THE SEARCH FOR THE HOLOGRAPHIC DUAL OF THE HETEROTIC STRING WORLDSHEET CFT

Josh Lapan, Aaron Simons & AS

IN PROGRESS

see also

Dabholkar & AS (unpublished)
Castro, Davis, Kraus & Larsen '07
Giveon & Kutasov '06

STRINGS '07 MADRID

OUTLINE

1) The question

2) Small black holes and small black strings

3) SUGRA analysis

4) W algebras and non-linear SCAs

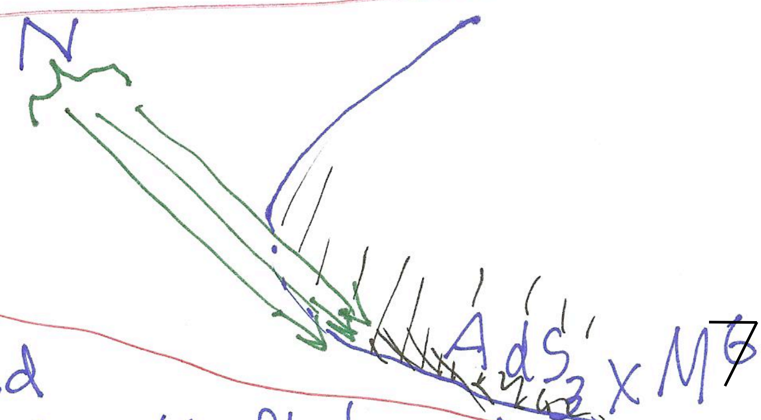
5) Worldsheet analysis

6) Closing comments

This is work in progress. The problem is not sewed up, but on the road several interesting characters already encountered.

THE QUESTION

Is there a spacetime solution of string theory holographically dual to the $(c_L, c_R) = (24N, 12N)$ 1+1 CFT living on N stretched, coincident heterotic strings — and if so, what is it?



First quantized N-stretched-string (in flat space) Hilbert space

=

Second-quantized heterotic string theory Hilbert space on $AdS_3 \times M^7$

Why should we expect such a beast?

i) General Maldacena scaling type argument $\Rightarrow$
~~nn~~horizon geometry $\Leftrightarrow$ LEFT on defect
In dual type I picture
$\rightsquigarrow$ open/closed duality.
Because heterotic ws is a CFT, expect AdS_3 factor.

ii) As we will see, surprising success of Dabholkar's (OS) small black hole picture would be explained (is implied?) by such a dual for T^5 compactifications. Henceforth largely specialize to ~~T8~~. T^5.

Some Ancient Facts

Gibbons Dabholkar Harvey & Ruiz-Ruiz '90

The leading SUGRA solution for N stretched heterotic strings in a T^5 compactification is

$$ds_{10}^2 = \frac{dx^+ dx^-}{\left(1+\frac{N}{r}\right)} + \underbrace{du^2 + dv^2 + dw^2 + \sum_{a=1}^{5} dx^a dx^a}_{\text{Flat!}}$$

T^5

$$e^{\Phi} = g_\infty \left(1+\frac{N}{r}\right)^{-\sqrt{\frac{2}{3}}}$$

$$r^2 = u^2 + v^2 + w^2$$

NOTE:

1. $\frac{1}{2}$ BPS $\Rightarrow$ 8 susies

2. Near-horizon $r \to 0$:

   i) $g_s \to 0$

   ii) $r(S^2) \to 0$

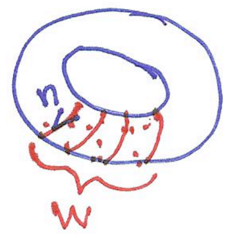
   $\Rightarrow$ α' corrections important ~~SUGRA~~ classical string theory good

Suggest worldsheet CFT methods applicable near-horizon.

No RR fields!

Small 4D BHs and why they are black

$\exists$ $\frac{1}{2}$ BPS DH states with (momentum, winding) = (n, w) on an S^1 in T^6,



and asymptotic degeneracies

$$\boxed{e^{S_{DH}} \sim e^{2\pi\sqrt{nw}}}$$

$\neq$ macroscopic BH ($S \sim Q^2$)

However, classical limit is

$$\hbar \to 0 \,;\quad w,\ \hbar n \ \text{fixed}$$

$$\Rightarrow \quad S_{DH} \sim \frac{1}{\sqrt{\hbar}} \,;\quad \frac{\delta S}{\delta w} \sim \sqrt{\frac{n}{w}} \sim \frac{1}{\sqrt{\hbar}} \to \infty$$

energy/light can't escape

$\Rightarrow$ event horizon

$\Rightarrow$ Exact string sol'n should have horizon

Small BH entropy computation

Dabholkar 05
Denef Dabholkar Moore & Pioline 05

Use IIA($K3 \times T^2$)/Het(T^6) map to $K3 \times S^1$-wrapped NS5. Entropy is

$$S_{BH} = 2\pi\sqrt{(\underbrace{D_{ABC}p^A p^B p^C}_{0} + \underbrace{c_{2A}p^A}_{w})\underbrace{q_0}_{n}}$$

$$= 2\pi\sqrt{nw}$$

Perfect match (and much more).

Beautiful but fishy!

Something must be up.

Suggests α' corrections give BH solution.

α' corrected solutions

Sen 06
Dabholkar Kallosh & Maloney 05

$R + R^2$ string gravity in 4D
has solutions with stringsized
near-horizon $AdS_2 \times S^2$ region.

But field redefinitions
$g_{\mu\nu} \to g_{\mu\nu} + \alpha' R_{\mu\nu}$.
takes nonsingular
$\to$ singular

Still, something looks right.
Would all make sense if
$\exists$ exact WS CFT.
No RR fields, lots of SUSY.

Small 4D black holes

→ Small 5D black strings

Near-horizon geometry

$\sim$ (KK U(1) fibered over AdS_2) $\times S^3$

$\sim$ (quotient of AdS_3) $\times S^3$

$\Rightarrow$ cover $= AdS_3 \times S^3$

$=$ near-horizon of 5D black string

So we turn to the simpler problem of ~~5D~~ the near-horizon geometry of stretched heterotic string in 5D.

This is why we specialized to T^5.

5D BPS small black strings

Castro, Davis, Kraus & Larsen '07

5D $N=4$ sugra w/ R^2 corrections has susy transformations

$$\delta\psi_{\mu i} = \nabla_\mu \varepsilon_i + \frac{1}{2}\Gamma_\mu{}^{\nu\lambda} T_{\nu\lambda}\varepsilon_i - \frac{1}{3}\Gamma_\mu \Gamma^{\alpha\beta} T_{\alpha\beta}\varepsilon_i + \ldots$$

$$\delta\chi = D\varepsilon_i - \Gamma^\mu \varepsilon_{\mu\nu\rho\lambda\sigma} T^{\rho\nu} T^{\lambda\sigma}\varepsilon_i + \ldots$$

D = aux. field

$T_{\mu\nu}$ = aux. fied $\to$ graviphoton

R^2 corrections appear mainly in aux. field e.o.m

RESULT

$\exists \frac{1}{2}$ BPS solutions with charges of N het. strings, $g_s^2 \sim \frac{1}{N}$ & $AdS_3 \times S^2$ near horizon.

A PUZZLE

What is the near horizon super-isometry group?

Must have 16 susies

$\varepsilon_i\,,\ \eta_i \equiv \mathcal{L}_K \varepsilon_i$

and contain $SL(2,R)$.

4 possibilities

	R-symmetry
$Osp(4^*\|4)$	$SU(2)\times Sp(4)$
$SU(1,1\|4)$	$SU(4)\times U(1)$
$F(4)$	$SO(7)$
$Osp(8\|2)$	$SO(8)$

But none of these groups extend to ~~~~ standard SCA. Biggest has 8. String-independent puzzle.

Brute computation

yields $Osp(4^*|4)$

SU(2) Killing vector

$$[\delta_\xi, \delta_\eta] \sim \bar{\xi}^i \Gamma^\mu \eta_i \nabla_\mu + \bar{\xi}^i \eta_j R^j{}_i + \ldots$$

Sp(4) R-symmetry generator

From 10D, $Sp(4) \sim SO(5) \sim T^5$ spin frame rotations. Not geometrical.

No group available for T^4, T^3!

Could

$$SL(2,R) \times SO(8) \in Osp(8|2)$$

arise from $AdS_3 \times S^7$ solution of $D=10$ stretched heterotic string?

Affine extension

According to Brown & Henneaux

$(L_{-1}, L_0, L_1) \in SL(2R)$ of AdS_3

$\to L_n$ Virasoro

= Asymptotic Symmetry Algebra

But

$Osp(4^*|4) \to$???

$\exists$ "nonlinear SCA's or "W-algebras"

containing $Osp(4^*|4)$

Knizhnik '86
Bershadsky '86

$$[L_m, L_n] = (m-n) L_{m+n} + \frac{c}{12} m^3 \delta_{m+n}$$

$$\{G^\alpha_m, G^\beta_n\} \sim 2\delta^{\alpha\beta} L_{m+n} + (m-n) R^{\alpha\beta}_{m+n} + \underbrace{\sum_p (R^{\alpha}{}_{\gamma})_{m+n-p} R^{\gamma\beta}_p}_{\text{non-linear term}} + \ldots$$

$SU(2)\times Sp(4)$ current

...

This "$\widehat{Osp}(4^*|4)$" and cousins have not had many applications in strings or elsewhere.

W algebra = ASA of AdS_3

Henneaux, Maoz & Schwimmer '99

AdS_3 admits boundary conditions for which $ASA = \widehat{Osp}(4^*|4)$.

So it all seems to fit so far

1) With R^2 corrections, N stretched heterotic strings have $AdS_3 \times S^2 \times T^5$ near horizon geometry
2) Superisometry group = $Osp(4^*|4)$
3) $ASA = \widehat{Osp}(4^*|4)$ (reps?)
   W-algebra

Note: no obvious consistent story for T^3 or T^4 compactification, but also no small black hole (string) story. Dimension specific.

COMMENT

The situation with unitarity of $\hat{O}sp(4^*|4)$ reps is unclear (at least to us). There are probably larger W algebras (w/ higher spin fields) with global $Osp(4^*|4)$ subgroups, but it is not apparent that these can be an ASA for AdS_3.

SO WHAT IS THE EXACT WS CFT FOR A MICROSCOPIC HETEROTIC STRING IN THE NEAR-HORIZON $AdS_3 \times S^2 \times T^3$ GEOMETRY?

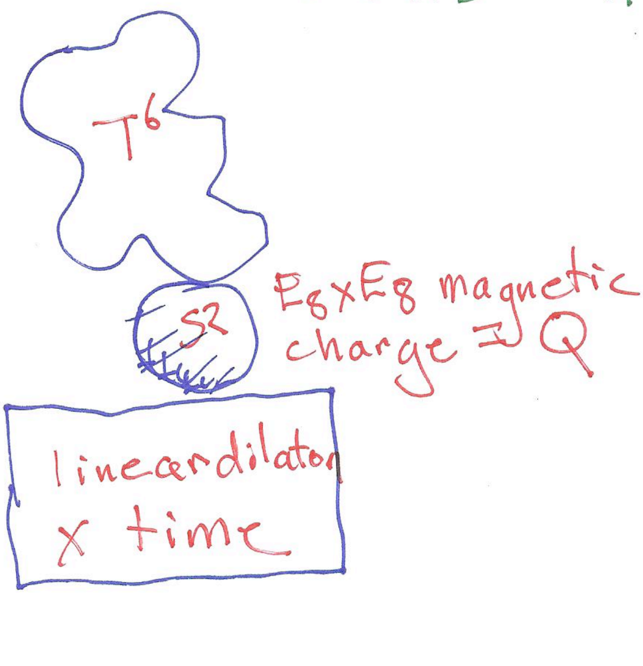
How to get an S^2 CFT

S^3 factors are easy: c.f. $SU(2)_{WZW}$ of NS5 throat.

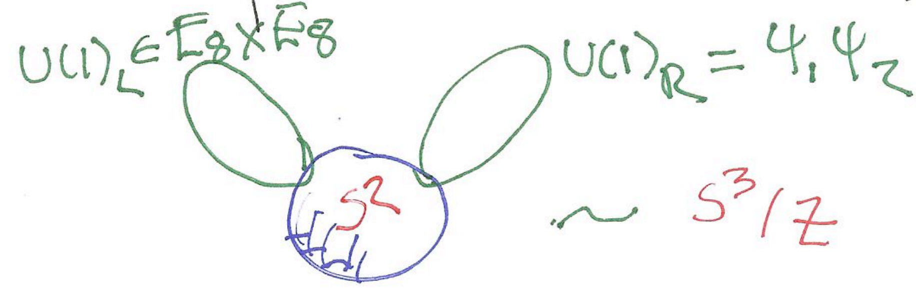
S^2 does not support H-torsion.

S^2 appears as near-horizon of 4D non-BPS magnetic black holes

Garfinkle, Horowitz and AS '91



Finding the S^2 WS CFT for the heterotic magnetic black hole was surprisingly subtle and difficult. It is an asymmetric orbifold,



$\sim S^3/Z$

of S^3:

$$\frac{SU(2)_{2Q^2-1}}{Z_{2Q+2}}$$

SURPRISE: a <u>neutral remnant</u> with <u>Q=0</u>

$$\frac{SU(2)_2}{Z_2^{\text{left}}}$$

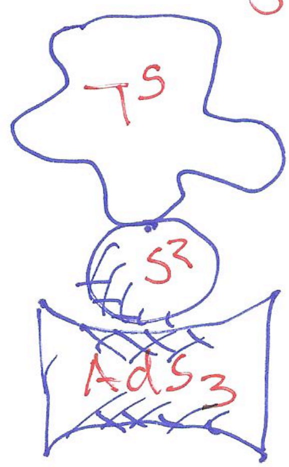
Giddings, Polchinski and AS '93

A Modification

$T^6 \to T^5$

R^2 linear dilaton $\to AdS_3 =$ (0,1) $SL2R_{k+2}$

$\Rightarrow$ 4D BH $\to$ 5D magnetic + fund string



Conjecture

$\boxed{\text{For } k=2 \text{ (neutral remnant)} \sim \text{near-horizon WS CFT for } N \sim \frac{1}{g_s^2} \text{ heterotic strings}}$

Comments

1) Related conjecture in
Giveon & Kutasov '07
(also Dabholkar & AS '05))

2) We dont yet have the
16 spin fields.

some very relevant literature
Giveon, Kutasov & Seiberg '98
Giveon & Pakman '03
describes the lift
worldsheet $^{\text{super}}$Virasoro
→ spacetime $^{\text{super}}$Virasoro
but must be altered for
nonlinear SCAs. Want
"stringy" Brown-Henneaux
formula.

Conclusion

Several arrows point to the existence of an $AdS_3 \times S^2 \times T^5$ dual for the CFT of N heterotic strings, with an exotic super-W algebra as an ASA. Several conjectures exist for an exact WS CFT description. More work remains.