Quantum Vortex Strings:

A Review of Gauge Theory Solitons

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The Plan

- Review of Classical Solitons
  - Instantons, monopoles, vortices and domain walls
  - Composite solitons:
- Dynamics of Vortex Strings
  - N=(2,2) dynamics for N=2 vortex strings
  - N=(0,2) dynamics for N=1 vortex strings
- Further Topics
  - Relationships between moduli spaces
  - Open-string description of field theory solitons

Classical Solitons: Instantons

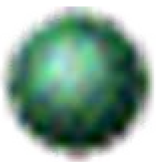
BPST

U(N) Gauge Field

$$S = \int d^4x \; \frac{1}{2e^2} \, \mathrm{Tr} \, F_{\mu\nu} F^{\mu\nu}$$

Instanton Equations: $F_{\mu\nu} = {}^\star F_{\mu\nu}$

Instanton Action: $S = \dfrac{8\pi^2 k}{e^2}$



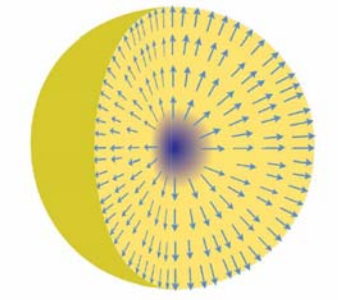
Classical Solitons: Monopoles

't Hooft, Polyakov

Adjoint Scalar

$$S = \int d^4x \; \frac{1}{2e^2} \,\mathrm{Tr}\, F_{\mu\nu}F^{\mu\nu} + \frac{1}{e^2}(\mathcal{D}_\mu \phi)^2$$

Vacuum: $\langle \phi \rangle = \vec{\phi} \cdot \vec{H}$



Monopole Equations: $B_i = \mathcal{D}_i \phi$

Monopole Mass: $M = \frac{4\pi}{e^2}\, \vec{\phi} \cdot \vec{g}$

The Higgs Phase

Fundamental Scalars

$$S = \int d^4x \; \frac{1}{2e^2} \,\mathrm{Tr}\, F_{\mu\nu}F^{\mu\nu} + \frac{1}{e^2}(\mathcal{D}_\mu \phi)^2 + \sum_{i=1}^{N_f} |\mathcal{D}_\mu q_i|^2$$

$$- \sum_{i=1}^{N_f} q_i^\dagger \phi^2 q_i - \frac{e^2}{2}\,\mathrm{Tr}\,(\sum_{i=1}^{N_f} q_i q_i^\dagger - v^2)^2$$

D-term, with FI parameter

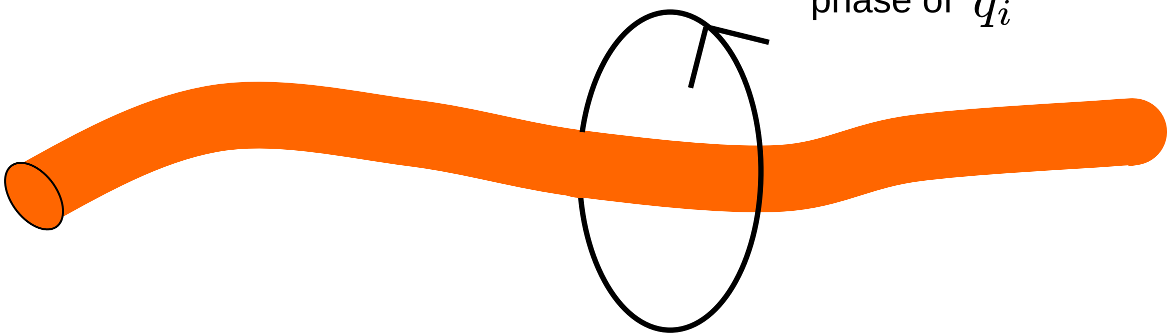
Vacuum: (requires $N_f \geq N_c$) $\langle \phi \rangle = 0$ and $\langle q \rangle \sim v$

The theory now lives in the Higgs phase. The U(N) gauge group is fully broken.

For $N_f > N_c$ there is a Higgs branch of vacua.

Vortex Strings

Nielsen, Olesen



Vortex Equations: $B_3 = \frac{e^2}{2}(\sum_i q_i q_i^\dagger - v^2)$

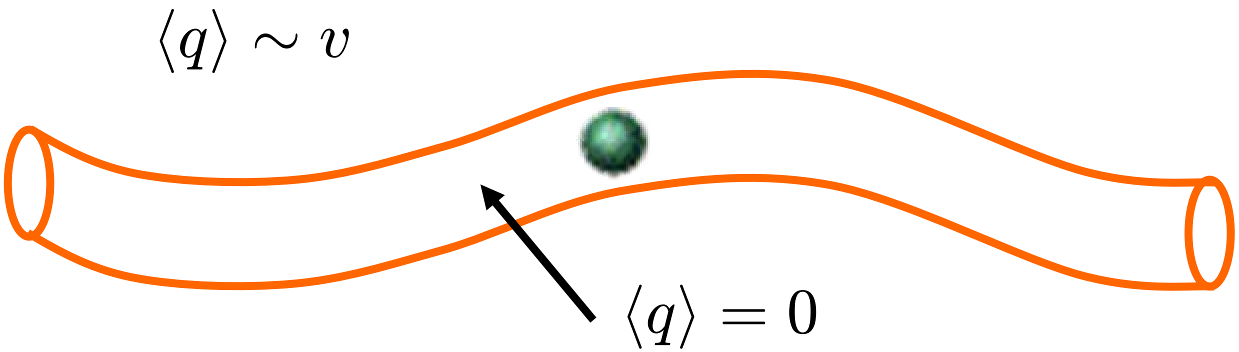
$\mathcal{D}_z q_i = 0$

Vortex Tension: $T = 2\pi k v^2$

Trapped Instantons

Hanany and Tong, '04

In the Higgs phase, the instanton can nestle inside the vortex string.



The configuration is ¼-BPS

$$F_{12} - F_{34} = \frac{e^2}{2}\left(\sum_i q_i q_i^\dagger - v^2\right)$$

$$F_{14} = F_{23} \qquad F_{13} = F_{24}$$

$$\mathcal{D}_z q_i = 0 \qquad \mathcal{D}_{\bar{w}} q_i = 0$$

Adding Masses

$$S = \int d^4x \; \frac{1}{2e^2}\,\mathrm{Tr}\, F_{\mu\nu}F^{\mu\nu} + \frac{1}{e^2}(\mathcal{D}_\mu\phi)^2 + \sum_{i=1}^{N_f} |\mathcal{D}_\mu q_i|^2$$

$$- \sum_{i=1}^{N_f} q_i^\dagger(\phi - m_i)^2 q_i - \frac{e^2}{2}\,\mathrm{Tr}\,(\sum_{i=1}^{N_f} q_i q_i^\dagger - v^2)^2$$

add masses

Vacuum: $\langle\phi\rangle = \mathrm{diag}(m_{i_1}, m_{i_2}, \ldots, m_{i_{N_c}})$

and $\langle q\rangle \sim v$

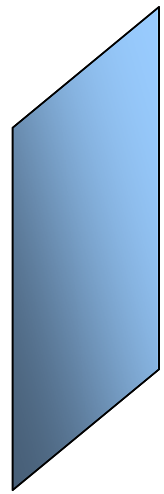
$\Longrightarrow \begin{pmatrix} N_f \\ N_c \end{pmatrix}$ isolated vacua

The theory now has a mass gap for all N_f

Domain Walls

Abraham and Townsend '91

$$\vec{\phi} = \vec{m}$$



$$\vec{\phi} = \vec{m}'$$

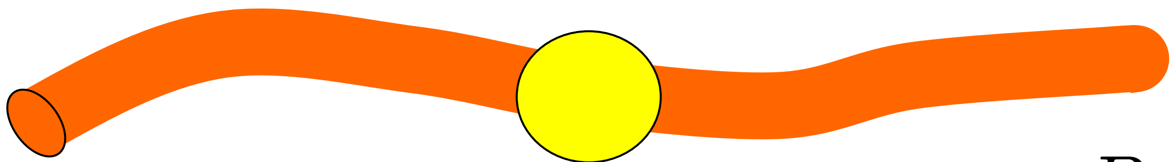
Domain Wall Equations:
$$\mathcal{D}_3\phi = \frac{e^2}{2}\left(\sum_i q_i q_i^\dagger - v^2\right)$$
$$\mathcal{D}_3 q_i = (\phi - m_i) q_i$$

Domain Wall Tension:
$$T = v^2 \Delta\vec{\phi} \cdot \vec{g}$$

Confined Monopoles

Tong '03

In the Higgs phase, with masses, the monopole is confined.



$$B \sim \begin{pmatrix} \mathbf{1} \\ \mathbf{0} \end{pmatrix} \qquad\qquad B \sim \begin{pmatrix} \mathbf{0} \\ \mathbf{1} \end{pmatrix}$$

The configuration is again ¼-BPS:

$$B_1 = \mathcal{D}_1 \phi \qquad B_2 = \mathcal{D}_2 \phi$$

$$B_3 = \mathcal{D}_3 \phi + \frac{e^2}{2}\left(\sum_i q_i q_i^\dagger - v^2\right)$$

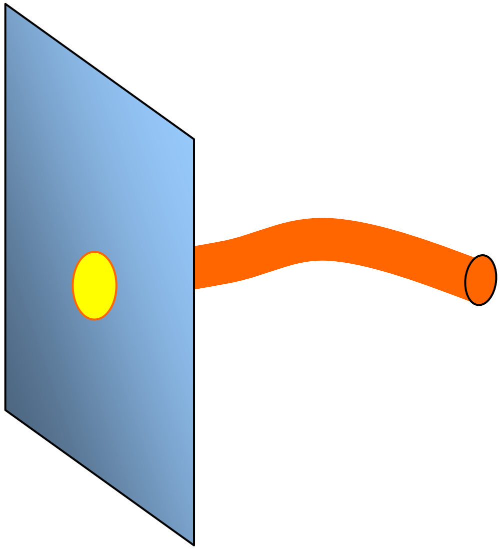
$$\mathcal{D}_1 q_i = i \mathcal{D}_2 q_i \qquad \mathcal{D}_3 q_i = -(\phi - m_i) q_i$$

D-Branes

Gauntlett, Portugues, Tong and Townsend, '00

Shifman and Yung,'03

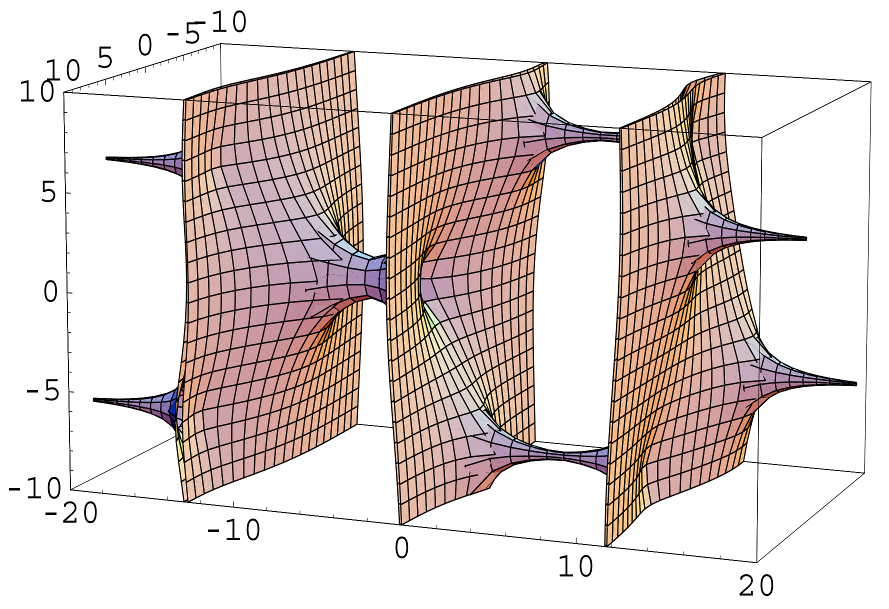
These same equations, with different boundary conditions, also have solutions describing vortex strings ending on domain walls.



Boojum: where the vortex intersects the wall, there exists a negative mass, half-monopole. Sakai and Tong '05

D-Branes

- There exists an analytic solution for a single string ending on a domain wall (in the $e^2 \to \infty$ limit). Other numerical solutions have been found.



Isozumi, Nitta, Ohashi and Sakai '04

Summary of Classical BPS Solitons

- Pure Yang-Mills
  - Instanton
- Yang-Mills + Adjoint Scalar
  - Monopole
- Yang-Mills + Massless Fundamental Scalars
  - Vortex String
  - Trapped Instanton
- Yang-Mills + Adjoint Scalar + Massive Fund. Scalars
  - Domain Wall
  - Confined Monopole
  - D-Brane

+ Others…

Eto, Nitta, Ohashi, Tong '05
Sakai et al; K. Lee et al

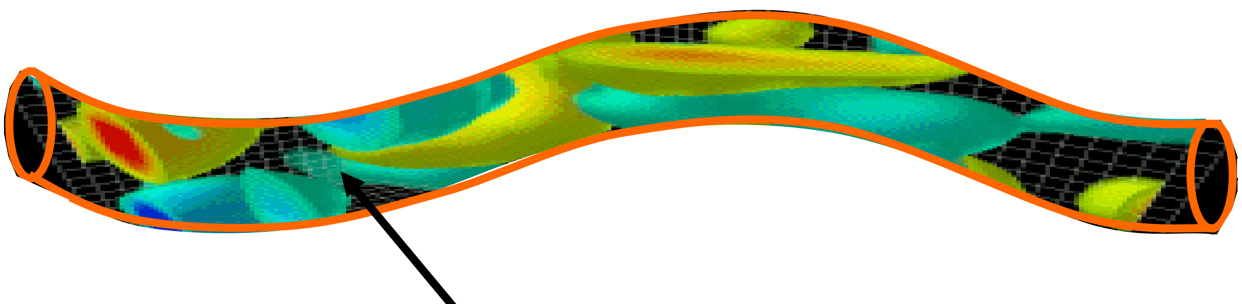
Part II

- The Dynamics of Vortex Strings

Dynamics of Vortex Strings

- Basic Idea:
  - The interior of the string sits in the non-abelian phase. This can be strongly coupled, even when the Higgs phase is weakly coupled.

$\langle q \rangle \neq 0$ The Higgs phase



$\langle q \rangle = 0$ The unbroken phase

  - The vortex string should know something about this strongly coupled phase.

Graphics by Leinweber

Dynamics of Vortex Strings

- Recipe:

$$\boxed{\text{Vortex string: 4d Gauge Theory} \rightarrow \text{2d Sigma Model}}$$

  - Pick your favourite d=3+1 U(N) gauge theory with fundamental matter
  - Push it into the Higgs phase where it admits vortex strings
  - Determine the d=1+1 theory on the string.

- We will describe:
  - Classical String Dynamics
    - N=(2,2) dynamics for vortices in N=2 theories
    - N=(0,2) dynamics for vortices in N=1 theories
  - Quantum String Dynamics
    - N=(2,2) dynamics for vortices in N=2 theories

Simplest Example

Hanany and Tong '03
Auzzi et al. '03

- $\mathcal{N}=2\ \ U(N_c)$ gauge theory with $N_f = N_c$ flavors
- Embed Abelian Vortex in U(N) gauge group in different ways gives orientational modes for vortex string
- Low-Energy dynamics is d=1+1 $\mathcal{N}=(2,2)\ \mathbf{CP}^{N_c-1}$ sigma-model, with Kahler class

$$\boxed{r = \frac{2\pi}{e^2}}$$

- In terms of a gauged linear sigma-model, the worldsheet dynamics is a U(1) gauge theory with N_c chiral multiplets of charge +1

Deformations Preserving N=2

Hanany and Tong, '03,'04
Shifman and Yung, '04

- We can add terms to 4d which preserve N=2 susy
- This is reflected in the worldsheet dynamics

<u>4d Deformation</u>		<u>Worldsheet Deformation</u>
$N_f > N_c$ fund. flavors	⟺	U(1) with N_c chirals, charge +1 $N_f - N_c$ chirals, charge -1
Masses *m* for hypermultiplets	⟺	Twisted masses *m* for chiral multiplets

Deformations to N=1

D.Tong and M. Edalati
To Appear

- We can also add deformations which break the 4d theory to N=1

- In general, the vortices are non-BPS. But the worldsheet theory has *spontaneously broken* N=(0,2) susy with

<u>4d Deformation</u> | <u>Worldsheet Deformation</u>

"Dijkgraaf-Vafa" type superpotential

$$\mathcal{W}(\Phi) \quad \Longleftrightarrow \quad \Xi\,\frac{\partial \mathcal{W}(\Sigma)}{\partial \Sigma}$$

N=(0,2) Superpotential

Change hypermultiplet coupling

$$\sqrt{2}\lambda\,\tilde{Q}_i\Phi Q_i \quad \Longleftrightarrow \quad \bar{D}_+\Gamma_i = \sqrt{2}\lambda\Sigma\Phi_i$$

N=(2,2) Goldstino

N=(2,2) σ field

Change Chiral Couplings

Quantum Vortex String

Dorey '98
Dorey, Hollowood, Tong '99
Hanany and Tong '04
Shifman and Yung '04

- Lets return to the N=2 4d theory, with N=(2,2) worldsheet theory.
- The worldsheet theory captures the 4d Seiberg-Witten solution. In particular, the two theories have the same spectrum
  - 4d theory: N=2 U(N) with N_f flavors, hypermultiplet masses m_i in Coulomb branch vacuum $\phi = \mathrm{diag}(m_1, m_2, \ldots, m_N)$
  - 2d theory: N=(2,2) U(1) with N chiral multiplets, charge +1
    N_f –N chiral multiplets, charge -1
    with twisted masses m_i

Quantum Vortex String

- The first hint that the vortex string knows about the 4d quantum theory comes from the beta functions. The relationship

$$\boxed{r = \frac{2\pi}{e^2}}$$

is preserved under RG flow.

$$r(\mu) = r_0 - \frac{2N_c - N_f}{2\pi} \log\left(\frac{M_{UV}}{\mu}\right)$$

- The strong coupling scale Λ of the string worldsheet is the same as that of the unbroken 4d theory.

The Quantum Spectrum

- The elementary string excitations correspond to W-boson and quark fields. The masses in 4d and on the worldsheet coincide.

- Kinks on the worldsheet correspond to (confined) magnetic monopoles, whose mass coincides with that of free monopoles

$$M_{\mathrm{kink}} = r\Delta m = \frac{2\pi}{e^2}\langle\phi\rangle = M_{\mathrm{mono}}$$

- In the regime $m \gg \Lambda$ both the 4d theory and 2d theory are weakly coupled. Quantum corrections to the soliton mass have expansion

$$M = M_{\mathrm{classical}} + M_{1-\mathrm{loop}} + \sum_{n=1}^{\infty} M_{\mathrm{n-instanton}}$$

- You can compute these quantum corrections in 2d or in 4d, summing over 2d instantons or 4d instantons. They agree.

Part III

- Further Topics

Relationship Between Moduli Spaces

Hanany and Tong, ’03, ‘05

- Instantons vs Vortices
  - Hyperkahler Instanton Moduli Space, dim 4kN
  - Kahler Vortex Moduli Space, dim 2kN

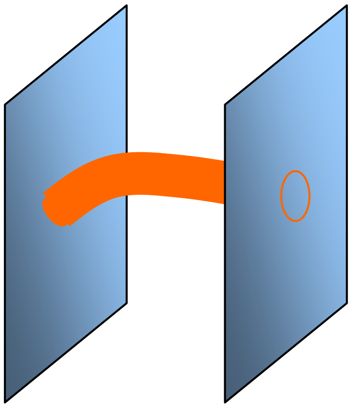
  - Vortex Moduli Space $\subset$ Instanton Moduli Space

- Monopoles vs Domain Walls
  - Hyperkahler Monopole Moduli Space
  - Kahler Domain Wall Moduli Space

  - Domain Wall moduli space $\subset$ Monopole Moduli Space

D-Branes in Field Theory

Tong '05



The domain walls are D-branes for the vortex string.

The classical scattering of two domain walls is described by a cigar-like moduli space

There also exists an open string description. The string between two walls gives rise to a chiral multiplet and associated Chern-Simons terms on the walls. The d=2+1 quantum dynamics reproduces the classical scattering of domain walls.

Summary

- New BPS classical solitons in Yang-Mills-Higgs theories.
- Quantum dynamics of vortex strings captures quantum dynamics of 4d N=2 theories.
  - Work in Progress: Extension to 4d N=1 theories, and the associated N=(0,2) worldsheet.
- Open string description of field theory soliton dynamics.