, X^j, \mathcal{L}_5] = 0.$$

where

$$\ell^2 = \frac{\mu g_s}{R_-} l_s^2$$

is the tiny graviton scale.

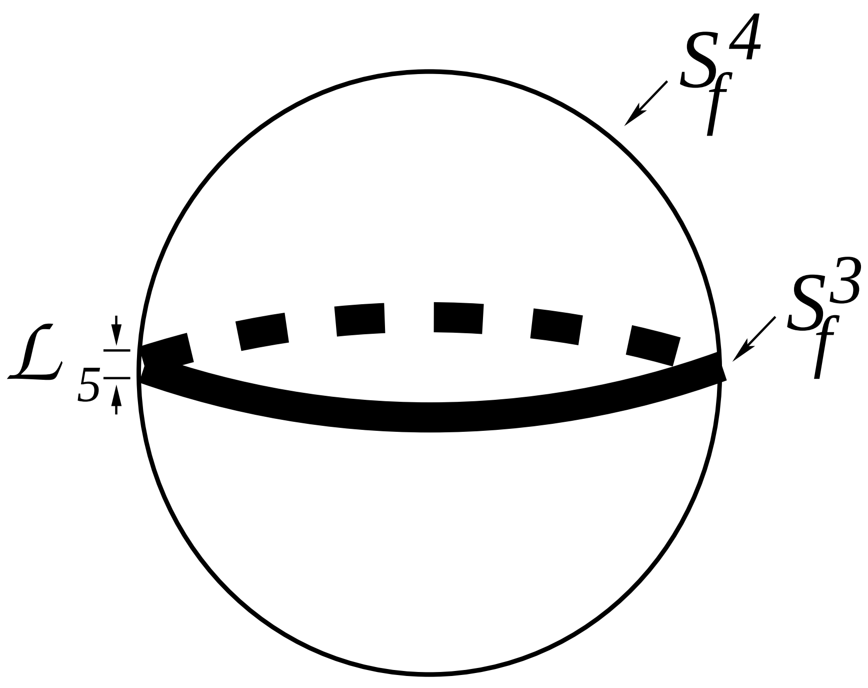
• They are all in the form of Fuzzy three Spheres S_F^3 either in X^i and/or X^a directions. As some particular examples, consider

$$X^a = 0\ , \quad [X^j, X^k, X^l, \mathcal{L}_5] = \epsilon_{ijkl}\frac{\mu g_s}{R_-}\ X^i$$

• These sol'ns are classified by $J \times J$ representations of $SO(4)$. For the irreducible reps, that is a single fuzzy sphere of radius (in units of l_s):

$$R^2 = \ell^2 J = \mu p^+ g_s.$$

Recall that for giant three sphere gravitons we also had a similar relation between radius and the ℓ. Therefore, this 1/2 BPS solution of the TGMT is nothing but a spherical three brane. In the continuum (string theory) limit it recovers the commutative giant 3-brane graviton.



- The reducible reps, generically give concentric giants, their radii squared sum to $\ell^2 J$. i.e.

the problem of classification of all 1/2 BPS states is equivalent to the problem of partition of a given integer J into a set of non-negative integers $\{J_i\}$, such that

$$\sum_i J_i = J.$$

These solutions are labeled by representations of group of permutations of J objects, $\mathcal{S}_J$ which can be represented as Young Tableaux of J boxes. Therefore,

There is a one-to-one correspondence between the half BPS states of $\mathcal{N} = 4$ $U(N)$ SYM with R-charge J, the chiral primary opts, 1/2 BPS sugra sol'ns, the LLM geometries and the fuzzy sphere soln's of the TGMT [M.M. Sh-J, M. Torabian, hep-th/0501001].

⊛ One can analyze less BPS states of TGMT. These are generically of the form of three-branes of various shape.

- One class of them are rotating spherical branes. This class is connected to the 1/2 BPS solutions when we turn off the angular momenta. Here we have 1/4 or 1/8 BPS rotating branes. These configurations are of the form of deformed spherical branes. Among this class we have configurations which are not geometric deformation of branes, they rather correspond to turning on gauge fields on the three brane.

- The second class are those which are not (small) deformations of 1/2 BPS solutions such as three-branes of hyperbolic shape.

It has been shown that there is a one-to-one correspondence between the TGMT 1/4 and 1/8 BPS states and those of $\mathcal{N} = 4$ $U(N)$ SYM. [M. Ali-Akbari, M.M. Sh-J, M. Torabian, hep-th/0606117]

⊛ Spectrum of fluctuations about the single giant fuzzy sphere solution has been worked out in [hep-th/0406214] and shown that
it exactly matches that of a spherical D3-brane in the plane-wave (or $AdS_5 \times S^5$) background. (The latter has been worked out in
[D. Sadri, M.M. Sh-J, hep-th/0312155].)
This is a non-trivial and crucial test, because the TGMT action was obtained from the DBI action in which the gauge field on the brane was not included.
Intuitively, the gauge field has appeared as a result of having the $\mathcal{L}_5$ [M. Ali-Akbari, M.M. Sh-J, M. Torabian, hep-th/0606117].
In a sense $\mathcal{L}_5$ accounts for the internal, non-geometric degrees of freedom on the brane.

⊛ One can also work out the effective coupling of these fluctuation modes [hep-th/0406214]. The effective coupling about the single giant vacuum is:

$$g_{eff} = \frac{R_-}{\mu\sqrt{g_s}}\frac{1}{J} = \frac{1}{\mu p^+\sqrt{g_s}}$$

The same result has been obtained in
[D. Sadri, M.M. Sh-J, hep-th/0312155]
for a round three sphere giant graviton (in the continuum limit).

It is worth noting that, expressed in terms of the $\mathcal{N} = 4$ $U(N)$ SYM parameters,

$$g_{eff}^2 = \frac{N}{J^2} = \frac{1}{g_2}$$

where g_2 is the effective coupling for strings on plane-wave S. Minwalla, et.al.[hep-th/0205089].

Q: What about the $X = 0$ vacuum?!
Where are type IIB fundamental strings?

■ 2nd part of the conjecture

In the string theory limit the $X = 0$ vacuum becomes strongly coupled and Fundamental type IIB strings are non-perturbative objects about the $X = 0$ vacuum.

(Remark: This is very similar to the M5-brane giants in the BMN matrix model, [J. Maldacena, M.M. Sh-J, M. van Raamsdonk, hep-th/0211139].)

⊛ Evidence: the spectrum of small BPS fluctuations (in a $\frac{R_-}{\mu}$ expansion) about the $X = 0$ vacuum, exactly matches with the spectrum of SUGRA modes (BPS spectrum of strings) in the plane-wave bg.

More works in this direction is under way .....

■ Summary and a short to-do-list

§ Tiny Gravitons may be used as “D0-branes” to give a matrix theory, $0+1$ dim. gauge theory, formulation for string/M- theory on the curved backgrounds, such as AdS-spaces and the plane-waves.

For the 11d. case, that is:

The BMN (Plane-Wave Matrix Model) is equivalently the

Tiny Graviton (Membrane) Matrix Theory.

§ The plane-wave string theory, in the DLCQ description, admits a matrix theory formulation, the TGMT. It is a SUSY gauge theory (though not a SYM!) with SUSY $PSU(2|2) \times PSU(2|2) \times U(1)_H$.

★ As the causal boundary of the plane-wave is one-dimensional light-like, the DLCQ description is the holographic description.

Therefore,

TGMT is the holographic formulation of string/M theory on the plane-wave.

★ Conceptually the TGMT and the BFSS (and its variants) are sharing the fact that in both cases this is the gravitons or gravity waves or metric fluctuations which are used to formulate a quantum gravity.

In other words, it leads to the motto that

Theory of Quantum Gravity is nothing but theory of quantized gravitons.

§ DLCQ vs. Covariant formulation?!

$$dim(\text{plane} - \text{wave Isometries}) = dim(AdS_5 \times S^5).$$

This is to be compared with the flat or the BFSS case. In the plane-wave we do not have J^{+-} and J^{+I} boosts and p^+ commutes with all the SUSY generators.

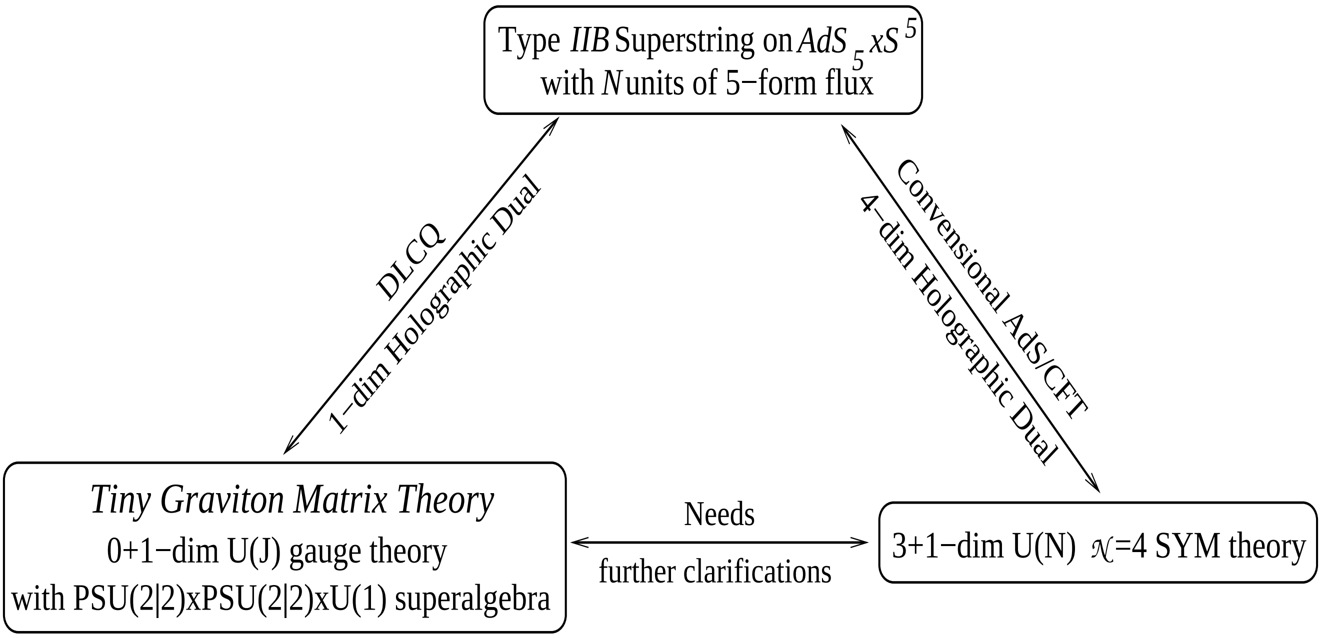
Therefore, in comparison to the BFSS case, TGMT has a better chance of capturing the covariant information of string on $AdS_5 \times S^5$.

§ In the formulation of TGMT, we introduced an extra traceless $J \times J$ matrix $\mathcal{L}_5$, which squares to identity.

The $\mathcal{L}_5$ is reminiscent of the eleven dimensional origin of the type IIB theory. It is related to the 11^{th} circle [hep-th/0501001].

Moreover, to recover the internal (gauge) d.o.f on the three brane vacua, one should introduce the $\mathcal{L}_5$ [hep-th/0606117].

★ How and where does TGMT fit in the AdS/CFT duality?!



★ Interesting observation: the tiny graviton scale or the fuzziness ℓ,

$$\ell^4 = \frac{1}{N} l_p^4 .$$

That is, the $1/N$ expansion has now a geometric meaning.

Q: $\frac{1}{J}$ vs. $\frac{1}{N}$ expansion?!

Q: Is $\mu \to 0$ (flat space) limit a smooth one? Does the TGMT in the $\mu \to 0$ limit recover the IKKT or $(2+1)$ SYM$/T^2$ (Susskind-Sethi model)?

Q: Lower SUSY and other D-brane solutions?

This has been partially addressed in [hep-th/0606117].

Q: Connection to Verlinde's String Bit Model? Or to the DVV Matrix String Theory?!

Q: For finite R_- we expect strings to have winding modes, where are they?

Q: Does TGMT satisfy the duality requirements, in particular the $SL(2,\mathbb{Z})$ of type IIB?

Q: Can we "quantize" an M5-brane theory in the same way we did for a 3-brane?
That is, by replacing the Nambu five brackets which appear in the M5-brane Hamiltonian
[e.g. see, hep-th/0211139],
by the Nambu six brackets introducing an $\mathcal{L}_7$?

Are the quantized giant M5-branes in the form of S^5_F?!
There are hints that the answer to this question is YES.
With in the above approach, one can obtain a matrix theory formulation of six dimensional $(0,2)$ theory (on $R \times S^5$)?

There are much more things to be done on the TGMT.......

De tour: How to obtain TGMT from a 3-brane DBI action on the plane-wave bg

$$S = \frac{1}{l_p^4}\int d\tau d^3\sigma\sqrt{-\det\left(G_{\mu\nu}\partial_a X^\mu \partial_b X^\nu\right)} + \int C_4$$

where $X^\mu = X^\mu(\sigma^r, \tau)$, $r = 1, 2, 3$ and

$\mu \in \{+, -, I\}$, $I = 1, 2, \cdots, 8$.

Note that we have turned off the gauge field on the brane to zero. To be discussed further later.....

Fixing the light-cone gauge:

$$X^+ = \tau$$

$$g_{0r} = G_{\mu\nu}\partial_0 X^\mu \partial_r X^\nu = 0$$

Note: the latter leads to "level matching" condition.

- The momenta:

$$p^+ = \frac{\partial L_{BI}}{\partial(\partial_\tau X^+)} \;;\; P^I = \frac{\partial L_{BI}}{\partial(\partial_\tau X^I)}$$

$$H_{l.c.} = -\frac{\partial L_{BI}}{\partial(\partial_\tau X^-)}$$

• p^+ is a constant of motion and may be used to eliminate $\partial_\tau X^-$ for other d.o.f, the X^I's.

$$H = \frac{1}{2p^+}\int d^3\sigma \Big[P_I^2 + \frac{1}{g_s^2}\det g_{rs} + (\mu p^+)^2 X_I^2 - \frac{\mu p^+}{3g_s}\Big(\epsilon_{ijkl}X^i\{X^j, X^k, X^l\} + \epsilon_{abcd}X^a\{X^b, X^c, X^d\}\Big)\Big]$$

where

$$g_{rs} = \partial_r X^I \partial_r X^I = \partial_r X^i \partial_r X^i + \partial_r X^a \partial_r X^a$$

$$\{F, G, H\} = \epsilon_{rps}\partial_r F\ \partial_p G\ \partial_s H$$

are the Nambu 3-brackets, a direct generalization of the Poisson bracket, and

$$\det g_{rs} = \frac{1}{3!}\Big(\{X^I, X^J, X^K\}\{X_I, X_J, X_K\}\Big)\,.$$

(One may add fermionic terms as well. For more details of LC gauge fixing see [D. Sadri & M.M.Sh-J, hep-th/0312155].)

After adding the fermions and the gauge fields, the full Hamiltonian enjoys the $PSU(2|2)\times PSU(2|2)\times U(1)_H$ invariance.

To quantize the above action, similarly to the membrane case, it is enough to quantize the Nambu brackets.

(De tour on Nambu brackets:

A Nambu p-bracket is defined as:

$$\{A_1, A_2, \cdots, A_p\} = \epsilon^{r_1 r_2 \cdots r_p} \frac{\partial A_1}{\partial \sigma^{r_1}} \frac{\partial A_2}{\partial \sigma^{r_2}} \cdots \frac{\partial A_p}{\partial \sigma^{r_p}}$$

where $A_i = A_i(\sigma^r)$, $r = 1, 2, \cdots, p$.

- Properties of N.B.

1) Cyclicity & Exchange property:

$$\{A_1, A_2, \cdots, A_p\} = -\{A_2, A_1, \cdots, A_p\}$$
$$\{A_p, A_1, \cdots, A_{p-1}\} = (-1)^{p-1}\{A_2, A_1, \cdots, A_p\}$$

(Note that $\epsilon^{i_1 i_2 \cdots i_p} = (-1)^{p-1}\epsilon^{i_p i_1 \cdots i_{p-1}}$.)

2) Jacobi Identity:

$$\epsilon^{i_1 i_2 \cdots i_{2p-1}} \times$$
$$\times \left\{F_{i_1}, F_{i_2}, \cdots, F_{i_{p-1}}, \{F_{i_p}, F_{i_{p+1}}, \cdots, F_{i_{2p-1}}\}\right\} = 0$$

3) Associativity:

$$\{F_1, F_2, \cdots, F_{p-1}, F_p G_p\} =$$
$$\{F_1, F_2, \cdots, F_{p-1}, F_p\} G_p + \{F_1, F_2, \cdots, F_{p-1}, G_p\} F_p$$

4) Trace property:

$$\int d^p\sigma \ \{F_1, F_2, \cdots, F_{p-1}, F_p\} = 0$$

5) By-part Integration:

$$\int d^p\sigma \{F_1, F_2, \cdots, F_{p-1}, F_p\} G_p =$$
$$- \int d^p\sigma \{F_1, F_2, \cdots, F_{p-1}, G_p\} F_p$$

Note: 5) is a result of 3)+4).

Quantization of Nambu Brackets

It is not possible to perform quantization for $p \geq 2$ and maintain all the five properties of the classical Nambu brackets. So we should make a compromise. With the following prescription one can maintain by-parts and trace properties for EVEN brackets but the associativity is lost.

- Quantized Nambu EVEN brackets

i) $A(\sigma_i) \longleftrightarrow \widehat{A}$ (matrices or operators).

ii) $\{F_1, F_2, \cdots, F_{2p}\} \longleftrightarrow [\widehat{F}_1, \widehat{F}_2, \cdots, \widehat{F}_{2p}]$

$$\equiv \frac{i^p}{(2p)!} \epsilon^{i_1 i_2 \cdots i_{2p}} \, \widehat{F}_{i_1} \widehat{F}_{i_2} \cdots \widehat{F}_{i_{2p}}.$$

iii) $\int d^{2p}\sigma \, \star \longleftrightarrow Tr \widehat{\star}.$

- Trace and by-part properties survive, as

$$\epsilon^{i_1 i_2 \cdots i_{2p}} = -\epsilon^{i_{2p} i_1 \cdots i_{2p-1}}.$$

- Quantized Nambu ODD brackets

Obviously the above procedure for even N.B. cannot be extended to odd N.B. while keeping the trace property.

The way out [hep-th/0406214]

Replace the Nambu $(2p-1)$-bracket with a Nambu $2p$-bracket:

$$\{F_1, F_2, \cdots, F_{2p-1}\} \longleftrightarrow [\widehat{F}_1, \widehat{F}_2, \cdots, \widehat{F}_{2p-1}, \mathcal{L}_{2p+1}]$$

$\mathcal{L}_{2p+1}$ is a given matrix (operator) closely related to the chirality operator in $2p$ dimensions. In particular, for the case of our interest, $p = 2$:

$$\{A, B, C\} \longleftrightarrow [A, B, C, \mathcal{L}_5] = \frac{1}{4!}\Big([A, B][C, \mathcal{L}_5] + [C, \mathcal{L}_5][A, B] - [B, \mathcal{L}_5][A, C] - [A, C][B, \mathcal{L}_5] + [A, \mathcal{L}_5][B, C] + [B, C][A, \mathcal{L}_5]\Big).$$

End of De tour on Nambu brackets)

Hence for Quantization of the LC Hamiltonian, replace:

$$p^+ \longleftrightarrow \frac{J}{R_-}$$

$$X_I(\sigma),\ P_I(\sigma) \longleftrightarrow X^I,\ J\Pi^I \ (J \times J \text{ matrices})$$

$$\psi_{\alpha\beta}(\sigma),\ \psi_{\dot\alpha\dot\beta}(\sigma) \longleftrightarrow \sqrt{J}\psi_{\alpha\beta},\ \sqrt{J}\psi_{\dot\alpha\dot\beta}$$

$$\frac{1}{p^+}\int d^3\sigma \,\star \longleftrightarrow R_-\ Tr\widehat{\star}$$

$$\{F, G, K\} \longleftrightarrow \frac{1}{J}[\widehat{F}, \widehat{G}, \widehat{K}, \mathcal{L}_5]$$

End of De tour From DBI to TGMT]