D-brane Instantons in Supersymmetric 4D String Vacua

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Motivation

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Program: **Systematic investigation of string instanton effects** for various classes of $\mathcal{N} = 1$ string vacua
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Program: Systematic investigation of string instanton effects for various classes of $\mathcal{N} = 1$ string vacua

(Most of the work so far was for world-sheet instantons in Type II and heterotic string theory and for M-brane instantons)

(Dine, Seiberg, Wen, Witten), (Becker$^2$, Strominger), (Harvey, Moore), (Witten), (Green, Gutperle), (Antoniadis, Gava, Narain, Taylor), (Rocek, Saueressig, Theis, Vandoren), (Berglund, Mayr), (Kashani-Poor, Tomasiello), (Tsimpis), (Halmagyi, Melnikov, Sethi), (Grimm) ...
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Strings 2007, 25.06.2007 – p.3/27
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• Stringy derivation of field theory instanton effects
Reminder: GS mechanism

Gauge group $Y^a U(N^a) = Y^a SU(N^a) U(1)^a$ in general contains anomalous $U(1)^a$ symmetries. Anomaly cancellation via the 4D Green-Schwarz mechanism yields anomalous $U(1)^a$s become massive and survive as global perturbative symmetries. Only specific linear combinations of $U(1)^a$s are massless and remain as unbroken gauge symmetry (like $U(1)^a Y$).

Global $U(1)^a$ forbid some desirable matter couplings, e.g. Majorana type neutrino masses, $SU(5)$ Yukawa couplings or $\text{--}^2$-terms.

Relation to M-theory on $G_2$ manifolds (?).
Reminder: GS mechanism

Gauge group

\[
\prod_{a} U(N_a) = \prod_{a} SU(N_a) \times U(1)_a
\]

in general contains **anomalous** \(U(1)_a\) symmetries
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- Anomalous \( U(1) \)s become massive and survive as global perturbative symmetries
- Only specific linear combinations of \( U(1) \)s are massless and remain as unbroken gauge symmetry (like \( U(1)_Y \))
- Global \( U(1) \) forbid some desirable matter couplings, e.g. Majorana type neutrino masses, \( SU(5) \) Yukawa couplings or \( \mu \)-terms \( \rightarrow \) relation to M-theory on \( G_2 \) manifolds(?)
Instanton corrections

Instanton corrections in string theory can break the axionic shift symmetries and therefore the global U(1) symmetries. (Bl, Cvetic, Weigand, hep-th/0609191), (Ibanez, Uranga, hep-th/0609213)

Consider: D2-brane (E2) instantons in Type IIA wrapping a sLag three-cycle on Calabi-Yau.

From E2-E2 open strings:

Generic 4 bosonic zero modes $X_i$ and 4 fermionic zero modes $\gamma_i$ and $\bar{\gamma}_i$.

Due to deformations, $b_1$ complex bosonic zero modes $Y_i$ and fermionic zero modes $\gamma_i$ and $\bar{\gamma}_i$. 
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Consider: D2-brane (E2) instantons in Type IIA wrapping a sLag three-cycle $\Xi$ on Calabi-Yau.
From E2-E2 open strings:

- Generic 4 bosonic zero modes $X_\mu$ and 4 fermionic zero modes $\theta^\alpha$ and $\bar{\theta}^{\dot{\alpha}}$
- Due to deformations, $b_1(\Xi)$ complex bosonic zero modes $Y_i$ and fermionic zero modes $\mu_i^\alpha$ and $\bar{\mu}_{i\dot{\alpha}}$
F-terms via E2-Instantons

The two zero modes are projected out by the E2 must be invariant under and must be an $O(1)$ instanton (instead of $SP(2)$ or $U(1)$) (Argurio, Bertolini, Ferreti, Lerda, Petersson), (Ibanez, Schellekens, Uranga), (Bianchi, Fucito, Morales). The two zero modes can be absorbed elsewhere, like for instantons on top of D6-brane: $x x x x \theta \beta (3/8, -3/2) \beta (3/8, 3/2) (0, 0) D6 E2 E2$!
F-terms possible only if

- The two $\theta^{\dot{\alpha}}$ zero modes are projected out by $\Omega \overline{\sigma}$. For this the E2 must be invariant under $\overline{\sigma}$ and must be an $O(1)$ instanton (instead of $SP(2)$ or $U(1)$) (Argurio, Bertolini, Ferreti, Lerda, Petersson), (Ibanez, Schellekens, Uranga), (Bianchi, Fucito, Morales)
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- The two $\theta^\alpha$ zero modes can be absorbed elsewhere, like for instantons on top of D6-brane:

\[
\begin{array}{c}
\theta^{(3/8,-3/2)} \\
\downarrow
\end{array}
\]

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E2 \\
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b_{(0,0)} \\
\end{array}
\]

$\rightarrow$ fermionic ADHM-constraints (Billo et al., hep-th/0211250),
Instanton Recombination and Fluxes

After recombination the resulting object does not have zero modes, but additional fermionic zero modes appear spoiling the generation of an F-term.

\[
\begin{align*}
E_2 \cdot E_2 &= 0 \\
E_2 \cdot E_2 &= 1
\end{align*}
\]

are soaked up and \( m \); zero modes survive (deformations of the instantons)!

generation of Beasley/Witten type multi-fermion couplings (Beasley, Witten)

Fluxes are known to lift \( E_3 \)-instanton zero modes (Witten), (Tripathy, Trivedi), (Bergsho et al.), (Lust et al.)

In Type IIB \( I_6 \) \( F \)-orientifolds a primitive \( G_{2,1} \) flux does not lift the zero modes of an \( U(1) \) instanton
Instanton Recombination and Fluxes

preliminary results of (Bl, Cvetic, Richter, Weigand, to appear)

E2-E2’ instanton recombination:

\[ \text{After recombination the resulting object does not have zero modes, but additional fermionic zero modes appear spoiling the generation of an F-term.} \]

\[ \text{After recombination are soaked up and new zero modes survive (deformations of the instantons) generation of Beasley/Witten type multi-fermion couplings (Beasley, Witten).} \]

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E2-E2′ instanton recombination:

- $E^2 \circ E^2′ \neq 0$: After recombination the resulting object does not have $\bar{\theta}$ zero modes, but additional fermionic zero modes appear spoiling the generation of an F-term.
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- $[E2 \cap E2']^\pm = 1$: After recombination $\bar{\theta}$ are soaked up and $m, \bar{\mu}_{\bar{\alpha}}$ zero modes survive (deformations of the instantons) → generation of Beasley/Witten type multi-fermion couplings (Beasley, Witten)
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Type IIA Space-time Instantons

Instanton action:

\[ W_{np} = S_E^2 = \exp \left( \frac{2}{\beta} \right) \]

is not gauge invariant under \( U(1) \).

Indeed \( S_E^2 \) is not equal to \( e^{i Q_a (E^2)} \),

where \( Q_a (E^2) = N_a - (a_0 a_0) \).
Type IIA Space-time Instantons

Instanton action:

\[ W_{np} \propto e^{-S_{E2}} = \exp \left[ -\frac{2\pi}{\ell_s^3} \left( \frac{1}{g_s} \int_\Xi \Re(\Omega_3) - i \int_\Xi C_3 \right) \right] \]

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Indeed

\[ e^{-S_{E2}} \rightarrow e^{i Q_a(E2) \Lambda_a} e^{-S_{E2}}, \]

where

\[ Q_a(E2) = N_a \mathcal{X} \circ (\Pi_a - \Pi'_a). \]
Type IIA Space-time Instantons

Consequence: If

\[ Q^a \left( E^2 \right)^6 = 0 \]

for some \( a \), no terms

\[ W = e^{S_2} \]

possible but:

\[ W = Y_i e^{S_2} \]

with

\[ X_i Q^a (i) + Q^a (E^2) = 0 \]

i.e. non-perturbative breakdown of global

\( U(1) \)

symmetries.

see also e.g.: (Achucarro, Carlos, Casas, Doplicher, hep-th/0601190), (Haack, Kre, Lust, Van Proeyen, Zagermann, hep-th/0609211)

How can we understand this selection rule in terms of fermionic zero modes?
Type IIA Space-time Instantons

Consequence: If $Q_a(E2) \neq 0$ for some $a$, no terms $W = e^{-S_{E2}}$ possible but:

$$W = \prod_i \Phi_i e^{-S_{E2}} \quad \text{with} \quad \sum_i Q_a(\Phi_i) + Q_a(E2) = 0 \quad \forall a$$

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Instanton zero modes

...
**Instanton zero modes**

Additional Zero modes charged under $U(1)_a$:

Strings between $E2$ and $D6_a$ have DN-boundary conditions in 4D and mixed boundary conditions along $CY_3 \rightarrow 1/2$ complex fermionic zero mode $\lambda_a$ (Ganor, hep-th/9612077)

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Total $U(1)_a$ charge of all zero modes:

$$Q_a(E2) = N_a \Xi \circ (\Pi_a - \Pi'_a).$$
Instanton calculus
E2-instantons are described by open strings → computation of stringy instanton correlation functions should be possible in (boundary) conformal field theory. (Gutperle, Green, hep-th/9701093), (Billo et al., hep-th/0211250)
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As a first step we would like to compute (rigid) E2-contributions to the charged matter field superpotential

$$W_{np} \simeq \prod_{i=1}^{M} \Phi_{a_i,b_i} e^{-S_{E2}}.$$ 

with $\Phi_{a_i,b_i} = \phi_{a_i,b_i} + \theta \psi_{a_i,b_i}$ denoting chiral matter superfields at the intersection of $\Pi_{a_i}$ with $\Pi_{b_i}$ (suppress Chan-Paton labels for simplicity).
Instanton calculus: Summary
Probe superpotential by correlator

\[
\langle \Phi_{a_1,b_1} \cdots \Phi_{a_M,b_M} \rangle_{E2-\text{inst}} = \frac{e^{\frac{\kappa}{2}} Y_{\Phi_{a_1,b_1}, \ldots, \Phi_{a_M,b_M}}}{\sqrt{K_{a_1,b_1} \cdots K_{a_M,b_M}}}
\]

\[
\langle \Phi_{a_1,b_1}(x_1) \cdots \Phi_{a_M,b_M}(x_M) \rangle_{E2-\text{inst}} = \\
= \int d^4x \ d^2\theta \ \sum_{\text{conf.}} \Pi_a \left( \prod_{i=1}^{[\Xi \cap \Pi_a]^+} d\lambda_a^i \right) \left( \prod_{i=1}^{[\Xi \cap \Pi_a]^+} d\bar{\lambda}_a^i \right) \exp(-S_{E2}) \times \exp(Z_0') \\
\times \langle \Phi_{a_1,b_1}[\vec{x}_1] \rangle^{\text{tree}}_{\lambda_{a_1},\bar{\lambda}_{b_1}} \cdots \langle \Phi_{a_L,b_L}[\vec{x}_L] \rangle^{\text{tree}}_{\lambda_{a_L},\bar{\lambda}_{b_L}} \times \\
\prod_k \langle \Phi_{c_k,c_k}[\vec{x}_k] \rangle^{\text{loop}}_{A(E2,D6_{c_k})}
\]
Recall: loop-amplitudes (no $a$-insertion).

Factor out vacuum loops involving at least one $E_2$ boundary:

$$Z_{A}(E_2;D_6; a) = \int_0^{\Lambda_0^2} \, t \, Tr \, E_2; D_6; a \, e^{\frac{2}{\alpha} t L_0} D_6 = 0$$

and likewise

$$Z_{M}(E_2;O_6) = 0$$

but

$$Z_{A}(E_2;E_2) = 0$$

(due to bose-fermi deg.).

Therefore

$$\exp \left( Z_0 \right) = \exp \left( \sum_{a} Z_{A}(E_2;D_6; a) + Z_{M}(E_2;O_6) \right) !$$

One-loop determinants!
Instanton calculus: 1-loop

Recall: loop-amplitudes uncharged (no $\lambda_\alpha$-insertion)
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- Factor off vacuum loops involving at least one $E2$ boundary:

\[
Z^A(E2, D6_a) = c \int_0^{\infty} \frac{dt}{t} \text{Tr}_{E2,D6_a} \left( e^{-2\pi t L_0} \right) \neq 0
\]

and likewise $Z^M(E2, O6) \neq 0$ but $Z^A(E2, E2) = 0$ (due to bose-fermi deg.).
Recall: loop-amplitudes uncharged (no $\lambda_a$-insertion)

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Therefore

$$\exp (Z_0) = \exp \left( \sum_a Z^A(E2, D6_a) + Z^M(E2, O6) \right)$$

One-loop determinants!
Instanton calculus: 1-loop
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Diagrammatically we have the relation (for even spin structures)

\[ E_{2a} D_b = D_a x F_a x D_b \]

(Abel, Goodsell), (Akerblom, Bl, Lüst, Plauschinn, Schmidt-Sommerfeld)

Open problem: Computation of odd spin-structure \( E2 - D6 \) amplitude.
Instanton calculus: 1-loop
Stringy one-loop amplitudes are known to include the holomorphic Wilsonian part and non-holo. contributions from wave-function normalisation

\[(Shifman, Vainshtein), (Kaplunovsky, Louis)\]

\[Z_0(E2_a) = -\text{Re}(f_W^a)_{1\text{-loop}} - \frac{b_a}{2} \ln \left[ \frac{M_p^2}{\mu^2} \right] - \frac{c_a}{2} K_{\text{tree}} \]

\[-\ln \left( \frac{V_3}{g_s} \right)_{\text{tree}} + \sum_b \frac{|I_{ab}N_b|}{2} \ln [\det Z(r)]_{\text{tree}}\]

with

\[b_a = \sum_b \frac{|I_{ab}N_b|}{2} - 3, \quad c_a = \sum_b \frac{|I_{ab}N_b|}{2} - 1.\]
Instanton calculus: 1-loop

The CFT disc amplitudes combine non-holomorphic and holomorphic pieces:

\[ h_{ab} \left[ x \right] i_{ab} = e^{K_{ab}} \]

Therefore, all the non-holomorphic piece including the instanton cancel out and one gets the holomorphic quantity:

\[ Y_{a_1b_1} = \cdots \]

Higher loop only contribute to corrections of Kahler potentials.
The CFT disc amplitudes combine non-holomorphic and holomorphic pieces

\[
\langle \hat{\Phi}_{a,b}[x] \rangle_{\lambda_a, \bar{\lambda}_b} = \frac{e^{K_{a,b}} Y_{\lambda_a} \hat{\Phi}_{a,b}[x] \bar{\lambda}_b}{\sqrt{K\lambda_{a,a} \hat{K}_{a,b}[x] K_{b,\bar{\lambda}_b}}}. 
\]
Instanton calculus: 1-loop

The CFT disc amplitudes combine non-holomorphic and holomorphic pieces

\[ \langle \hat{\Phi}_{a,b}[\vec{x}] \rangle_{\lambda_a,\lambda_b} = \frac{e^{\frac{\kappa}{2} Y_{\lambda_a} \hat{\Phi}_{a,b}[x] \lambda_b}}{\sqrt{K_{\lambda_a,a} \hat{K}_{a,b}[x] K_{b,\lambda_b}}} . \]

Therefore, all the non-holomorphic piece including the instanton cancel out and one gets the holomorphic quantity

\[ Y_{\Phi_{a_1,b_1},...,\Phi_{a_M,b_M}} = \sum_{\text{conf.}} \exp(-S_{E2})_{\text{tree}} \exp(-f_{W}^a)_{1-\text{loop}} \]

\[ Y_{\lambda_{a_1} \hat{\Phi}_{a_1,b_1}[\vec{x}_1] \lambda_{b_1}} \cdots Y_{\lambda_{a_L} \hat{\Phi}_{a_L,b_L}[\vec{x}_L] \lambda_{b_L}} . \]

Higher loop only contribute to corrections of Kähler potentials.
Applications: Moduli potential

For E2-instantons with no matter field zero modes corrections to the uncharged closed/open string moduli superpotential can be generated:

$$W = A(T; U) e^U$$

Vacuum destabilisation

KKLT like stabilisation of closed string moduli

(Baumann et. al. hep-th/0607050)
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Applications : matter couplings

For appropriate E2-instantons, important perturbatively excluded matter couplings can be generated. Majorana masses for right-handed neutrinos (Bl, Cvetic, Weigand, hep-th/0609191), (Ibanez, Uranga, hep-th/0609213), see also (Bianchi, Kiritsis), (Cvetic, Richter, Weigand), (Ibanez, Schellekens, Uranga), (Antusch, Ibanez, Macri).

Non-pert. Majorana coupling:

\[ W_M = M_M N_R c N_R c \]

with

\[ M_M = x M_s e^{-2 \frac{3}{2} s g s V / E_2} \]

The natural mass scale is \( M_s' \approx M_{GUT} \) so that \( M_M \) is non-pert. suppressed w.r.t. to \( M_s >> M_{weak} ! \)
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SU(5) Yukawa couplings

Consider SU(5) GUT model via intersecting D6-branes. sector number U(5)

U(1) a U(1) b reps.

U(1) X (a0; a0) 3 10 (2; 0) 1 2 (a; b0)

3 5 (1; 1) 3 2 (b0; b0) 3 1 (0; 2) 5 2

H(1; 1) + 5 H(1; 1) i

Perturbative Yukawa couplings

h10 (2; 0) 5 (1; 1) 5 (0; 2) 5 H(1; 1) i

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\[
W_Y = Y^{\alpha\beta}_{\langle 10 \ 10 \ 5_H \rangle} \epsilon_{ijklm} \ 10^\alpha_{ij} \ 10^\beta_{kl} \ 5^H_m
\]

Flipped $SU(5)$: hierarchy between $(d, s, b)$ and $(u, c, t)$ by E2-instanton, flavour hierarchy by world-sheet instantons
Applications: The ADS superpotential
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$\mathbb{N}=1$ SQCD with $N_f = N_c - 1$ flavours

(Akerblom, Blumenhagen, Lüst, Plauschinn, Schmidt-Sommerfeld, hep-th/0612132)

(Florea, Kachru, McGreevy, Saulina, hep-th/0610003)
The ADS superpotential

Fermionic zero modes:
\[ L_{\text{ferm}} = c + f_e + \ldots \]

Bosonic zero modes:
\[ L_{\text{bos}} = b + c + \ldots \]

ADHM constraints

Eventually one arrives at
\[ SW'Z^4 x^2 N_c N_f \det[M_{ff}]_0 : \]

Higher corrections?
generalisations (Argurio, Bertolini, Ferreti, Lerda, Petersson), (Bianchi, Fucito, Morales)
The ADS superpotential

Issues:

• Fermionic zero modes:

\[ \mathcal{L}_{\text{ferm}} = \beta_c \Phi \lambda_f + \lambda_f \tilde{\Phi} \tilde{\beta}_c. \]
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\[
S_W \simeq \int d^4x \ d^2\theta \ \frac{\Lambda^{3N_c-N_f}}{\det[M_{ff'}]}.\]

- Higher \(\alpha'\) corrections?

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Instanton corrections to $f$

Holomorphy dictates that for D6-branes the holomorphic gauge kinetic function must look like:

$$f = X^I M^{Ia} u^c I + f_{1}\text{loop} e^T c_i + f_{np} e^T U c_I e^T T c_i$$

For intersecting D6-branes on $T^6$ the holomorphic one-loop gauge threshold corrections are: (Lust, Stieberger), (Akerblom, Bl, Lust, Schmidt-Sommerfeld)

- $N = 1$ sector: $f^{(1)} = 0$
- $N = 2$ sector: $f^{(1)} = \ln \left(\frac{\alpha^I T^c}{I T c^I}\right)$

World-sheet instanton corrections come from world-sheets with two boundaries! Expect $E_2$-instantons from non-rigid ones with $b_1() = 1$. 
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Zero modes: $Y_i$, $\mu^\alpha$, $\bar{\mu}^{\dot{\alpha}}$. Distinguish two cases depending on how the anti-holomorphic involution $\bar{\sigma}$ acts on the open string modulus $Y$

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The zero mode measure reads

$$\int d^4x \, d^2\theta \, d^2y \, d^2\bar{\mu} \, e^{-S_{E2}} \ldots, \quad \text{for } \bar{\sigma} : y \rightarrow y$$

and

$$\int d^4x \, d^2\theta \, d^2\mu \, e^{-S_{E2}} \ldots, \quad \text{for } \bar{\sigma} : y \rightarrow -y.$$ 

(dual to world-sheet instantons studied by Beasley-Witten)
Instanton corrections to $f$
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An instanton wrapping a 3-cycle with $b_1(\Xi) = 1$ and no additional zero modes can generate a correction to the $SU(N_a)$ gauge kinetic function.

$$\langle F_a(p_1) F_a(p_2) \rangle_{E2} = \int d^4x d^2\theta d^2\mu \exp(-S_{E2}) \exp(Z_0'(E2)) A_{F_a^2}(E2, D6_a)$$
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\]

where $A_{F_a^2}(E2, D6_a)$ is the annulus diagram
Corrections to F1 terms

Classically
\[ a = Z \]

If \( a = 0 \) classically for all branes, then no F1-term is generated at one-loop. (Lawrence, McGreevy, hep-th/0409284)

But if \( b_6 = 0 \) then a F1-term is generated on a D6-brane at one-loop.

Expect also E2-brane instanton corrections!

Stability of D-branes
Corrections to FI terms

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\[ \xi_a = \int_{\Pi_a} \mathcal{S}(\Omega_3). \]

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Corrections to FI terms

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