Super Yang Mills Scattering amplitudes at strong coupling

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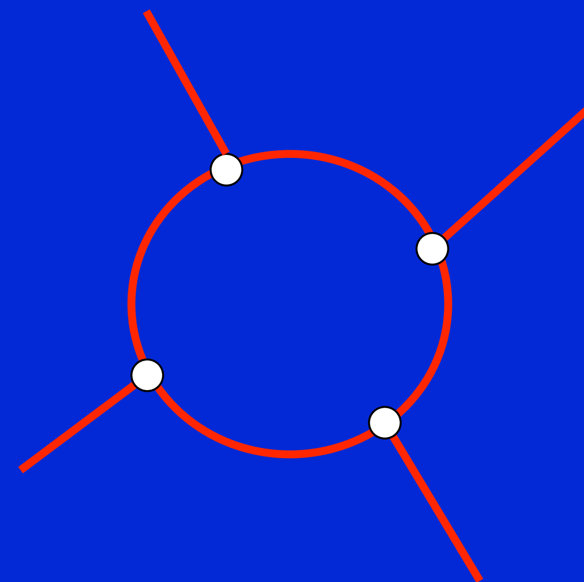
Based on L. Alday & JM **arXiv:0705.0303** [hep-th] & to appear

Length of talk: 30 minutes

Goal

- Compute planar, color ordered, gluon scattering amplitudes at strong coupling using strings on $AdS_5 x S^5$



$$\mathcal{A} = \sum_P Tr[T^{a_1} \ldots T^{a_n}] \mathcal{A}_n(k_1, \cdots, k_n)$$

- Yes, they are IR divergent.
- We can introduce an IR regulator.
- They are building blocks for closely related IR finite observables → Jet observables.
- There is a conjecture for the all order form of these amplitudes (in the MHV case):

$$\mathcal{A} = \mathcal{A}_{Tree} \exp\{ f(\lambda) a(s,t,\mu) \}$$

(more precise version later…)

Bern, Dixon, Smirnov 2005

Also … Anastasiou, Bern, Dixon, Kosower; Catani

Final Answer

The scattering process is described by a classical string worldsheet in AdS

$$\mathcal{A} \sim \exp\{-\frac{R^2}{2\pi\alpha'}(\mathrm{Area})\}$$

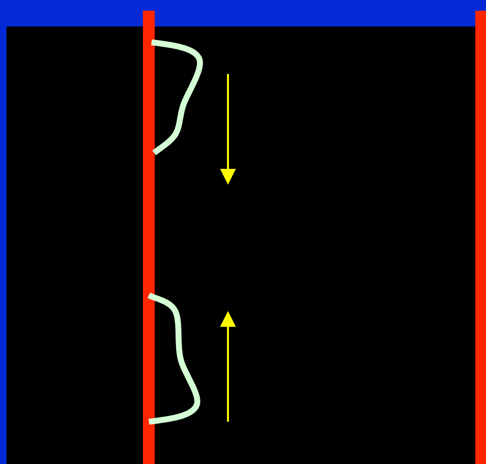
$$\mathcal{A} \sim \exp\{-\sqrt{\lambda} S_{cl}(s,t,\mu)\}$$

Two IR regulators

- 1st regulator: Brane in the IR → for motivation
- 2nd regulator: Dimensional regularization → for computations

1^{st} IR regulator

- D-brane in the bulk



Z_{IR}

Z=0
(boundary
of AdS)

Scatter with fixed gauge theory energy→ proper energy is very large→

high energy scattering in the bulk → classical string solution

$$ds^2 = \frac{dx^2}{z^2}$$

$$p_{\rm proper} = zp$$

Gross Mende

(see also Polchinski Strassler)

T-dual AdS space

It is convenient to introduce the T-dual coordinates

$$ds^2 = \frac{dx^2 + dz^2}{z^2}$$

$$dy = *\frac{dx}{z^2}, \qquad r = \frac{1}{z}$$

$$ds^2 = \frac{dy^2 + dr^2}{r^2}$$

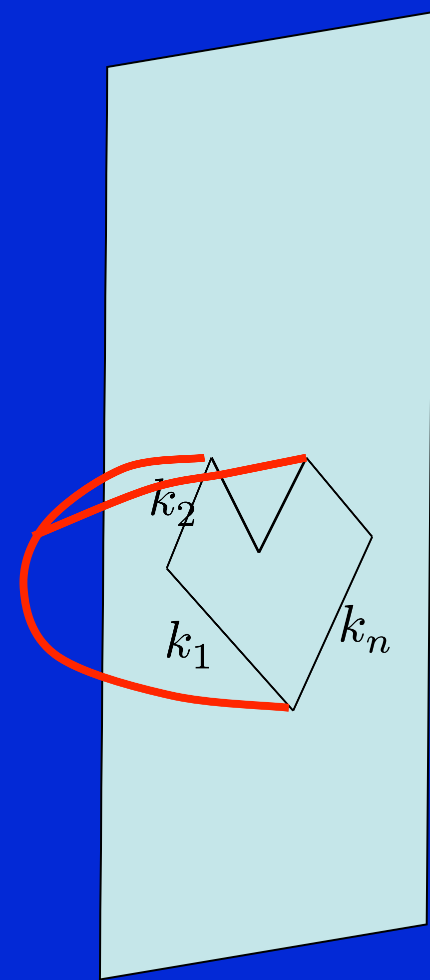
(Kallosh-Tseytlin)

We get again AdS but boundary and IR are exchanged
Strings wind in *y* by an amount proportional to the momentum of the gluon. y represents momentum space.

Final Prescription

Gluon momenta, define sequence of light like segments on the boundary. Two consecutive vertices are separated by the momentum k_i .

Sequence given by color ordering.



Formally similar to the computation of light-like Wilson loops

This is all in the T-dual coordinates.

We have temporarily removed the IR regulator

- Leading order answer is independent of the polarization of the gluons.
- The dependence on the polarizations should appear at the next order.
- Same for MHV and non-MHV

Symmetries

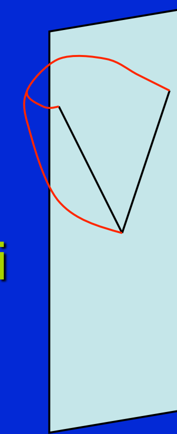
- The classical problem is invariant under SO(2,4) of the "momentum space" or T-dual coordinates.
- Is not a symmetry of the full theory (there is a non-invariant dilaton in the T-dual AdS).
- Was observed at weak coupling in the planar limit.

Drummond, Henn, Smirnov, Sokatchev

Finding the worldsheet

-Start with the cusp in poincare coordinates

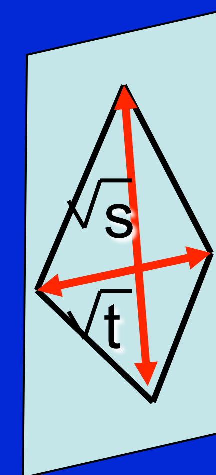
Kruczenski



- Apply a general conformal transformation

(not the same as as a conformal transformation in the original coordinates)

- Get the solution for four light-like segments



$$-Y_{-1}^2 - Y_0^2 + Y_1^2 + Y_2^2 + Y_3^2 + Y_4^2 = -1$$

$$Y_0 Y_{-1} = Y_1 Y_2 \ , \qquad Y_3 = Y_4 = 0$$ (for s=t)

- (The n gluon solution would require more work or ingenuity)

- Area is divergent….

2nd Regulator: Dimensional regularization

Amplitudes are usually defined in the gauge theory using dimensional regularization to $4+\epsilon$ dimensions

Dimensional reduction from 10d SYM to $4+\epsilon$ dimensions

Use the metric for a D-p-brane with $p=3+\epsilon$ in 10 dim.

$$ds^2 = f^{-1/2} dx^2_{4+\epsilon} + f^{1/2}(dr^2 + d\Omega^2_{5-\epsilon})$$

where $$f = c_p g_p^2 N \frac{1}{r^{4-\epsilon}}$$

Introducing a scale $g_p^2 \sim g^2 \mu^{-\epsilon}$

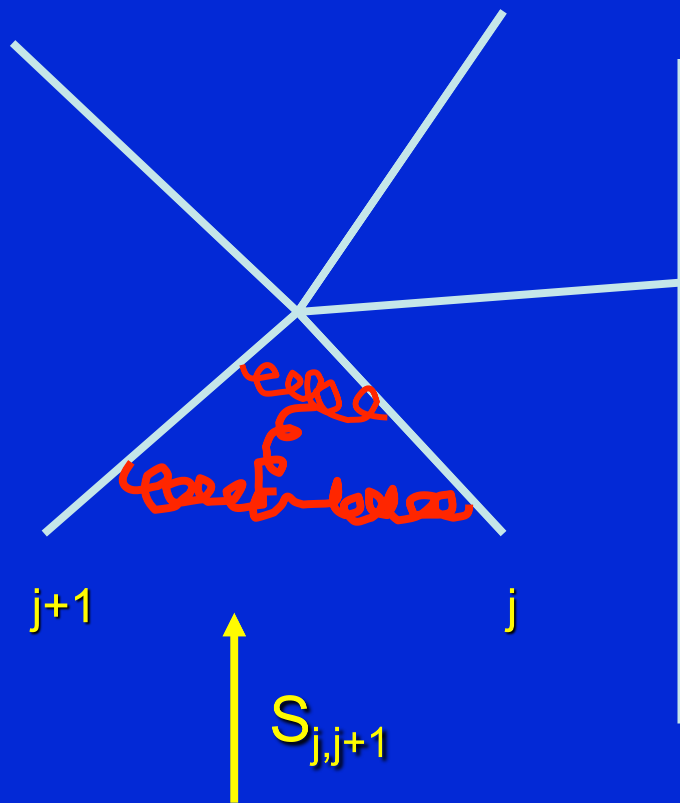
We then find

$$iS = \text{div} + \frac{\sqrt{\lambda}}{4\pi}(\log \frac{s}{t})^2 + \text{constant}$$

IR Divergence

Sudakov, A. Sen
Korchemsky, Catani,
Sterman, Magnea, Tejeda Yeomans
Bern, Dixon, Smirnov

We get one per “pizza slice”:



$$Div = \sum_{j=1}^{n} iS_{div}(s_{j,j+1})$$

$$iS_{div}(s) = -\frac{1}{2\epsilon^2} F\left(\frac{\lambda(-s)^{\epsilon/2}}{\mu^\epsilon}\right) + \frac{1}{2\epsilon} G\left(\frac{\lambda(-s)^{\epsilon/2}}{\mu^\epsilon}\right)$$

$$\left(\lambda \frac{d}{d\lambda}\right)^2 F(\lambda) = f(\lambda)\ , \qquad \left(\lambda \frac{d}{d\lambda}\right) G(\lambda) = g(\lambda)$$

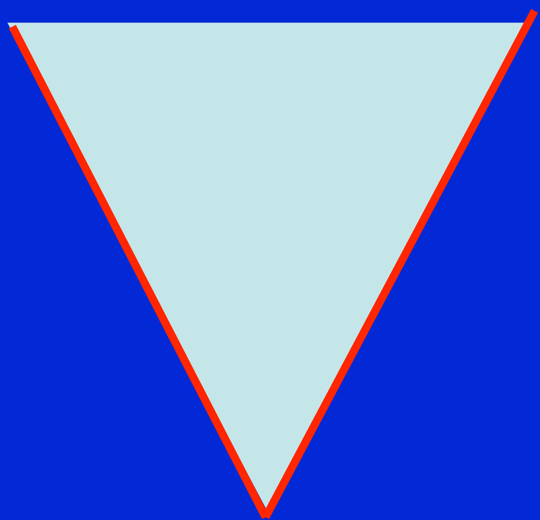
Cusp anomalous dimension

($\Delta - S = f(\lambda) \log S + \cdots$)

They are characterized by two functions of the coupling

They suppress the amplitude → it is very improbably *not* to emit other gluons

Why this form?



Replace gluons by Wilson lines

Configuration invariant under two non compact symmetries:
- Boosts
- Scale transformations

$$\mathcal{A} \sim \langle W \rangle \sim \exp\{-f(\lambda)\Delta\tau\Delta\chi\} \sim \exp\{-f(\lambda)(\log \mu^2/s)^2\}$$

Both are cutoff in the UV direction by the momenta or s.

Each gives a factor of log .

The second function, *g(λ)*, characterizes the subleading IR divergencies

It was computed to 3 loops at weak coupling Bern, Dixon, Smirnov

It also seems to obey a “maximal transcendentality” principle

We can compute it at strong coupling and we find

$$\mathrm{g}(\lambda) \sim \frac{\sqrt{\lambda}}{2\pi}(1-\log 2)$$

The Bern-Dixon-Smirnov ansatz

4 gluons:

$$\log \frac{\mathcal{A}}{\mathcal{A}_{Tree}} = \mathrm{div} + f(\lambda)(\log \frac{s}{t})^2$$

Agrees with our answer once we use the strong coupling form for f(λ) .

Gubser, Klebanov, Polyakov ; Kruczenski
Beisert, Eden, Staudacher

4 gluon amplitude is determined by momentum space conformal invariance and the form of the IR divergencies.

For n gluons BDS propose an explicit but more complicated function of the kinematic invariants.

Conclusions

- We gave a prescription for computing n gluon scattering amplitudes in AdS
- We checked the Bern Dixon Smirnov ansatz at strong coupling for four gluons.
- We observed the presence of a "momentum space" conformal symmetry.
- We computed the function $g(\lambda)$ at strong coupling
- We can also consider processes involving local operators → n-gluons. Form factors.

Future

- Compute the solutions for n gluons and check the BDS ansatz for n gluons.
- Can the function *g(λ)* be computed for all values of the coupling using integrability?
- Explicit solutions for form factors.
- Understand the crossover from weak to strong coupling in jet physics.