The Cosmic String Inverse Problem

Joe Polchinski
KITP, UCSB

JP & Jorge Rocha, hep-ph/0606205
JP & Jorge Rocha, gr-qc/0702055
Florian Dubath & Jorge Rocha, gr-qc/0703109
Florian Dubath, JP & Jorge Rocha, work in progress
JP, review in preparation

Strings 07, Madrid, 6/29/07

There are many potential cosmic strings from string compactifications:

- The fundamental strings themselves
- D-strings
- Higher-dimensional D-branes, with all but one direction wrapped.
- Solitonic strings and branes in ten dimensions
- Solitons involving compactification moduli
- Magnetic flux tubes (classical solitons) in the effective 4-d theory: the classic cosmic strings.
- Electric flux tubes in the 4-d theory.

A network of any of these might form in an appropriate phase transition in the early universe.

• What are the current bounds, and prospects for improvement?

• Too what extent can we distinguish different kinds of cosmic string?

• What are the current bounds, and prospects for improvement?

• Too what extent can we distinguish different kinds of cosmic string?

The cosmic string inverse problem:

Observations → Microscopic models

• What are the current bounds, and prospects for improvement?

• Too what extent can we distinguish different kinds of cosmic string?

The cosmic string inverse problem:

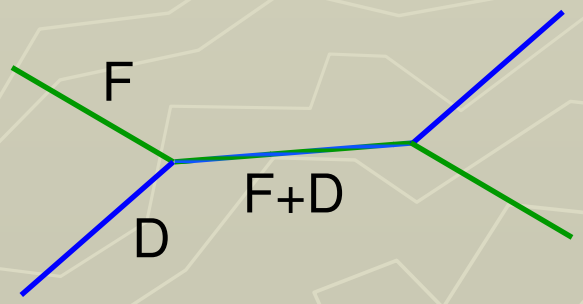
There is an intermediate step:

Observations → Macroscopic parameters → Microscopic models

Macroscopic parameters:

- Tension μ
- Reconnection probability P:

- Light degrees of freedom: just the oscillations in 3+1, or additional bosonic or fermionic modes?
- Long-range interactions: gravitational only, or axionic or gauge as well?
- One kind of string, or many?
- Multistring junctions?

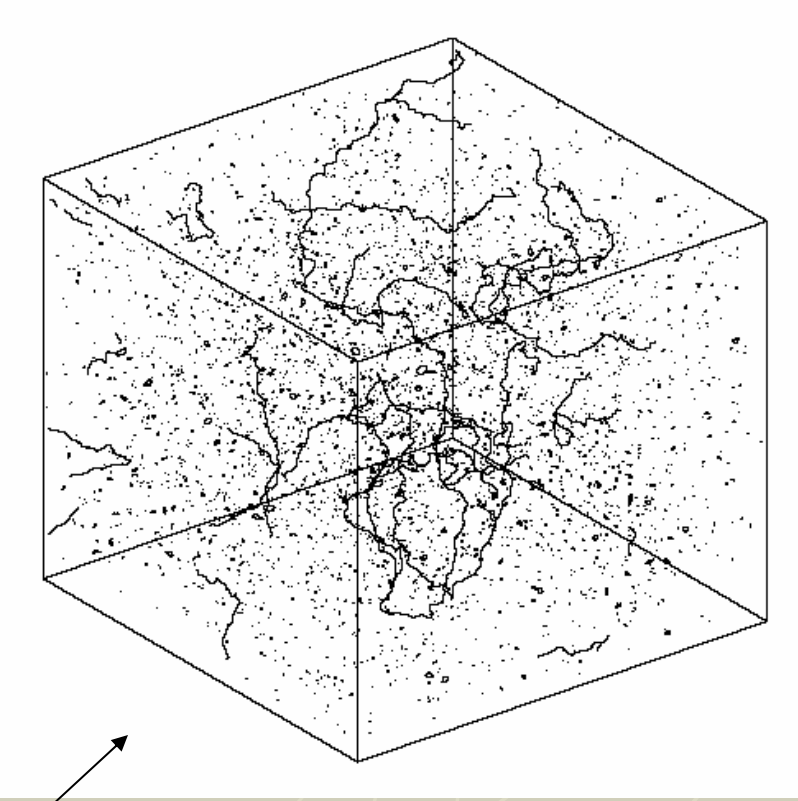


Vanilla Cosmic Strings:

- $P = 1$
- No extra light degrees of freedom
- No long-range interactions besides gravity
- One kind of string
- No junctions

Even for these, there are major uncertainties.

A simulation of vanilla strings (radiation era, box size ~ $.5t$):

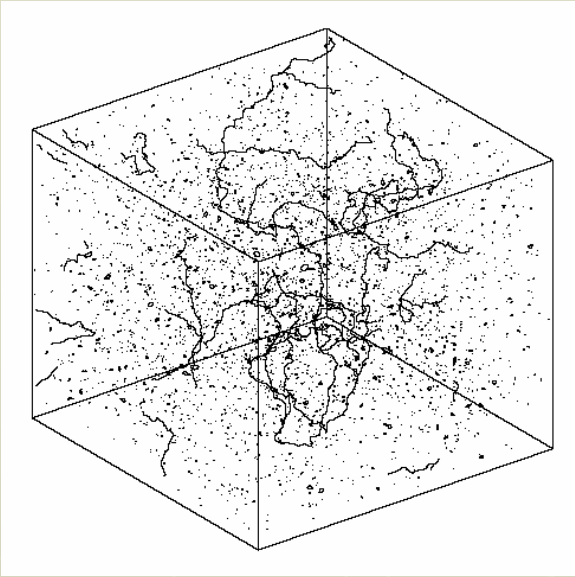


Simple arguments suggest that t ~ Hubble length ~ Horizon length is the only relevant scale.

If so, simulations (Albrecht & Turok, Bennett & Bouchet, Allen & Shellard, ~ 1989) would have readily given a quantitative understanding.

However, one sees kinks and loops on shorter scales (BB). Limitations: UV cutoff, expansion time.

Estimates of the sizes at which loops are produced range over more than *fifty* orders of magnitude, in a completely well-posed, classical problem.



Since the problem is a large ratio of scales, try an RG-like approach: use simulations at horizon scale, but then scale to shorter scales analytically.

Not exactly like the RG: since the comoving scale increases more slowly than t, structure flows from long distance to short.

However, with the aid of recent simulations, we have perhaps understood what the relevant scales are, and why.

Outline:

- Review of network evolution*
- A model of short distance structure
- Signatures

*Good references:
Vilenkin & Shellard, *Cosmic Strings and Other Topological Defects*;
Hindmarsh & Kibble, hep-ph/9411342.

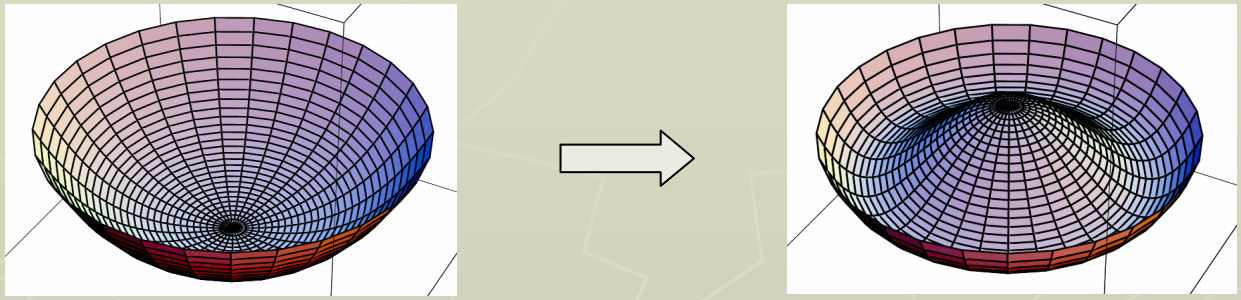
I. Review of Vanilla Network Evolution

Processes:

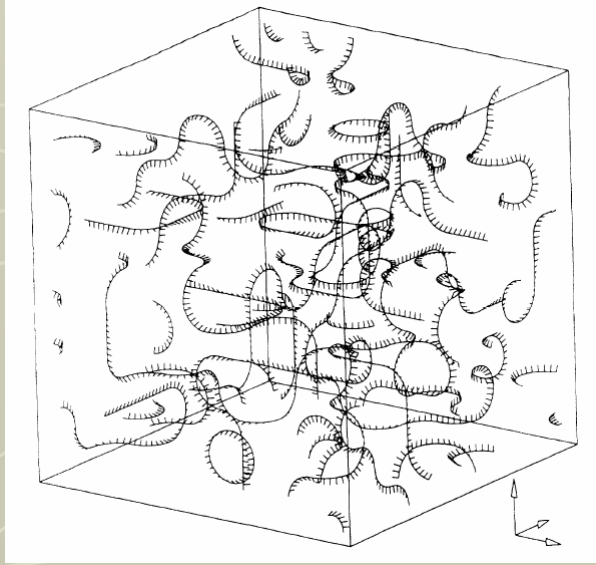
1. Formation of initial network in a phase transition.
2. Strings must be (meta)stable against breakage and axion confinement.
3. Stretching of the network by expansion of the universe.
4. Long string intercommutation.
5. Long string smoothing by gravitational radiation.
6. Loop formation by long string self-intercommutation.
7. Loop decay by gravitational radiation.

1. Network creation

String solitons exist whenever a $U(1)$ is broken, and they are actually produced whenever a $U(1)$ *becomes* broken during the evolution of the universe (Kibble):



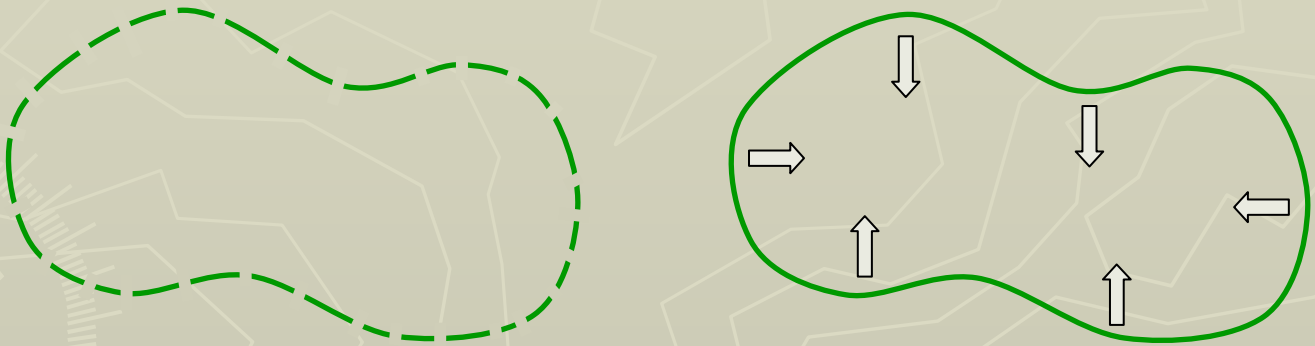
Phase is uncorrelated over distances > horizon. $O(50\%)$ of string is in infinite random walks.



(Dual story for other strings - e.g. brane/antibrane inflation - Sarangi & Tye).

2. Stability

We must assume that the strings are essentially stable against breakage and axion domain wall confinement (model dependent).



3. Expansion

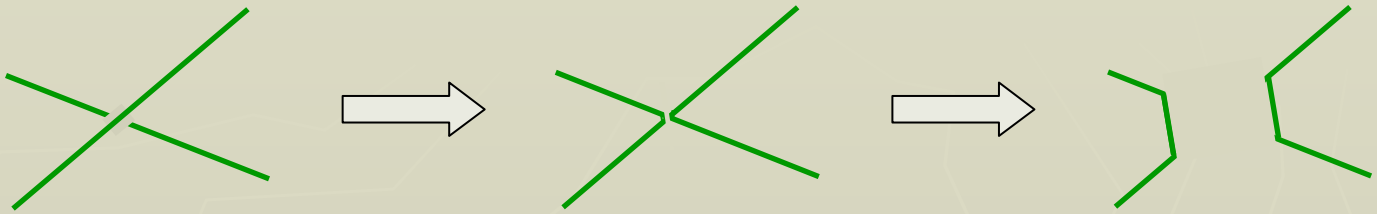
FRW metric: $$ds^2 = -dt^2 + a(t)^2 d\mathbf{x} \cdot d\mathbf{x}$$

As in flat spacetime, the motion can be expressed in terms of a pair of unit vectors, $\mathbf{p}_\pm \equiv \dot{\mathbf{x}} \pm \frac{1}{\epsilon}\mathbf{x}'$, which are essentially the right- and left-moving tangent vectors, but unlike flat spacetime they interact:

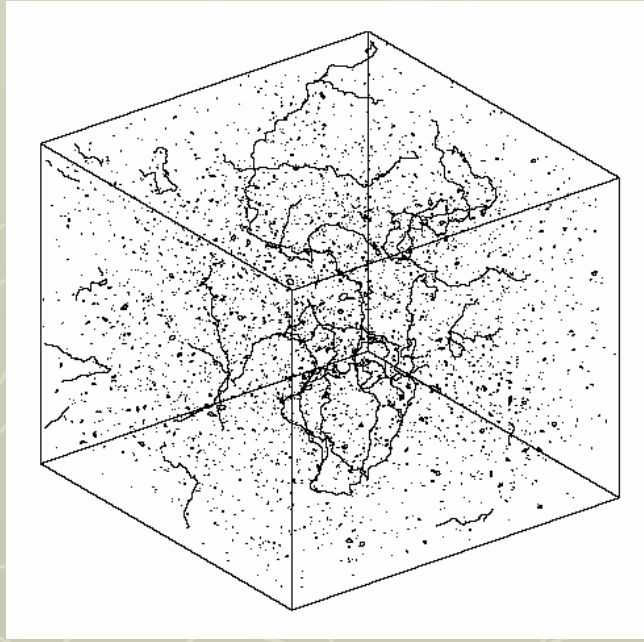
$$\dot{\mathbf{p}}_\pm \mp \frac{1}{\epsilon}\mathbf{p}'_\pm = -\frac{\dot{a}}{a}\left[\mathbf{p}_\mp - (\mathbf{p}_+ \cdot \mathbf{p}_-)\,\mathbf{p}_\pm\right]$$

$$\epsilon \equiv \left(\frac{\mathbf{x}'^2}{1 - \dot{\mathbf{x}}^2}\right)^{1/2}$$

4. Long string intercommutation



Produces L- and R-moving kinks. Expansion of the universe straightens these slowly, but more enter the horizon (BB).

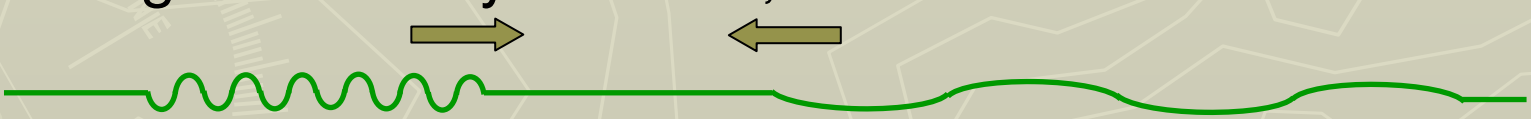


5. Long string gravitational radiation

This smooths the long strings at distances less than some scale l_{G}.

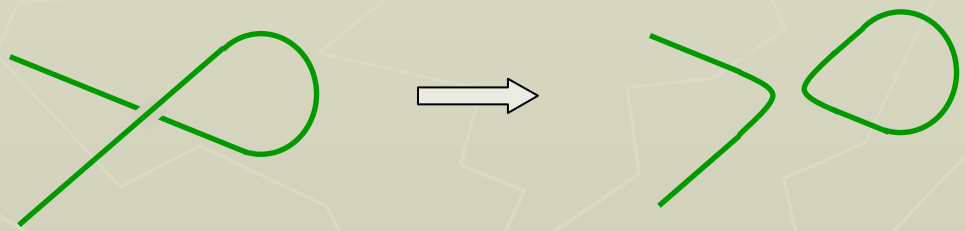
Simple estimate gives $l_{\mathrm{G}} = \Gamma G\mu t$, with $\Gamma \sim 50$.

Subtle suppression when L- and R-moving wave-lengths are very different,



so in fact $l_{\mathrm{G}} = \Gamma (G\mu)^k$, where estimates of k vary from 1.2 to 2.5 (Siemens & Olum; … & Vilenkin; JP & Rocha).

6. Loop formation by long string self-intersection



7. Loop decay by gravitational radiation

Dimensionally, for a loop of length l, the rate of gravitational wave emission is

$$\dot{E} = \Gamma G \mu^2$$

A loop of initial length l_{i} (energy μl_{i}) decays in time

$$\tau = l_{\mathrm{i}} / \Gamma G \mu$$

A loop of size $l_{\mathrm{i}} = \Gamma G \mu t$ lives around a Hubble time.

Scaling hypothesis:

All statistical properties of the network are constant when viewed on scale t (Kibble).

If only expansion were operating, the long string separation would grow as $a(t)$. With scaling, it grows more rapidly, as t, so the various processes must eliminate string at the maximum rate allowed by causality.

Simulations, models, indicate that the scaling solution is an attractor under broad conditions (m & r) (more string $\rightarrow$ more intercommutions $\rightarrow$ more kinks $\rightarrow$ more loops $\rightarrow$ less string). Washes out initial conditions.

Review:

1. Formation of initial network in a phase transition.
2. Stability against decays.
3. Stretching of the network by expansion of the universe.
4. Long string intercommutation.
5. Long string smoothing by gravitational radiation.
6. Loop formation by long string self-intercommutation.
7. Loop decay by gravitational radiation.

(Simulations replace grav. rad. with a rule that removes loops after a while)

Estimates of loop formation size

$0.1\,t$: original expectation, and some recent work (Vanchurin, Olum & Vilenkin)

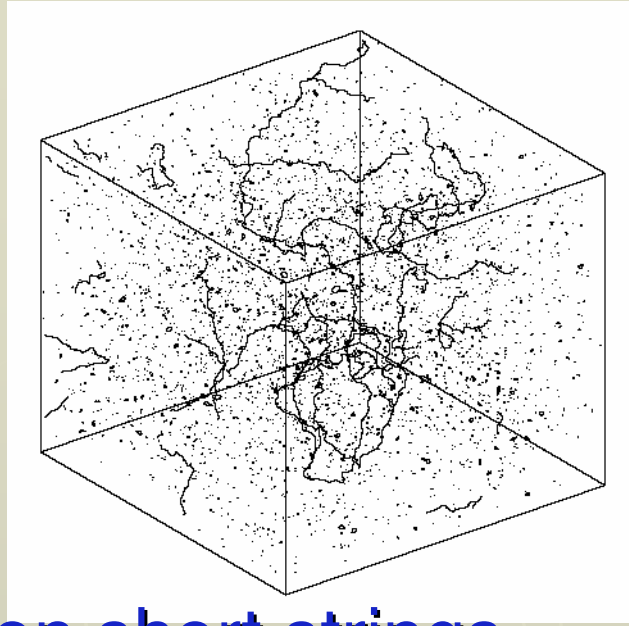
$10^{-3}\,t$: other recent work (Martins & Shellard, Ringeval, Sakellariadou & Bouchet)

$\Gamma G\mu\, t$: still scales, but dependent on gravitational wave smoothing (Bennett & Bouchet)

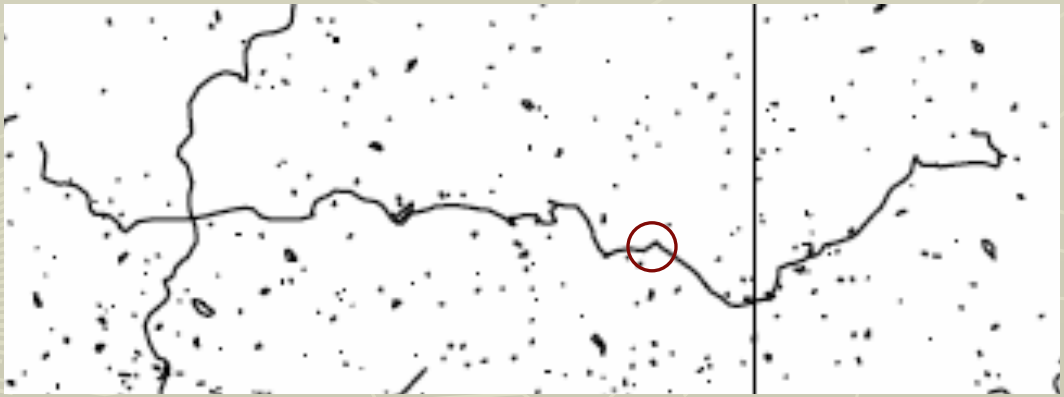
$\Gamma(G\mu)^k\, t,\ 1.2 \leq k \leq 2.5$: corrected gravitational wave smoothing (Siemens, Olum & Vilenkin; JP & Rocha)

τ_{string}: the string thickness - a fixed scale, not $\propto t$ (Vincent, Hindmarsh & Sakellariadou)

II. A Model of Short Distance Structure

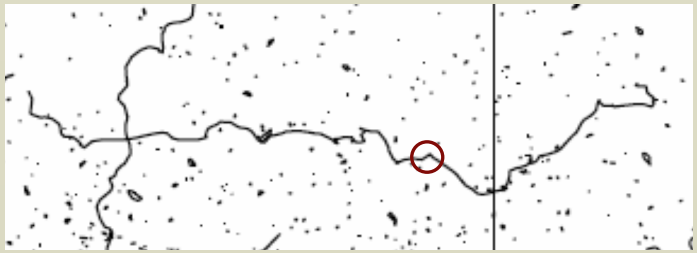


1. Small scale structure on short strings



Strategy: consider the evolution of a small (right- or left-moving) segment on a long string.

Evolution of a short segment, length l. Possible effects:



1. Evolution via Nambu-FRW equation
2. Long-string intercommutation
   very small probability, $\propto l$
3. Incorporation in a larger loop
   controlled by longer-scale configuration, will not change mean ensemble at length l^*
4. Emission of a loop of size l or smaller
   ignore? not self-consistent, but again controlled by longer-scale physics
5. Gravitational radiation
   ignore until we get to small scales

Nambu-FRW equations simplify for a short segment. Separate segment into its mean and a (small) fluctuation:

$$\mathbf{p}_\pm(\tau,\sigma) = \mathbf{P}_\pm(\tau) + \mathbf{w}_\pm(\tau,\sigma) - \frac{1}{2}\mathbf{P}_\pm(\tau) w_\pm^2(\tau,\sigma) + \ldots$$

where $P_\pm^2 = 1 \text{ and } \mathbf{P}_\pm \cdot \mathbf{w}_\pm = 0$

Then $$\dot{\mathbf{w}}_+ - \frac{1}{\epsilon}\mathbf{w}'_+ = -(\mathbf{w}_+ \cdot \dot{\mathbf{P}}_+)\,\mathbf{P}_+ + \frac{\dot{a}}{a}\underbrace{(\mathbf{P}_+ \cdot \mathbf{P}_-)}\,\mathbf{w}_+$$

just precession

average over Hubble time, $\mathbf{P}_+ \cdot \mathbf{P}_- = 2\overline{v}^2 - 1$

$$\mathbf{w}_{+,-} \propto a^{2\,\overline{v}^2 - 1}$$

In flat spacetime, virial theorem gives $\overline{v}^2 = 1/2$, but redshifting reduces this to 0.41 (radiation era) and 0.35 (matter era), from simulations.

For $a = t^r$,

$$\left\langle [\mathbf{w}_+(\sigma,\tau) - \mathbf{w}_+(\sigma',\tau)]^2 \right\rangle = t^{-2r(1-2\bar{v}^2)} f(\sigma - \sigma')$$

Initial condition when segment approaches horizon scale, gives

$$f(\sigma - \sigma') = 2A|\sigma - \sigma'|^{2\chi} \ , \quad \chi = \frac{r(1-2\bar{v}^2)}{1 - r(1-2\bar{v}^2)}$$

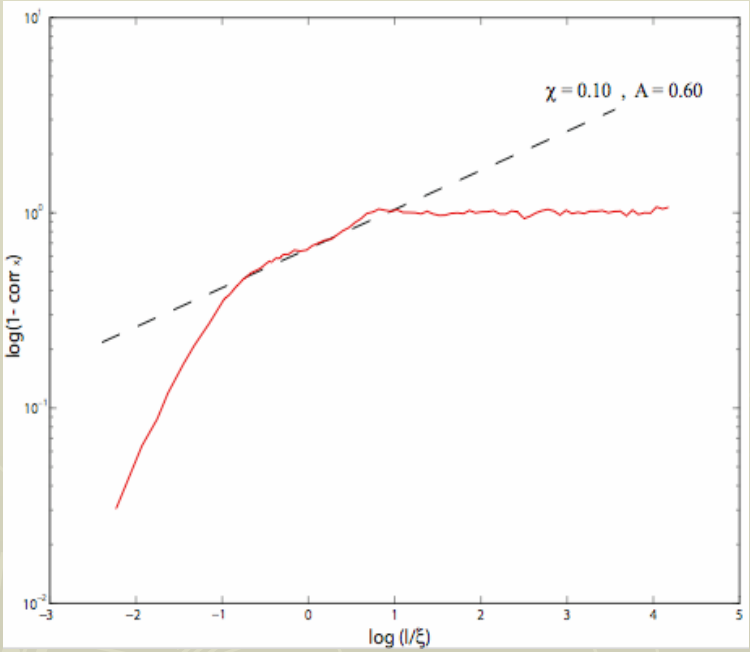
(A is another parameter that must be taken from simulations. Final result:

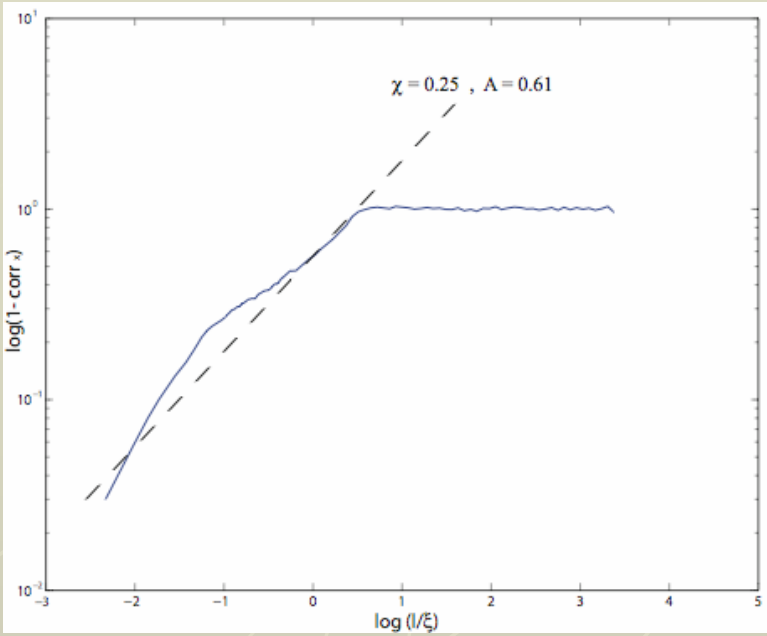
$$\left\langle [\mathbf{w}_+(\sigma,\tau) - \mathbf{w}_+(\sigma',\tau)]^2 \right\rangle = 2A(l/t)^{2\chi}$$

(l = physical length of segment)

$$\chi_{\mathrm{m}} = 0.25 \text{ and } \chi_{\mathrm{r}} = 0.10$$

Compare with simulations (Martins & Shellard):





Random walk at long distance.
Discrepancy at short distance - but the expansion factor is only 3 - transient.

2. Loop formation

Loops form whenever string self-intersects. This occurs when $\mathbf{L}_+(u,l) = \mathbf{L}_-(v,l)$, where

$$\mathbf{L}_+(u,l) = \int_u^{u+l} du\, \mathbf{p}_+(u)\ , \quad \mathbf{L}_-(v,l) = \int_v^{v+l} dv\, \mathbf{p}_-(v)$$

Rate per unit u, v, l:

$$\langle \det J\, \delta^3(\mathbf{L}_+(u,l) - \mathbf{L}_-(v,l))\rangle\ , \quad J = \frac{\partial^3(\mathbf{L}_+(u,l) - \mathbf{L}_-(v,l))}{\partial u\, \partial v\, \partial l}$$

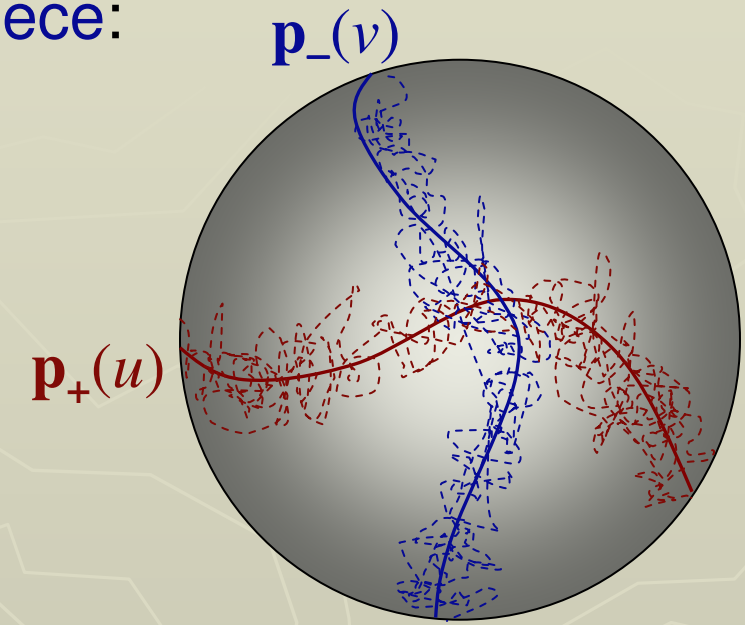
Components of L_+–L_- are of order l, l, $l^{1+2\chi}$.
Columns of J are of order l^{χ}, l^{χ}, $l^{2\chi}$.
Rate ~ $l^{-3+2\chi}$.

Rate of loop emission ~ $l^{-3+2\chi}$.
Rate of *string length* converted to loops, per unit world-sheet area and unit dl ~ $l^{-2+2\chi}$.

Total rate per world-sheet area = $\int dl\, l^{-2+2\chi}$: this diverges at the lower end for $\chi < 0.5$, even though the string is becoming smoother there.

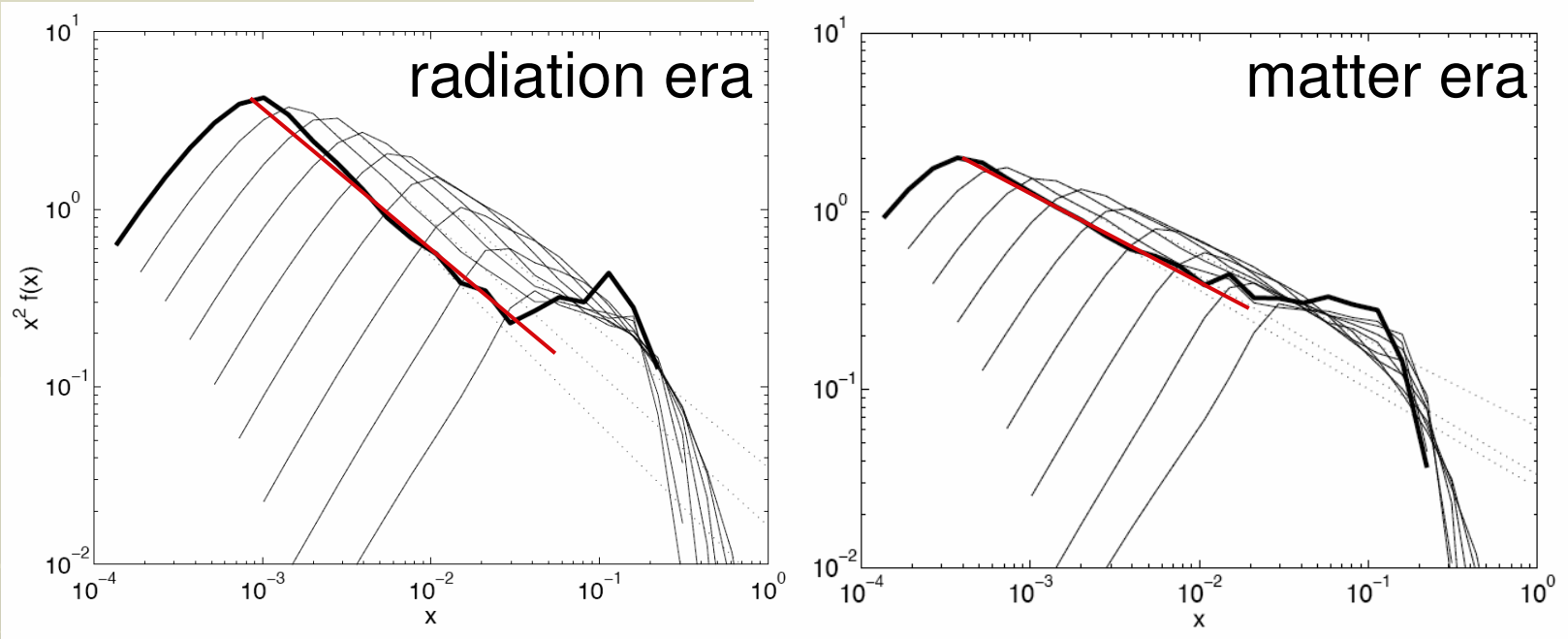
What cuts this off, and at what scale?

Resolving the divergence: separate the motion into a long-distance `classical' piece plus short-distance fluctating piece:



Loops form near the cusps of the long-distance piece. *All sizes form at the same time.* Get loop production function $l^{-2+2\chi}$, but with cutoff at gravitational radiation scale, $\Gamma_{\text{eff}}(G\mu)^{1+2\chi}t$.

Recent simulations (Vanchurin, Olum, Vilenkin) use a volume-expansion trick to reach larger expansion factors. Result:



Two peaks, ~ 0.1 horizon and ~ UV cutoff. VOV interpret the latter as transient, but this is the one we found.

What about the large loops? We can understand why these exist, but need simulations to determine fraction.

Scorecard on loop formation size

10-20% $0.1\,t$: original expectation, and some recent work (Vanchurin, Olum & Vilenkin)

$10^{-3}\,t$: other recent work (Martins & Shellard, Ringeval, Sakellariadou & Bouchet)

$\Gamma G\mu\, t$: still scales, but dependent on gravitational wave smoothing (Bennett & Bouchet)

80-90% $\Gamma(G\mu)^{1+2\chi}\, t$: corrected gravitational wave smoothing (Siemens, Olum & Vilenkin; JP & Rocha)

τ_{string}: the string thickness - a fixed scale, not $\propto t$ (Vincent, Hindmarsh & Sakellariadou)

III. Gravitational Signatures

Vanilla strings have only gravitational long-range interactions, so we look for gravitational signatures:

1. Dark matter.
2. Effect on CMB and galaxy formation.
3. Lensing.
4. Gravitational wave emission.

Key parameter: $G\mu$. This is the typical gravitational perturbation produced by string. In brane inflation models,

$$10^{-12} < G\mu < 10^{-6}$$

Normalized by $\delta T/T$. (Jones, Stoica, Tye)

1. Dark Matter: No: $\rho_{\text{string}}/\rho_{\text{matter}} \sim 100\, G\mu << 1$.

2. Perturbations of CMB: Primarily from *long strings.*
Current bounds from power spectrum: $G\mu < 2$ x 10^{-7}
" from non-gaussianity: $G\mu < 6$ x 10^{-7}
1 to 2 order of magnitude improvement over long term, from polarization and non-gaussianity.

3. Lensing: Primarily from *long strings.* $G\mu < 2$ x 10^{-7} implies $\delta < 1$". Future surveys may detect $G\mu$ to 10^{-8} (optical), 10^{-9} (radio). *Note:* string is rather straight at lensing scale (fractal dimension ~1 not 2).

4. Gravitational radiation: primarily from *loops.*
Two imporant distinctions:

Large loops vs. *small.*

- Large ($l > \Gamma G\mu t$) live > Hubble time.
- Small ($l < \Gamma G\mu t$) live < Hubble time.

Low harmonics vs. *high.*

- Low ($\omega \sim 4\pi/l$) get most of the energy, seen as stochastic superposition of many loops.
- High seen as stochastic superposition and/or individual cusps.

QuickTime™ and a
Animation decompressor
are needed to see this picture.

Both stochastic waves and cusps can be seen at interferometers (LIGO, LISA, etc.) and indirectly through their effect on the precise timing of pulsar signals.

Current interesting limit on stochastic background from pulsars: $\nu_0 \frac{d\Omega_{\rm GW}}{d\nu_0} < 4 \times 10^{-8}$ (frequency $\nu_0 \sim \mathrm{yr}^{-1}$)

From low harmonics of large loops:

$$\nu_0 \frac{d\Omega_{\rm GW}}{d\nu_0} = 0.0035 \gamma^{-3/2} (\alpha G\mu)^{1/2} f$$

α = loop size / $t \sim 0.1$
γ = initial boost of large loop ~ 1
f = string fraction in large loops ~ 0.1

Implies $G\mu < 2 \times 10^{-7}$, similar to CMB bound.

Existence of a significant fraction of large loops is very favorable for stochastic background.

Cusp signal less favorable than previously believed (Siemens, et. al.).

Two-peak distribution - effect on bounds:

	Low harmonics	High harmonics
Large loops:	Current (pulsar): 2×10^{-7} PPTA: 10^{-9} Advanced LIGO: 10^{-10} SKA, LISA: 10^{-11}	Advanced LIGO: ?? LISA: 10^{-13} SKA: 10^{-11}
Small loops:		Advanced LIGO: ?? LISA: 10^{-10}

The inverse problem:

Observation of low harmonics of large loops probably allows measurement of $G\mu$ only (through absolute normalization) - *if the networks are understood perfectly.* Confirmation of string interpretation from observation in both pulsar and LIGO bands, or from spectral slope.

Slightly less vanilla strings: $P \neq 1$: Normalization $\propto$ P^{-1}? P^{-2}? $P^{-0.6}$? : degenerate with $G\mu$.

Observation of high harmonics gives several independent measurements: measure $G\mu$, P, look for less vanilla strings.

Conclusions

- Long-standing problem of understanding networks perhaps nearing solution.
- Observations will probe most or all of brane inflation range.
- If so, there is prospect to distinguish different string models, maybe not until LISA. Observation of cosmic strings would just be the beginning.
- Precise understanding of string networks will require a careful meshing of analytic and numerical methods - an interesting kind of problem.

Advertisement:

KITP program “Fundamental Issues in Superstring Theory,” coordinators M. Becker, N. Berkovits, JP, Jan. - May 2009.

Also - late Jan. during the program there will be an informal one-day celebration of Stanley Mandelstam’s 80th birthday.