DIMENSIONAL - DUALITY

Dimensional duality.
Daniel Green (SLAC & Stanford U., Phys. Dept.) , Albion Lawrence (Brandeis U.) , John McGreevy (MIT, LNS) , David R. Morrison (Duke U., CGTP & UC, Santa Barbara) , Eva Silverstein (SLAC & Stanford U., Phys. Dept.) . NSF-KITP-07-56, SLAC-PUB-12439, SU-ITP-07-05, MIT-CTP-3829, BRX-TH-586, DUKE-CGTP-07-02, UCSB-MATH-2007-08, May 2007. 25pp. .
e-Print: **arXiv:0705.0550** [hep-th](PRD to appear)

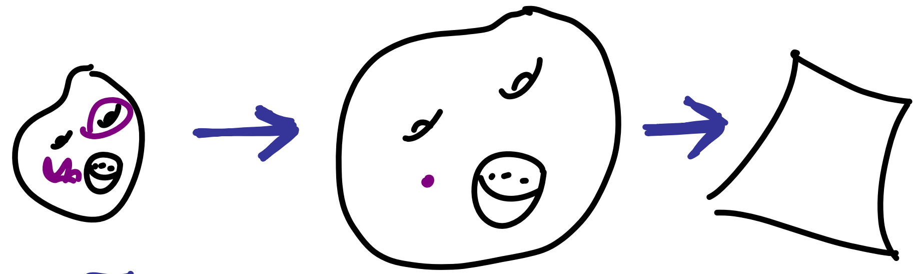
New dimensions for wound strings: The Modular transformation of geometry to topology.
John McGreevy (MIT, LNS) , Eva Silverstein, David Starr (SLAC & Santa Barbara, KITP) . SLAC-PUB-12250, SU-ITP-06-33, MIT-CTP-3795, Dec 2006. 29pp.
Published in **Phys.Rev.D75:044025,2007**.

Supercritical stability, transitions and (pseudo)tachyons.
Ofer Aharony, Eva Silverstein . SLAC-PUB-12243, SU-ITP-06-32, WIS-18-06-NOV-DPP, Dec 2006. 21pp.
Published in **Phys.Rev.D75:046003,2007**.

Dimensional mutation and spacelike singularities.
Eva Silverstein (SLAC & Stanford U., Phys. Dept.) . SU-ITP-05-26, SLAC-PUB-11510, Oct 2005. 15pp.
Published in **Phys.Rev.D73:086004,2006**.
e-Print: **hep-th/0510044**

cf
- Hellerman, Liu, Swanson
- Nakayama
- Kleban, Redi
- Maloney

- In String theory, spacetime topology and geometry (GR) "EMERGE"



$L \lesssim \sqrt{\alpha'}$	$L \gg \sqrt{\alpha'}$	$L \to \infty$
~~GR~~	$GR + \mathcal{O}\left(\frac{\alpha'}{L^2}\right)$	GR
topology change singularity resolution...		

FROM
- worldsheet CFT $g_s \ll 1$
- ⋮
- spacetime CFT AdS backgrounds
- ⋮
- ? generally / cosmologically

- The Dimensionality D is also an emergent quantity:

In the regime of perturbative string theory,

D is generalized to $\underline{D_{eff} \equiv \frac{2}{3} C_{eff}^{(B)}}$,

corresponding to the growth of the Hagedorn density of states:

Cardy
Kutasov/
Seiberg
...

$$\rho(m) \sim e^{m\sqrt{\alpha'}\,\pi\sqrt{\frac{2C_{eff}}{3}}} \qquad \left(m^2 \sim \tfrac{2}{\alpha'}\Delta\right)$$

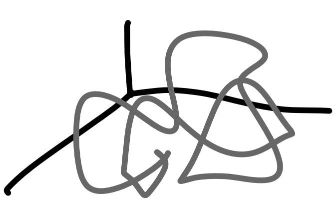
extracted from the 1-loop vacuum amplitude

$$\int \frac{d^2\tau}{4\tau_2}\, \underbrace{e^{-2\pi\tau_2(L_0+\bar{L}_0)}\, e^{2\pi i\tau_1(L_0-\bar{L}_0)}}$$

$$\xrightarrow[\text{UV: } (\tau_2\to 0)]{} e^{\frac{\pi C_{eff}}{6\tau_2}}$$

superstring:

$$\underset{\frac{3}{2}D_{eff}}{\overset{\shortparallel}{C_{eff}^{B}}} + C_{eff}^{F} = 0$$



More dimensions for string to probe $\Leftrightarrow$ higher density of states

"Supercritical" means $C_{eff} > C_{eff}^{crit}$
(sub) $(<)$

Modular Invariance $\Rightarrow$

$\tau_2 \to 0$ $\quad Z_{UV} \sim e^{\frac{\pi C_{eff}}{6\tau_2}}$ $\quad m_{min}^2 = -\frac{C_{eff}}{6\alpha'}$

$\tau_2 \to \infty$ $\quad Z_{IR} \sim e^{-\frac{\pi\alpha' m_{min}^2}{\tau_2}}$

So $\quad C_{eff}^B + C_{eff}^F > 0 \Rightarrow$ pseudo*-tachyon

* Negative mass^2 in time-dependent background
not automatically a catastrophic instability :

can have $\rho_{\langle T\rangle} \ll \left(\frac{\dot a}{a}\right)^2 M_p^2 = \frac{k}{a^2} + \rho_0$

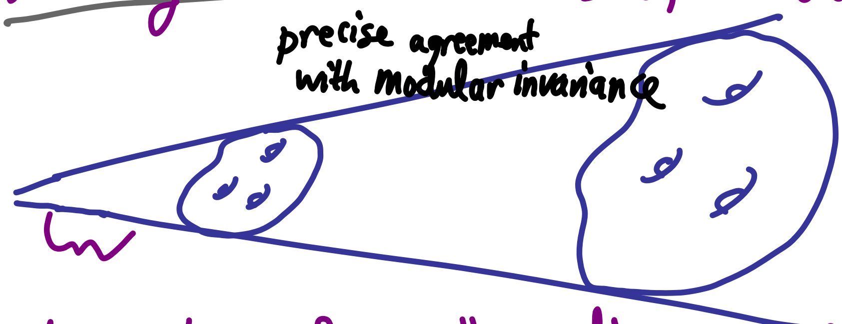
Energy density in condensing pseudo-tachyon

and we'll consider this stable case
(cf inflationary $\delta\rho/\rho$)

This Talk: Duality between "critical" strings on a compact negatively curved space $\mathcal{M}_n$ and supercritical strings $(c_{eff} > c_{eff}^{crit})$

Dimensions emerge from topology

① effective central charge $c_{eff} > c_{eff}^{critical}$ from exponential density of winding states $\longleftrightarrow$ IR perturbation spectrum



② Dual description of small-radius space: Current algebra $\Rightarrow$ Minimal Completion respecting $U(1)^{b_1}$ winding symmetry contains a T^{b_1}

③ AdS/CFT formulation also reveals T^{b_1}

Basic Motivations

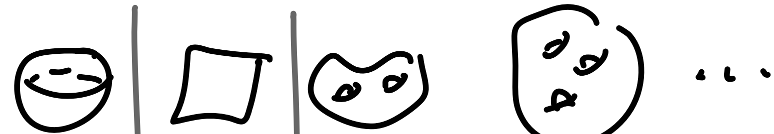
Curved target spaces are generic.

e.g. $d=2$

$\vdots$

$d=6$

$\vdots$



S^2	T^2	$h=2$	$h=3$	...
Freund-Rubin	Calabi-Yau	Everything else		

$N=1, N=0$ phenomenologically viable

and at large radius, evolve slowly with time controlled by GR e.g. constant curvature vacuum sol'n $-dt^2 + t^2 ds^2_{\mathbb{H}^n/\Gamma}$

Relevant for basic physical questions:

- compactifications (most are curved)
- 3d space may be globally compact

↪ initial singularity, measure?

★ • "Quantum" geometry: spacetime emergent, and so is its dimensionality

Consider moduli potential in a string compactification X of volume V

$$\underset{\substack{4d \\ \text{Einstein frame}}}{\mathcal{U}(g_s, V, \ldots)} = \frac{g_s^2 (D-D_c)}{V} + \frac{g_s^2}{V}\int_X \frac{\sqrt{G_X}(-R)}{V}$$

+ fluxes + orientifolds + branes + quantum

positive potential useful for dS model-building

ES '01 MSS '02 $(D>D_c)$ SS '04 $(R<0)$ ...

BP '00

Becker^2 ... GKP '01 KKLT '03

From the worldsheet point of view

$$\beta_{\log g_{eff}^2} \sim D-D_c + \frac{\int -R}{V} + \cdots$$

↪ Raises the question:

Q Are negatively curved compactifications of 10d string theory effectively supercritical at finite volume V?

Consider n-dimensional Compact space M_n

of negative sectional curvature :

$\star$ Milnor '68, Margulis '69, ... $\Rightarrow$ $\rho(\ell) \geq e^{(n-1)\frac{\ell}{\ell_0}}$

$\rho(\ell)$: density of periodic geodesics $\in \pi_1(M_n)$

$e^{(n-1)\frac{\ell}{\ell_0}}$: Exponential growth $\left(|R| > \frac{1}{\ell_0^2}\right)$

In string theory, this $\Rightarrow$ $\boxed{\rho_{winding} \sim e^{\frac{m_{winding}}{m_0}}}$

$$C_{eff} = C_{eff}^{crit} + \underline{\Delta C_{eff}} > C_{eff}^{crit}$$

ΔC_{eff}: from trace over winding modes in partition function

- Conversely, non-negative curvature

$\Rightarrow \quad \rho \leq \ell^n$

Detailed check : $\mathcal{M}_n \equiv \mathbb{H}_n / \Gamma$

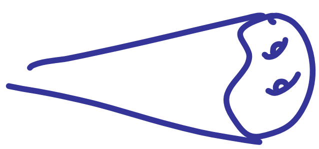
compact hyperbolic

simplest background solving vacuum equations (candidate worldsheet CFT)

$$ds^2 = -dt^2 + t^2 \, ds^2_{\left\{ \begin{matrix} \mathbb{H}_n \\ \mathbb{H}_n/\Gamma \end{matrix} \right.}$$

covering space = Minkowski space

compact hyperbolic $\mathcal{M}_n$



Late times $t \gg \sqrt{\alpha'}$: $H^2 = \left(\frac{\dot{a}}{a}\right)^2 \ll \frac{1}{\alpha'}$

semiclassical control

	UV	$\leftrightarrow$	IR	
$\mathbb{H}_n$:	$c_{eff} = c_{eff}^{crit}$		$m^2_{min} = 0$	spatial gap
$\mathbb{H}_n/\Gamma$	$c_{eff} > c_{eff}^{crit}$	$\rho(\ell) \sim e^{\ell/\ell_0}$	$m^2_{min} < 0$	no gap

IR) On $\mathbb{H}_n$: gap in spatial modes

$$\nabla^2_{\mathbb{H}_n} \eta = -k^2 \eta \qquad \text{Normalizeability} \Rightarrow \underline{\underline{k^2 > \frac{(n-1)^2}{4t^2}}}$$

$$ds^2_{\mathbb{H}_n} = dy^2 + \underbrace{\sinh^2 y}_{\text{exponential growth of volume}}\, ds^2_{S^{n-1}}$$

Compensated by t evolution

$$\nabla^2 \eta_k = \frac{1}{t^2}\left(-(t^2\partial_t^2 + n t \partial_t) - k^2\right)\eta_k$$

Complete basis $\eta_k(t) = t^{\frac{1-n}{2}} e^{i\omega \ln t}$

in terms of which the 1-loop amplitude is ($\hat{\omega} = i\omega$)

$$\int dt\, dy \cdots \sqrt{G} \int d\hat{\omega} \int_{k^2 > \frac{(n-1)^2}{4}} dk \cdots \int \frac{d\tau_2}{\tau_2}\, e^{-\pi\alpha' \frac{\tau_2}{t^2}\underbrace{\left(\hat{\omega}^2 + k^2 - \frac{(n-1)^2}{4}\right)}_{m^2_{min} = 0\ \checkmark}}$$

On $\mathcal{M}_n = \mathbb{H}_n/\Gamma$: compact,

$k = 0$ mode is Normalizeable

$$\Rightarrow \boxed{M^2_{min} = -\frac{(n-1)^2}{4t^2}}$$

psendo-tachyon : $\rho_T \propto \frac{1}{t^{2n}} << \left(\frac{\dot{a}}{a}\right)^2 \propto \frac{1}{t^2}$

UV) $$m^2_{min} = -\underbrace{\frac{(n-1)^2}{4t^2}} + \text{modular invariance}$$

requires $$c_{eff} = c_{eff}^{crit} + \underbrace{\frac{3\alpha'(n-1)^2}{2t^2}}$$

This is precisely matched by the winding spectrum.

cf Selberg trace

$$\mathrm{Tr}\, q^{L_0}\bar{q}^{\tilde{L}_0}\Big|_{\substack{\tilde{\tau}_2 \to 0 \\ \tilde{\tau}_2 = 1/\tau_2}} \sim \int d\ell\, \underbrace{\frac{1}{\left(\sinh\frac{\ell}{2t}\right)^{n-1}}}\, \rho(\ell)\, e^{-\frac{\alpha'\ell^2}{4\pi\tilde{\tau}_2}} \times \underbrace{e^{\frac{\pi c_{eff}}{6\tilde{\tau}_2}}}_{\text{usual oscillators}}$$

$$\rho(\ell) \sim e^{(n-1)\frac{\ell}{t}}$$

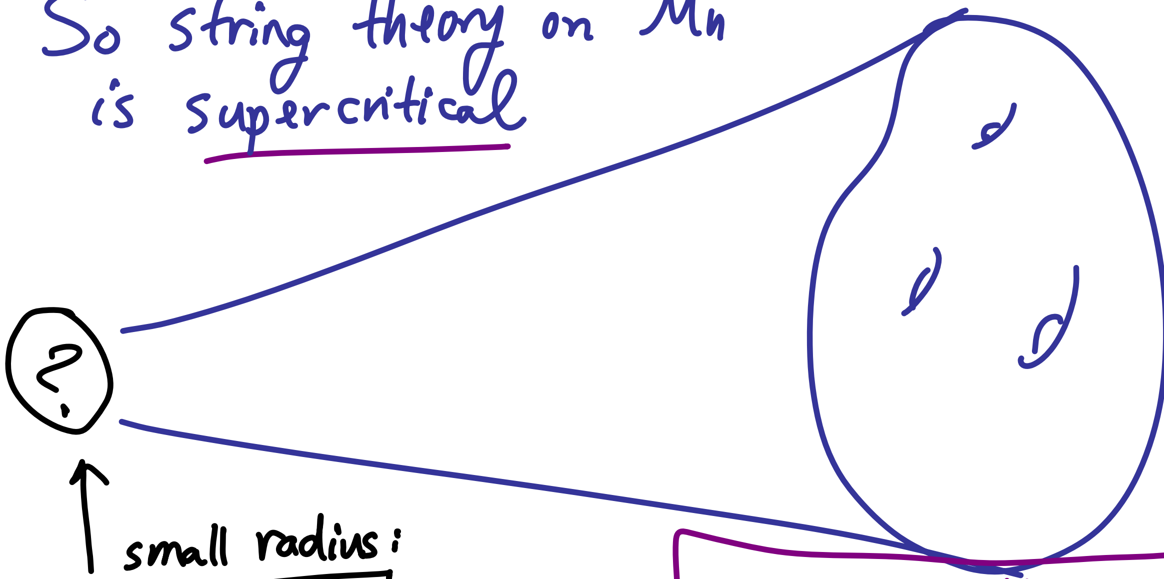
transverse motion

$$\left(\sum_{N=0}^{\infty} e^{-(N+\frac{1}{2})\frac{\ell}{t}}\right)^{n-1}$$

(cf Berkooz et al)

Harmonic oscillator in the N-1 transverse directions

So string theory on M_n is supercritical



small radius:
worldsheet nonlinear σ-model strongly coupled.
Is there a better, dual set of variables in terms of a more familiar presentation of supercritical string theory?

Myers
Polchinski
Bachas et al
Cooper/Susskind...

Large radius:

$$C_{eff} = C_{eff}^{crit} + \frac{3\alpha'(n-1)^2}{2t^2}$$

cf Non-perturbative string dualities

e.g. $IIA \xrightarrow[g_s \to \infty]{} 11\,d$ Hull/Townsend, Witten, ..

indicated by D0-brane spectrum

Here, the enhanced C_{eff} arises from perturbative winding strings

- $\mathcal{M}$ has a conserved winding symmetry $U(1)^{b_1}$ $\quad b_1 = \dim H_1$

$\hookrightarrow$ worldsheet theory has b_1 <u>indenpendent</u> conserved currents $J^I \equiv *\partial X^I \quad I=1\cdots b_1$ (where scalars X^I not free fields in general) $\rightarrow$ b_1-dimensional description

- D-brane on $\mathcal{M}$ has T^{b_1} classical moduli space

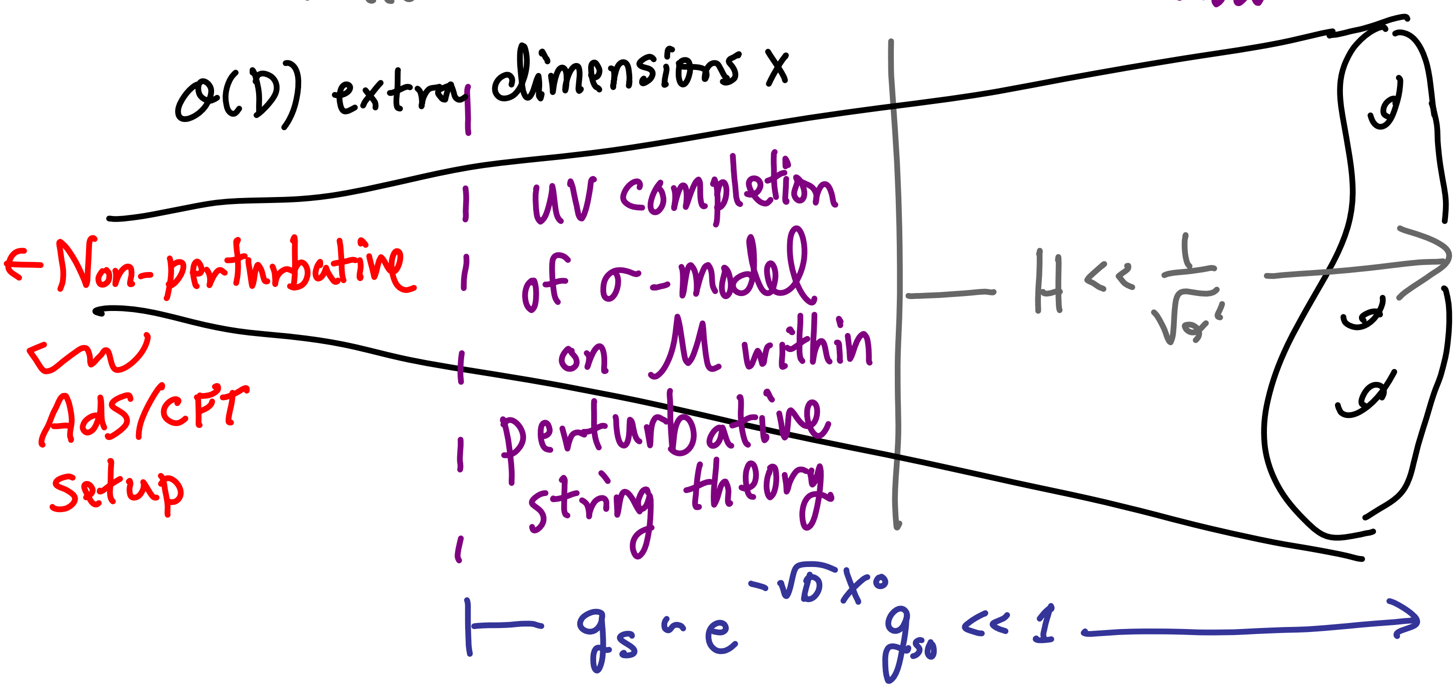
$\rightarrow$ Is there a formulation in terms of a T^{b_1} ?

<u>Yes</u>, in fact can show that the minimal possible completion preserving the $U(1)^{b_1}$ winding symmetries contains a T^{b_1} :

Start by considering a regime where time evolution is well approximated by RG flow: large dilaton friction in timelike linear dilaton background:

$$\ddot{\eta} + \underbrace{\dot{\eta}\,\underbrace{\dot{\Phi}}_{\sqrt{D-D_c}} = \beta_\eta}_{\text{1st order RG flow}}$$

Cooper/Susskind, Polchinski, Bachas et al, Freedman et al



$g_s \sim e^{-\sqrt{D}X^0} g_{s0} \ll 1$

dilaton slope adjusts as $\downarrow D_{eff}$

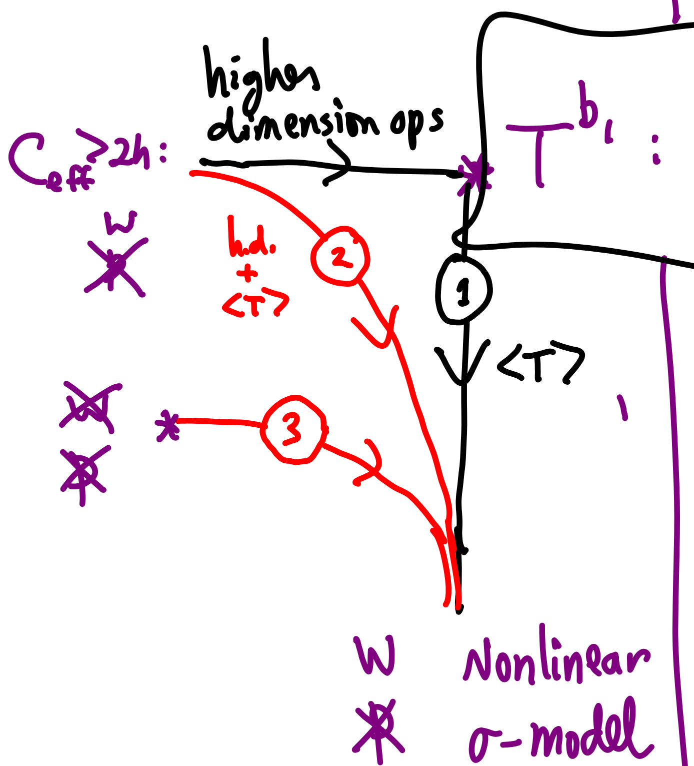
cf Liu Hellerman

A priori, there are many ways to UV complete the sigma model on $\mathcal{M}$ $\leftrightarrow$ Choices of initial conditions

Schematically:

$w \equiv$ winding symmetry

$p \equiv$ momentum symmetry



Minimal completion preserving $U(1)^{b_1}$ winding symmetry:

- minimal ⇒ reaches compact CFT as UV fixed point
- winding symmetry: spacetime gauge symmetry

Vertex operators

$$V^I_\mu = e^{ik\cdot Y}\left(\partial Y^\mu \bar{J}^I - J^I \bar{\partial} Y^\mu\right)$$

$I = 1 \dots b_1$

$$\left.\begin{array}{l} J^I \text{ primary } (1,0) \Rightarrow \bar{\partial} J = 0 \Rightarrow J^I = \partial X^I \\ \bar{J}^I \text{ primary } (0,1) \Rightarrow \partial \bar{J} = 0 \Rightarrow \bar{J}^I = \bar{\partial} X^I \end{array}\right\} X^I \text{ free, compact} \Rightarrow \boxed{T^{b_1}}$$

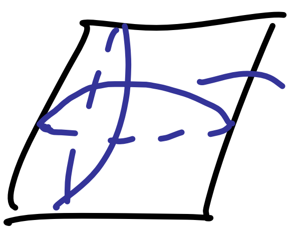
In the case of a genus h Riemann surface, the Jacobian torus T^{2h}

$$X^a = X^a + \oint_{a^b, \beta^b} \omega \qquad \forall b = 1 \cdots h$$

holomorphic 1-forms

is a natural starting point, with the transition $T^{2h} \xrightarrow[\langle T \rangle]{}$ Riemann Surface M mediated by a closed string tachyon condensate ($\cong$ worldsheet potential term) $T(X, X^0)$ restricting the worldsheet to $M \subset T^{2h}$ embedded via

$$\text{Abel-Jacobi} \quad X^a = \int_{z_0}^{z} \omega^a$$



$H_1(T^{2h}) \cong H_1(M)$

winding modes $\Updownarrow$ winding modes

Explicit dual pairs a la Buscher

Alvarez-Gaume et al
Giveon/Rabinovici
Rocek/Verlinde
Hori/Vafa Morrison/Plesser
Tong ...

e.g. (2,2) σ-model (dressed with RG time)

X, Z, $\Sigma = D_+ \bar{D}_- V$, $\check{\Sigma} = D_+ \bar{D}_- \check{V}$ twisted chiral

$$S_{GSO} = \int d^2\check{\theta}\, \tfrac{1}{2}\gamma_{ab}\left[\check{\Sigma}^a\left(X^b - \int^Z \omega^b\right) - \Sigma^a X^b\right] + \text{h.c.}$$

$$+ \int d^4\theta\, \gamma_{ab}\left[\tfrac{1}{2}V^a V^b - \frac{1}{2e^2}\check{\Sigma}^{a\dagger}\check{\Sigma}^b + \varepsilon Z^\dagger Z\right]$$

$\int DX$ first

$\int DV$ first

$$S_{RS} = -\int d^4\theta \left|\int^Z \omega\right|^2 + \mathcal{O}\left(\frac{\partial^2}{e^2}\right)$$

$$= \int d^2\sigma\, \gamma_{ab}\, \omega^a \partial_\sigma z\, \bar{\omega}^b \partial_\sigma \bar{z} + \ldots$$

$$\check{S} = \int d^2\sigma\, \gamma_{ab}\left\{\partial X^a \partial \bar{X}^b + \underline{e^2|\check{\sigma}|^2} + \underline{e\check{\sigma}\cdot\omega F_z} + \underbrace{\left(X - \int^z \omega\right)^a \check{F}^b + \frac{F}{e^2}}_{\to\, e^2\left(X^a - \int^z \omega^a\right)\left(\bar{X}^b - \int^{\bar{z}} \bar{\omega}^b\right)} + \ldots\right.$$

Caveats

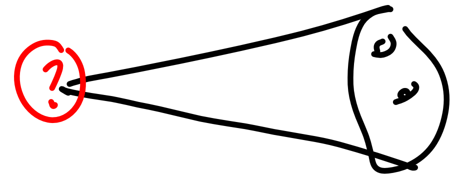
- To get type II GSO, we used extra multiplets $\check{\Sigma}$ in addition to $X \subset T^{2h}$

cf Hellerman/Swanson classification

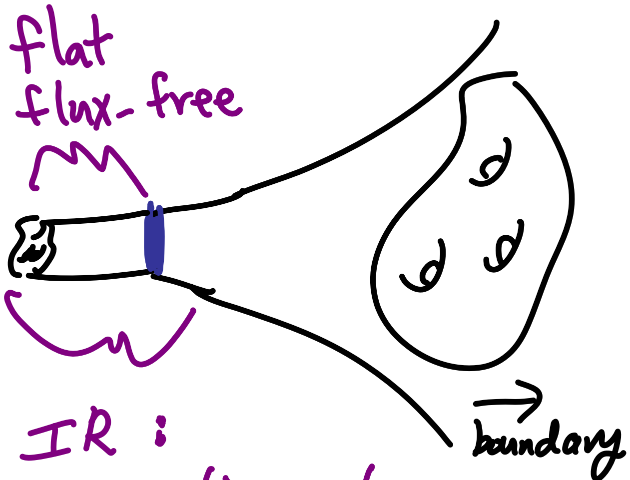
- Tachyon $T(X, \check{\sigma})$ is a nontrivial function of spatial position; need initial profile strong enough to avoid large particle production cf Nakayama

- Generic GSO-preserving $T(X, \check{\Sigma})$ gives $\underline{\underline{a}}$ Riemann surface. We further assume symmetry principle of unbroken $U(1)^{b_1}$.

D-duality and AdS/CFT :



flat
flux-free



$\cong$ CFT on $M_3 \times$ time out on Coulomb branch

Krans
Larsen
Trivedi

IR :
Wilson lines trace out

$$\boxed{(T^{b_1})^{S_N} / \mathbb{Z}_N}$$

$\left(\text{masses} \ll \frac{1}{L_{M_3}} \right)$

↑ appears naturally in IR

---

* 2 Conformal frames : cf Witten/Yau
Berenstein

$$ds^2 = d\eta^2 + ds^2_{\mathbb{H}_3/\Gamma}$$

$R\alpha'^2 < 0$

$\eta \to +\infty$ instability

$$\boxed{\eta \to -\infty \quad \text{no singularity}}$$

$$ds^2 = -dt^2 + t^2 ds^2_{\mathbb{H}_3/\Gamma}$$

$R = 0$

$$\boxed{t \to \infty \quad \text{FRW}}$$

$t \to 0$ singularity

Further Developments & applications

- "g theorem" : The modular invariance relation

UV $\longleftrightarrow$ IR

Enhanced $\rho(m)\Big|_{m \gg \frac{1}{\sqrt{\alpha'}}} \propto m^{\gamma} e^{\frac{m}{m_0}}$ Hubble expansion (affects IR spectrum)

suggests general lesson:

sufficiently strong Hubble expansion requires more worldsheet degrees of freedom (either via exponential or enhanced power law growth of $\rho(m)$)

$\hookrightarrow$ e.g. Nil 3-manifold $\xleftrightarrow{\text{T-duality}}$ H flux on T^3

negative scalar curvature (Kachru Schulz Triwedi)

$\rho(\ell) \sim \ell^4 > \rho(\ell)\Big|_{\text{non-negative}}$ $\rho(n) \sim n^4$

windings momenta (Landau levels)

(MS, ES) (Maloney)

- Model building:

Genericity + utility for dS construction motivates further exploration

- $\mathcal{N}=0$ & $\mathcal{N}=1$ phenomenologically viable
- inflation in metastabilized $\mathcal{M}$

cf Saltman/ES '04

- Mathematical connections (?)

– positive "virtual" b_1 conjecture
(b_1 of finite cover)

– absence of $\pi_1(\mathcal{M})$ of intermediate growth $\ell^\gamma < \rho < e^{\frac{\ell}{\ell_0}}$

– string-theoretic interpretations of trace formulas

$$\bigcap$$

$$Z_{\text{1-loop}}$$

(like Poisson resummation)

($\Leftrightarrow$ chaotic dynamics, ζ, ...)

<u>D-Duality</u> between "critical" strings

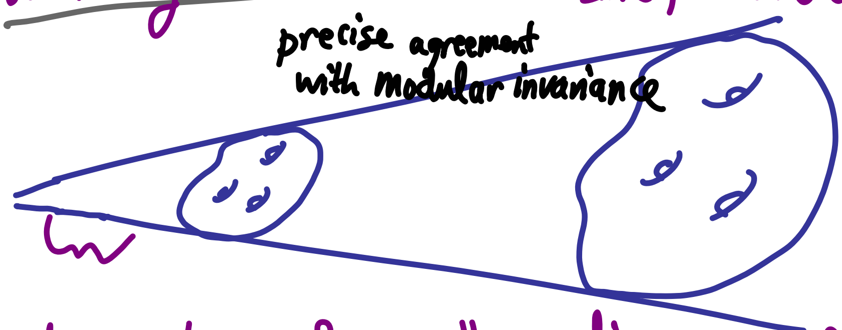
on a compact negatively curved space M_n

and supercritical strings $\left(c_{eff} > c_{eff}^{crit}\right)$

Dimensions emerge from topology

① effective central charge $c_{eff} > c_{eff}^{critical}$ from <u>exponential</u> <u>density</u> of <u>winding states</u> $\longleftrightarrow$ IR perturbation spectrum

precise agreement with modular invariance



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