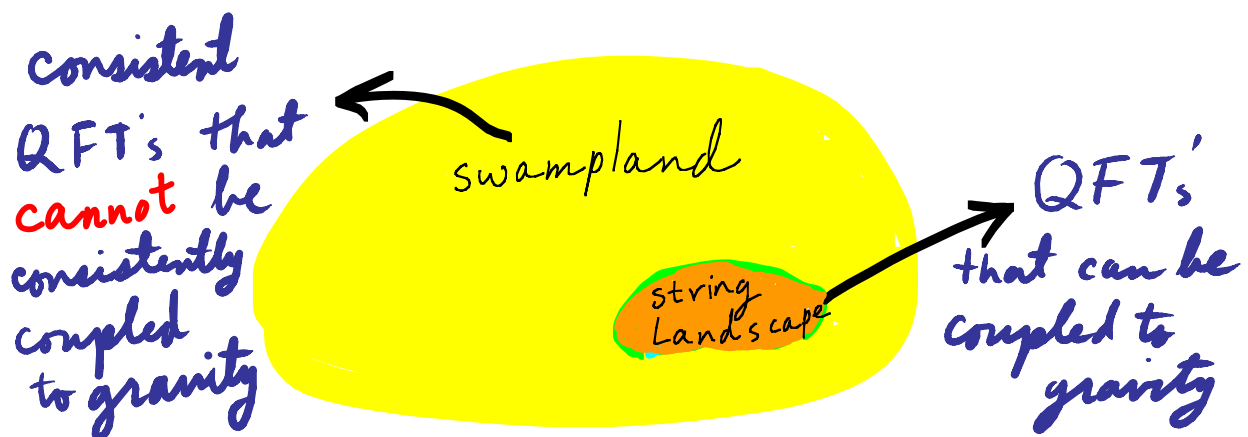


String Landscape + the Swampland



Suppose $\mathcal{L}_{\text{QFT}}$ is a consistent QFT.

Since string theory is the only consistent quantum theory of gravity known to date we will interpret this question as follows:



We conjecture a few criteria which distinguish the **String Landscape** from the **Swampland**.

We will not be able to prove these conjectures; we will find many non-trivial examples supporting these conjectures. These conjectures are about the space of flat directions $\mathcal{M}_X$ for compactification X , as well as the nature of gauge groups + gauge coupling constants + $\#$ of light fields:

The Conjectures

0 - $\mathcal{M}_X \longleftrightarrow \langle \phi \rangle$
 "No adjustable parameters"

1 - $\phi \in \mathcal{M}_X \longleftarrow$ "string landscape"

$$\mathcal{L} = \int \dots + |\nabla \phi|^2$$

$$\text{Vol}(\mathcal{M}_X) = \text{finite}$$

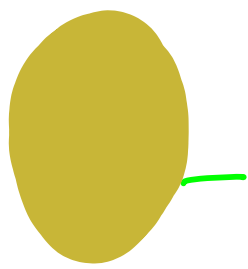
(more precisely $\mathcal{M}_X^\Lambda : (m \gg \Lambda)$)

2 - $\dots = \dots$

uniform bound
 (depending only on d)

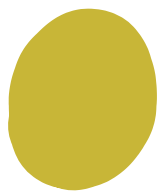
3 - Choices for gauge groups

$\dots$



Gauge coupling constants cannot

be too weak $\approx$
gravity is the **weakest** force



Diameter of
 $\text{diam}(\mathcal{M}_X) = \infty$

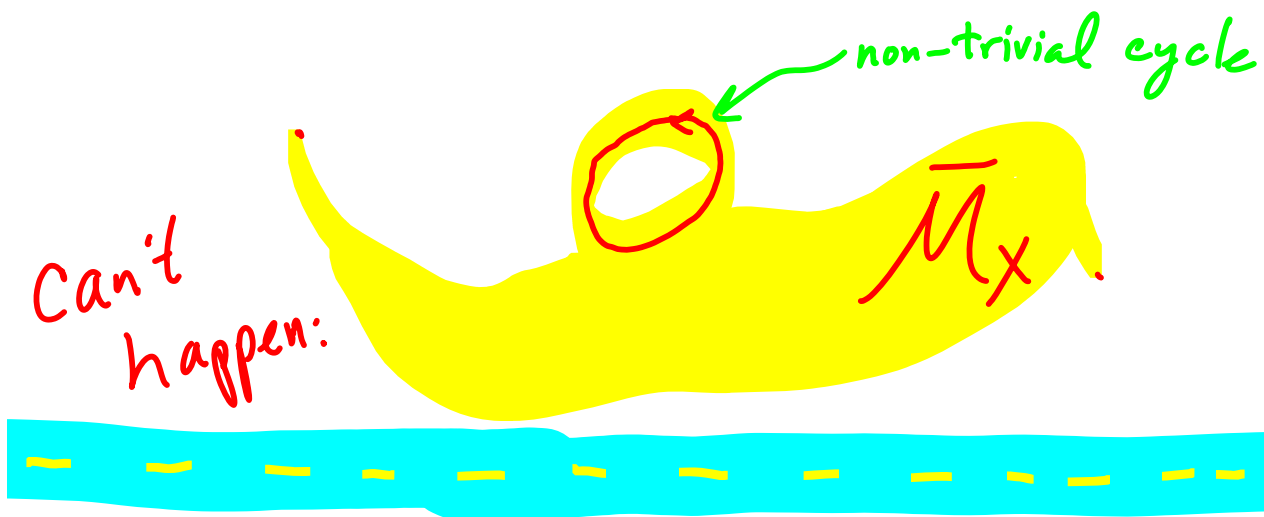
and pts at
 ∞ correspond to appearance
of a tower of light particles
with m :



$$m(d) \sim e^{-\propto d}$$

6- $R(d \rightarrow \infty) < 0$
(scalar curvature)

7- $\pi_1(\bar{M}_X) = \emptyset$



We provide evidence for these conjectures + show they can all be violated if we decouple gravity,
(for example by taking)

→ $\int$ $\cdot R$ → gravity decoupled

lower dim. $\parallel M_{pl}^{d-2} \rightarrow \infty$

Based on

Conjectures

hep-th/0509212

Title: **The String Landscape and the Swampland**

Authors: [Cumrun Vafa](#) </find/hep-th/1/au:+Vafa C

+

hep-th/0601001

Title: **The String Landscape, Black Holes and Gravity as the Weakest Force**

Authors: [Nima Arkani-Hamed](#) </find/hep-th/1/au:

+

hep-th/0605264

Title: **On the Geometry of the String Landscape and the Swampland**

Authors: [Hiroshi Ooguri](#) </find/hep-th/1/au:+Ooguri H/0/1/0/all/0/1>

(related work : Douglas et.al.
Kachru et.al.
Li et.al.
Motl et.al.)



$$- \mathcal{M} \leftrightarrow \langle \phi \rangle$$

is an obvious fact in string theory, but it did not have to be the case. Note that QFT's without gravity do have adjustable parameters.



Finiteness of Vol.

Examples of finiteness:

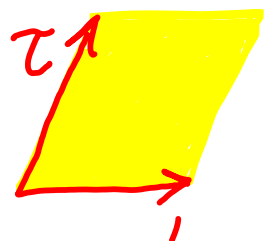
$$\text{Let } X = T^2 \times \dots$$

$$\text{Then } \phi \in \mathcal{M}_{T^2}^{\text{cplx}} \times \dots$$

$$\text{Vol}(\phi) = \int \frac{d^2 \tau}{\tau_2^2} < \infty$$



fundamental domain



Other examples

$$X = CY \text{ 3-fold}$$

$$\phi \in \mathcal{M}_{CY}^{cplx}$$

There are arguments (Todorov)
(Douglas et al.)

$$\text{Vol}(\mathcal{M}_{CY}^{cplx}) < \infty$$

More subtle applications: Kähler moduli

$$\text{Dualities} \left\{ \begin{array}{l} \text{T-duality} \\ \text{(mirror sym.)} \\ \text{S-duality} \end{array} \right\} \longleftrightarrow \begin{array}{l} \text{Vol}(M_x) \\ \text{finite} \end{array}$$

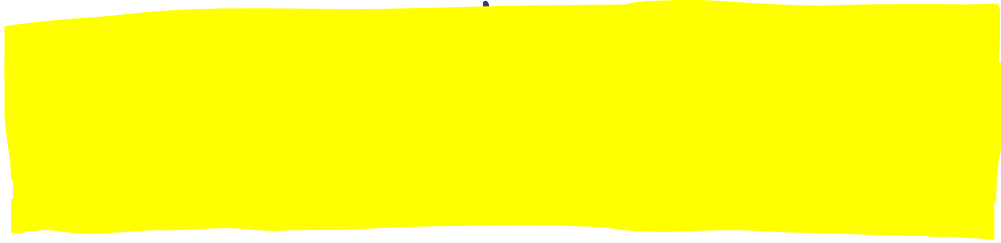
e.g. τ (coupling const) in Type IIB
($SL(2, \mathbb{Z}) \rightarrow \text{Vol}(\tau) = \text{finite}$)

This is a "gravity" effect.

If we decoupled "gravity" by

taking $X = \text{non-compact}$

then the volume is **not** finite,



(e.g. $\text{Vol}(\phi \in \text{QFT}) = \infty$)

$\phi \in \mathbb{R}^n$

(Note: A red arrow points from the ϕ in the second line to the ϕ in the first line.)

$$2 - (\dim M_X) = \text{bounded}$$

This is meant as a uniform bound. (fixed, for $\dim X = \text{fixed}$)

$$\text{Example: } X : \mathbb{C}Y^3 \\ (\dim_{\mathbb{C}} M_X) = h^{1,1} + h^{1,2}$$

The conjecture in this case suggests $h^{1,1}, h^{2,1}$ bounded.

(This has been conjectured by Douglas)

It is not known whether this mathematical conjecture is true or false. Currently there

is no counter example (i.e., no ∞ sequence of CY with increasing hodge $\#$'s).

Again this conjecture is true if we decouple gravity

The decoupling of gravity can be done by taking

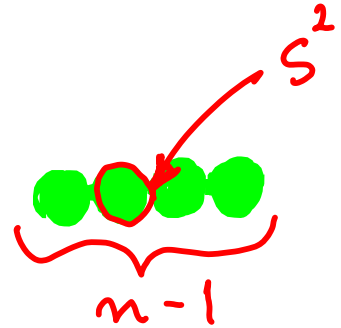
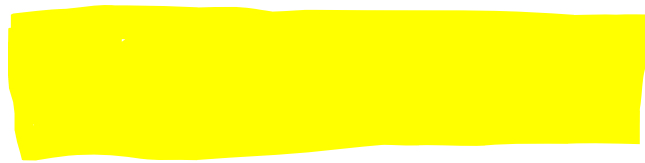
X : N m-compact

In this case $\exists$ counter-examples:

$$X = \mathbb{C}^2 / \mathbb{Z}_n$$

$$\begin{aligned} \mathbb{Z}_n : (\mathbb{Z}_1, \mathbb{Z}_2) &\rightarrow (\omega \mathbb{Z}_1, \omega^{-1} \mathbb{Z}_2) \\ \omega^n &= 1 \end{aligned}$$

This gives rise to A_{n-1} singularity
which can be resolved:



This finiteness of $\dim(M_x)$
has a counterpart conjecture
involving a  on
the $\dim(M_x)$ at least in
certain cases. For example if we
consider $N=2$ supersymmetric
compactifications to $d=4$, we always
seem to end up with 
scalar field. (CY case: $h_2 \geq 1$).

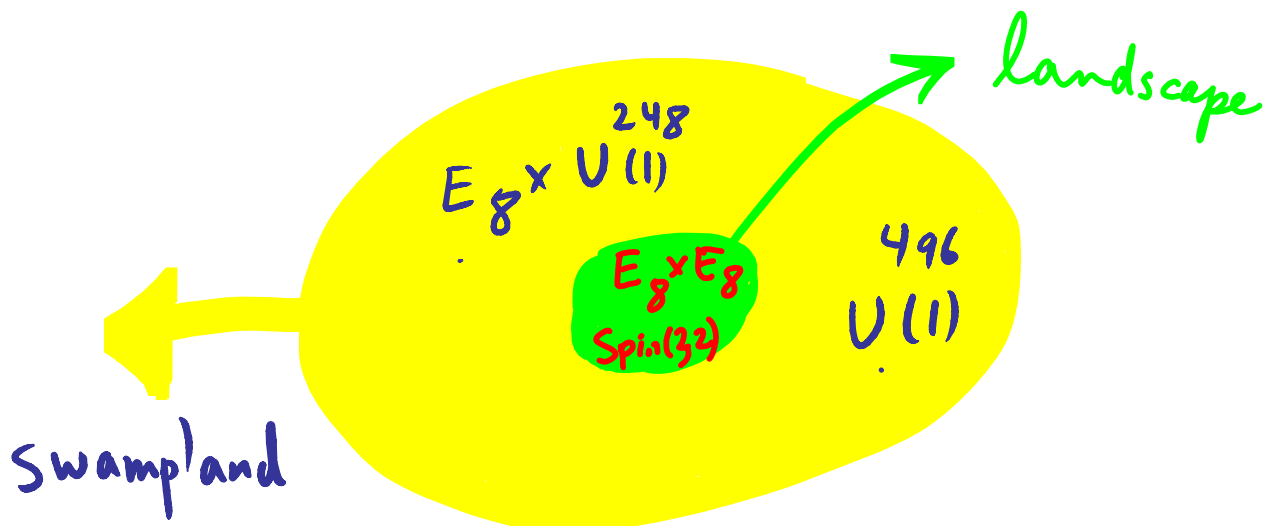


Conjecture $2 \Rightarrow \dim G < \infty$

But it seems there are more restrictions on G .

Examples:

$$\begin{array}{ll} d=10 \Rightarrow & G = E_8 \times E_8 \\ \mathcal{N}=1 \text{ susy} & \text{or} \\ & G = \text{Spin}(32)/\mathbb{Z}_2 \end{array}$$



In other words, even though $E_8 \times U(1)^{248}$ and $U(1)^{496}$ are semiclassically consistent choices for gauge group G , but they are not realized in string theory (nor in its non-perturbative completions, M - or F -Theories)

We conjecture that these choices are actually inconsistent as a full quantum theory.

There are similar restrictions in other dimensions. For example

$$d=4 \quad \text{locally} \quad \mathbb{C}^2 / \mathbb{Z}_n \Rightarrow G = U(n)$$

but in the context of a compact

$$\text{theory } \mathbb{C}^2 / \mathbb{Z}_n \subset K3$$

$$\Rightarrow \text{rank}(G) = 20$$

In addition

not all rank=20 gauge groups are realized.

$$(G \text{ realized} \Leftrightarrow \text{root lattice} \subset \text{self-dual even integral lattice})$$

Again these restrictions on G seem correlated with having a dynamical gravity, in particular for X : non-compact no restrictions known.

(Example: $\mathbb{C}^2 / \mathbb{Z}_n \rightarrow U(n)$
 $\quad \quad \quad \mathbb{A}^1_n$)

Strength of
gauge coupling const.

Consider a $U(1)$ gauge theory with coupling constant g

Claim: $\exists$ light charged particle

$$m \lesssim g M_{\text{Planck}}$$

$$\rightarrow g \gtrsim \frac{m}{M_{\text{Pl}}}$$

+

Checks

1) Heterotic strings:

$$M_s \sim g M_{\text{Planck}}$$

$\exists$ light states, **strings**,
and $\Lambda > M_s$ effective
gauge theory breaks down

2) KK reductions

$$g M_{\text{Pl}} \sim \frac{1}{R};$$

$\Lambda > \frac{1}{R}$ effective gauge theory breaks
down

Clearly the condition is
vacuous when $M_{Pl} \rightarrow \infty$

$$(g \geq \frac{m}{M_{Pl}} \sim 0)$$

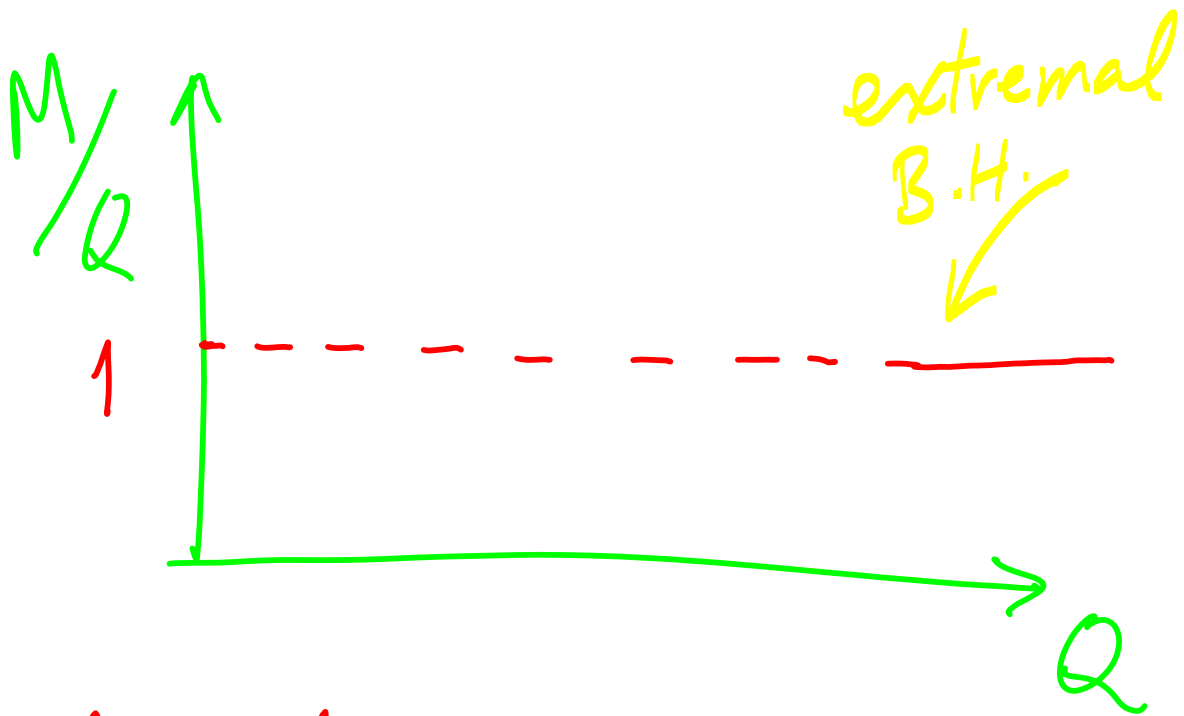
One way to motivate
this conjecture:

(Unless protected by some
charge):

~~✗~~ Stable particles $< \infty$

$$\left(\frac{M}{Q} \right)_{\min} < 1$$

Extremal Black holes

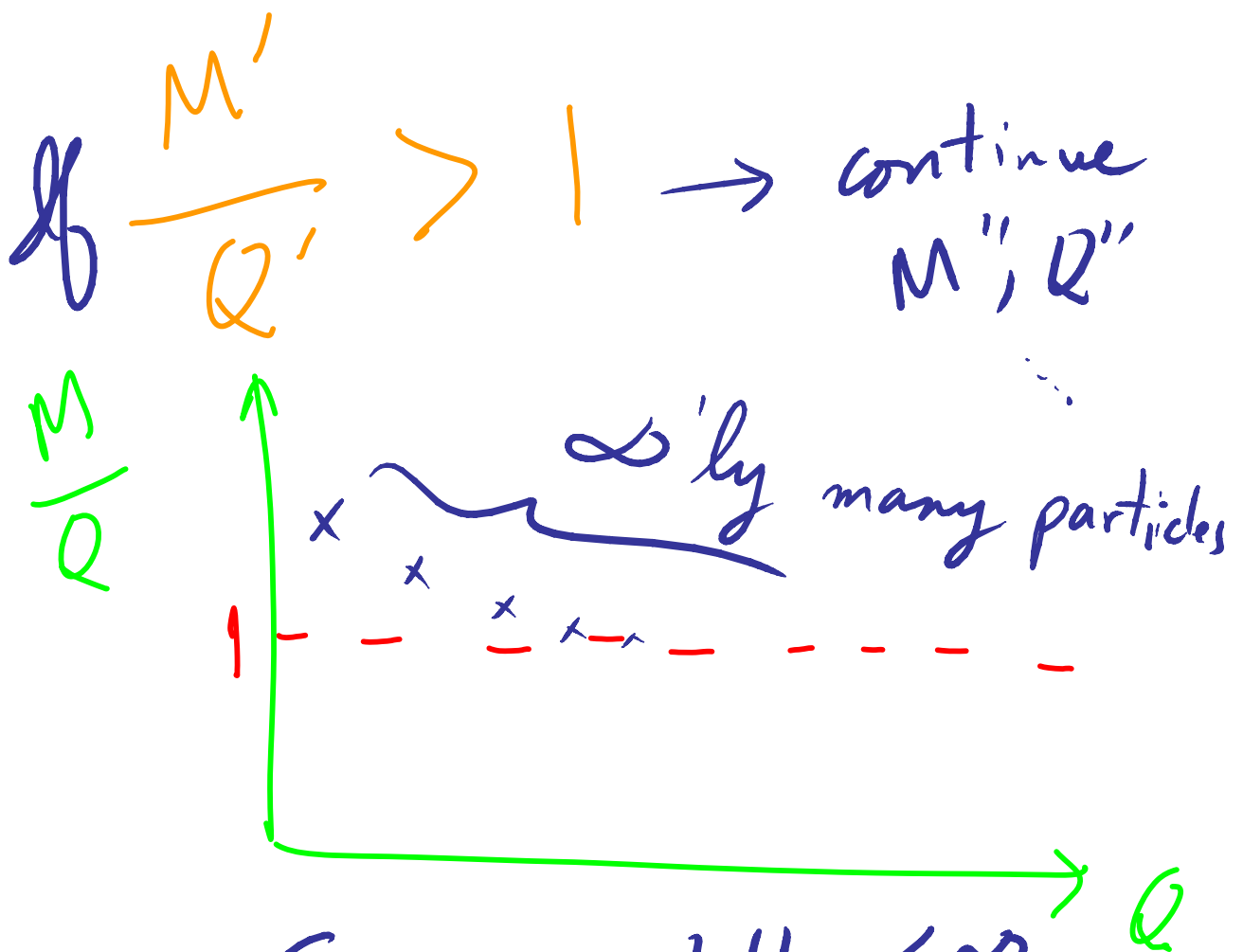


$$\text{If } \frac{M}{Q} > 1 \Rightarrow M > Q$$

$$\left(\begin{array}{c} \leftarrow \bullet \rightarrow \\ M \end{array} \quad \begin{array}{c} \leftarrow \bullet \rightarrow \\ M \end{array} \right) = \text{attractive force}$$

$\Rightarrow$ bound state

$$M' < 2M \quad Q' = 2Q$$



So $\nexists$ stable $< \infty$



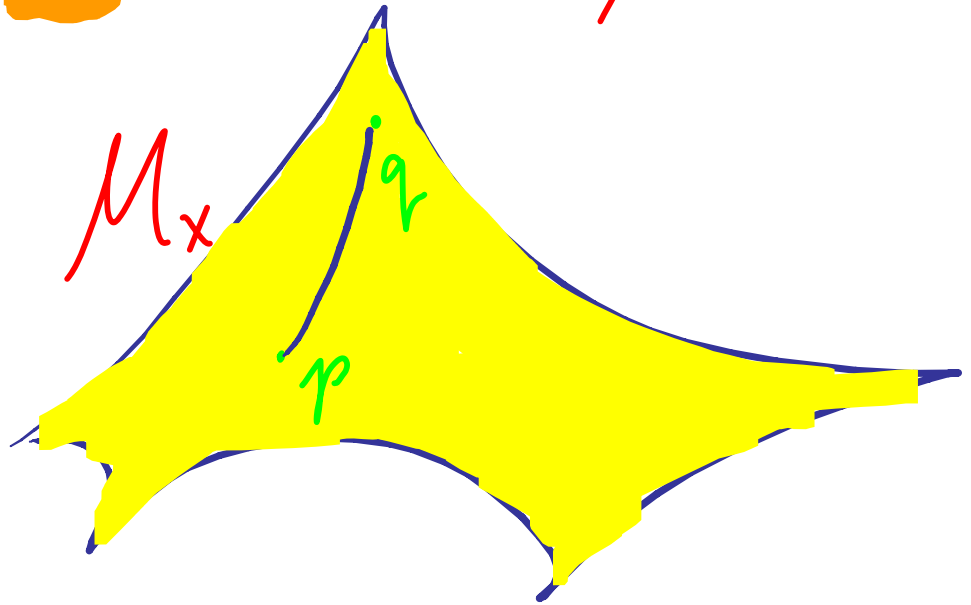
$$\exists \left(\frac{M}{Q} < 1 \right) \rightarrow$$

$$(M/Q)_{\min} < 1$$

Non-trivial example:

Non-susy BH in heterotic string compactifications.

 $\text{Diam.}(M_x) = \infty$

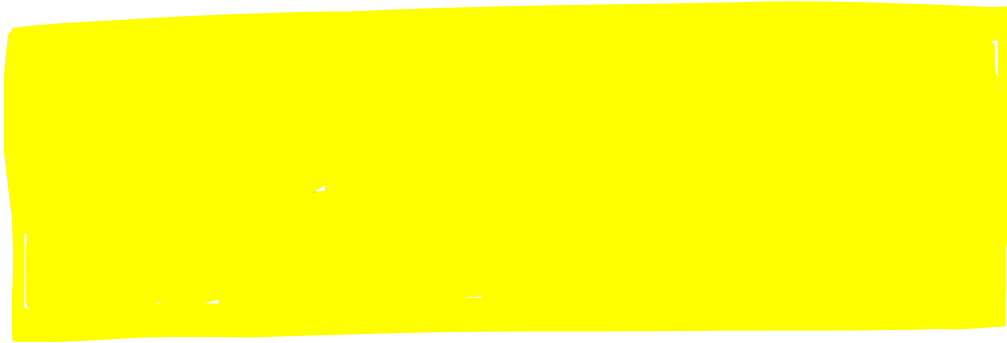


$$\text{Diam} \equiv \max_{p, q \in M_x} [d(p, q)]$$

$\text{Diam} = \infty$;

fix p ; look at all q
there are points at ∞ ;

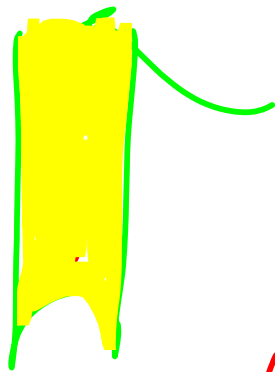
Moreover pts. at ∞ correspond to the regions with an ∞ tower of light particles :



as $d_{(P,q)} \rightarrow \infty$, for some α .

Examples: T^2 compactification

τ :

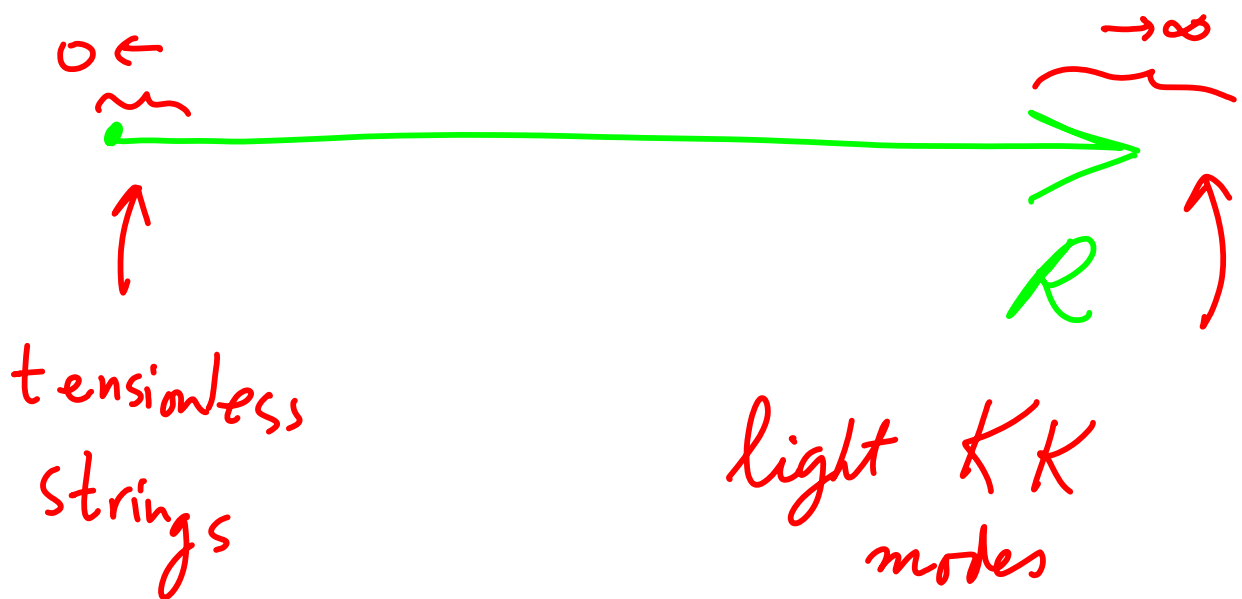


$$d = \int_{-\frac{1}{\Lambda}}^{\frac{1}{\Lambda}} d\tau_2 \sim \ln \frac{1}{\Lambda}$$

$$\Lambda \sim e^{-d}$$

Another example :

M-theory on S^1_R :



Counter examples if we decouple gravity:

$$\text{diam}(M_x) \neq \infty$$

$$M_x = G/H$$

$$\text{e.g. } G = SU(2) ; H = U(1)$$

Also in field theory
even if $\text{diam}(\mathcal{M}_x) = \infty$
 $\nRightarrow$ extra light modes:

e.g. $N=4$ YM
 $G=SU(2)$

$$\mathcal{M}_x = \mathbb{R}^6 / \mathbb{Z}_2 \ni \phi^i$$

$p \in \mathcal{M}_x$ at ∞ correspond
to $W^\pm$ infinitely massive,

no extra light particles appear.

$$\boxed{6} - R(d=\infty) < 0.$$

Note : $d = \infty$ } dash
 $Vol = \text{finite}$ }

$\rightarrow R|_{\infty} < 0$ is

almost a requirement.

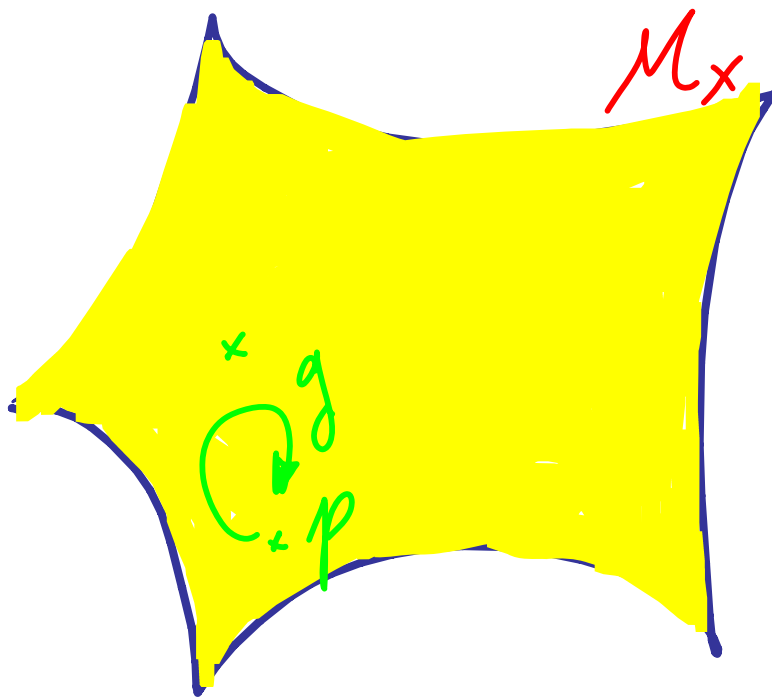
Example : $G(Z) \setminus G/H$

that appear in string theory
 have $R < 0$.

Condition violated for $N=2$
 for field theory: gauge theories
 w/o gravity $R > 0$

$$\square - \pi_1(\mathcal{M}_x) = \phi$$

Intuition: Duality group
is generated by gauge symmetry.

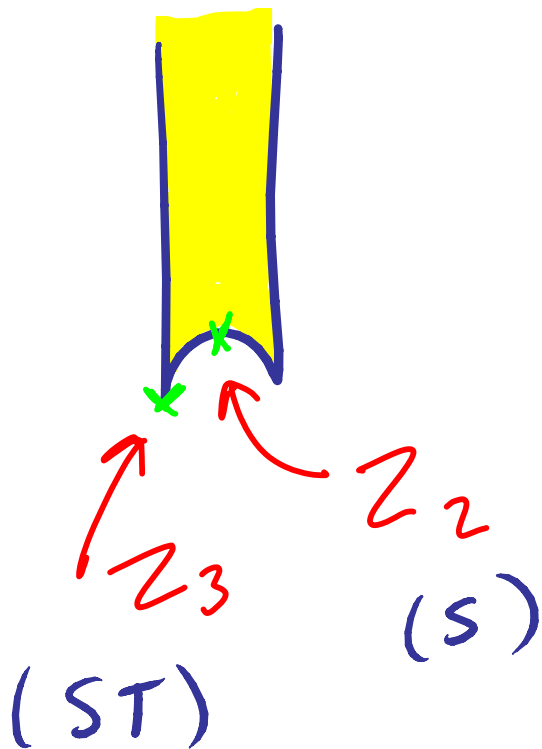


$$\mathcal{M}_x = T_x / \Gamma \quad \swarrow \text{duality group}$$

$p \in T_x$ fixed by $g \in \Gamma$

$\Rightarrow g \in \text{gauge group at } p$

Example:



$SL(2, \mathbb{Z})$
generated by
 S, ST

If Γ generated by elements
with fixed pts $\Rightarrow \pi_1(\mathcal{M}_x) = \emptyset$

(Another intuition: compactification

to $1+1 \Rightarrow$ global charge

$\Rightarrow$ not allowed in a
quantum gravity)

Again, counterexample
can be constructed if
gravity is decoupled:

eg Axion $\pi_1(\text{Axion}) = \mathbb{Z}$

(Suggests axion is always
accompanied by an extra
field when coupled to gravity)

Conclusion

Dynamical gravity puts restrictions on allowed theories

Mathematically

X compact vs. non-compact

↓
more restrictive

In particular:

$$\text{Vol}(X) < \infty \Rightarrow \text{Vol}(M_x) < \infty$$

Also: $\dim(M_x) < \infty$
(uniform)

$X = \text{singular} \Rightarrow G$:
restrictive

We have also seen
finiteness have other
consequences:

$$\left(\frac{M}{Q} \right)_{\min} < 1$$

(gravity is the weakest force)

$\Rightarrow \exists$ scale

$$\Lambda \sim g M_{\text{pl}}$$

$$E > \Lambda$$

effective field theory
breaks down

$$- \text{diam.} (\mathcal{M}_x) = \infty$$

$$(\text{points at } \infty) \Rightarrow (\text{Tower of } m \rightarrow 0)$$

$$- R \Big|_{d \rightarrow \infty} < 0$$

$$- \pi_1 (\mathcal{M}_x) = \emptyset$$

It would be important to:

- Try to find counterexamples
- or Try to understand why they are true.
- Broaden the list of conjectures.