Why $Z_{\mathrm{BH}} = |Z_{\mathrm{top}}|^2$.

Davide Gaiotto, AS and Xi Yin hepth/0602046;

****

Chris Beasley, Monica Guica, DG,

Lisa Huang, AS and XY; in progress

****

STRINGS 2006 BEIJING

1. BASIC IDEA

For a IIA Calabi-Yau black hole

$$\begin{aligned} Z_{BH} &= \sum \mathrm{near-horizon\ \ bound\ \ states} \\ &= \sum (\mathrm{wrapped\ \ M2-branes}) \times \\ &\qquad\quad (\mathrm{wrapped\ \ anti-M2-branes}) \\ &= Z_{\mathrm{top}} \bar{Z}_{\mathrm{top}}. \end{aligned} \tag{1.1}$$

We will see this in 6 steps.

2. STEP ONE

Write the elliptic genus of the D4-D2-D0 (p^A, q_A, q_0) black hole as a partition function of a wrapped M5 brane

$$Z_{BH} = Z_{CFT}(\tau, y^A) \tag{2.1}$$

$$= \mathrm{Tr}_R (-)^F q^{L_0 - \frac{c_L}{24}} \bar{q}^{\bar{L}_0 - \frac{c_R}{24}} e^{2\pi i y^A q_A} \tag{2.2}$$

a la MSW. q_A is a membrane charge(= momenta in Narain latttice) and y^A the conjugate potential.

3. STEP TWO

Use spectral flow to NS sector and AdS_3/CFT_2

$$Z_{CFT} = e^{\frac{-\pi i \tau c_L}{12}} Z_{sugra}(AdS_3 \times S^2 \times CY_3) \tag{3.1}$$

to reexpress as a sum over chiral primaries on the M5 attractor with

$$G_4 = \omega_{S^2} \wedge p^A \omega_A \tag{3.2}$$

$$\int_{\alpha^A} J = (2\pi)^2 \frac{p^A}{\ell} \tag{3.3}$$

4. STEP THREE

Find classical BPS wrapped M2-branes.

These have

$$L_0 = \overline{L}_0 = J^3 = \tfrac{1}{2} q_A p^A. \qquad (4.1)$$

MAIN POINT: CAN BE WRAPPED BRANES AT NORTH POLE OR WRAPPED ANTI M2-BRANES AT S POLE!!

Can also orbit.

5. STEP FOUR

Go quantum. After some work; there are N_{q_A, j_L, j_R} (Gopakumar-Vafa invariant) chiral primary hypermultiplets with

$$L_0 = \tfrac{1}{2} q_A p^A + 2m_L + l + 1 + J_\phi \quad (5.1)$$

$$-j_{L,R} \leq m_{L,R} \leq j_{L,R}, \quad l \geq 0, \quad J_\phi \geq 0. \quad (5.2)$$

6. STEP FIVE

Sum over dlilute gas of multiparticle chiral primary wrapped M2s

$$Z_{sugra} = \prod_{q_A,n,m_L,j_L,j_R} \left[1 - e^{2\pi i\tau(\frac{1}{2}q_A p^A + n + 2m_L)} e^{2\pi i q_A y^A}\right]^{(-)^{2j_R+2j_L} n(2j_R+1)N_{q_A,j_L,j_R}} \times (y^A \to -y^A), \tag{6.1}$$

where J_ψ and l are non-negative integers, n is a positive integer and $-j_{L,R} \leq m_{L,R} \leq j_{L,R}$.

7. STEP SIX

Modular transform Z_{CFT} and use GV-GW relation

$$
\begin{aligned}
&Z_{BH} \\
&= Z_{CFT}(\tau, y^A) \\
&= Z_{CFT}(-1/\tau, y^A/\tau) e^{-\frac{2\pi i}{\tau} y^2} \\
&= \exp\left[\frac{2\pi i}{\tau}\left(\frac{c_L}{24} - y^2\right)\right] Z_{sugra}(-1/\tau, y^A/\tau) \\
&= \mathrm{Tr}_R[(-)^F \bar{q}^{\bar{L}_0 - \frac{c_R}{24}} e^{-\phi^A q_A - \phi^0 (L_0 - \frac{c_L}{24})}] \\
&= \left| Z_{top}(g_{top} = \frac{4\pi^2}{\phi^0}, t^A = \frac{\phi^A - \pi i p^A}{\phi^0}) \right|^2 .
\end{aligned}
\tag{7.1}
$$

for purely imaginary $\tau = i\phi^0/2\pi$ and $y^A = i\phi^A/2\pi$. THIS IS OSV!

8. Direct derivation (in progress)

At high temperatures, $M \rightarrow IIA$, so we should look for a direct string theory worldsheet derivation. BGGHSY (in progress) propose

(i) Euclidean string partition function on a IIA attractor$=Z_{BH}$

(ii) This IIA partition function is (N pole worldsheet instantons)$\times$ (S pole worldsheet anti-instantons)

(iii)This gives $Z_{BH} = |Z_{top}|^2$

(iv)Seems to be working with Green-Schwarz formalism

THANK YOU!