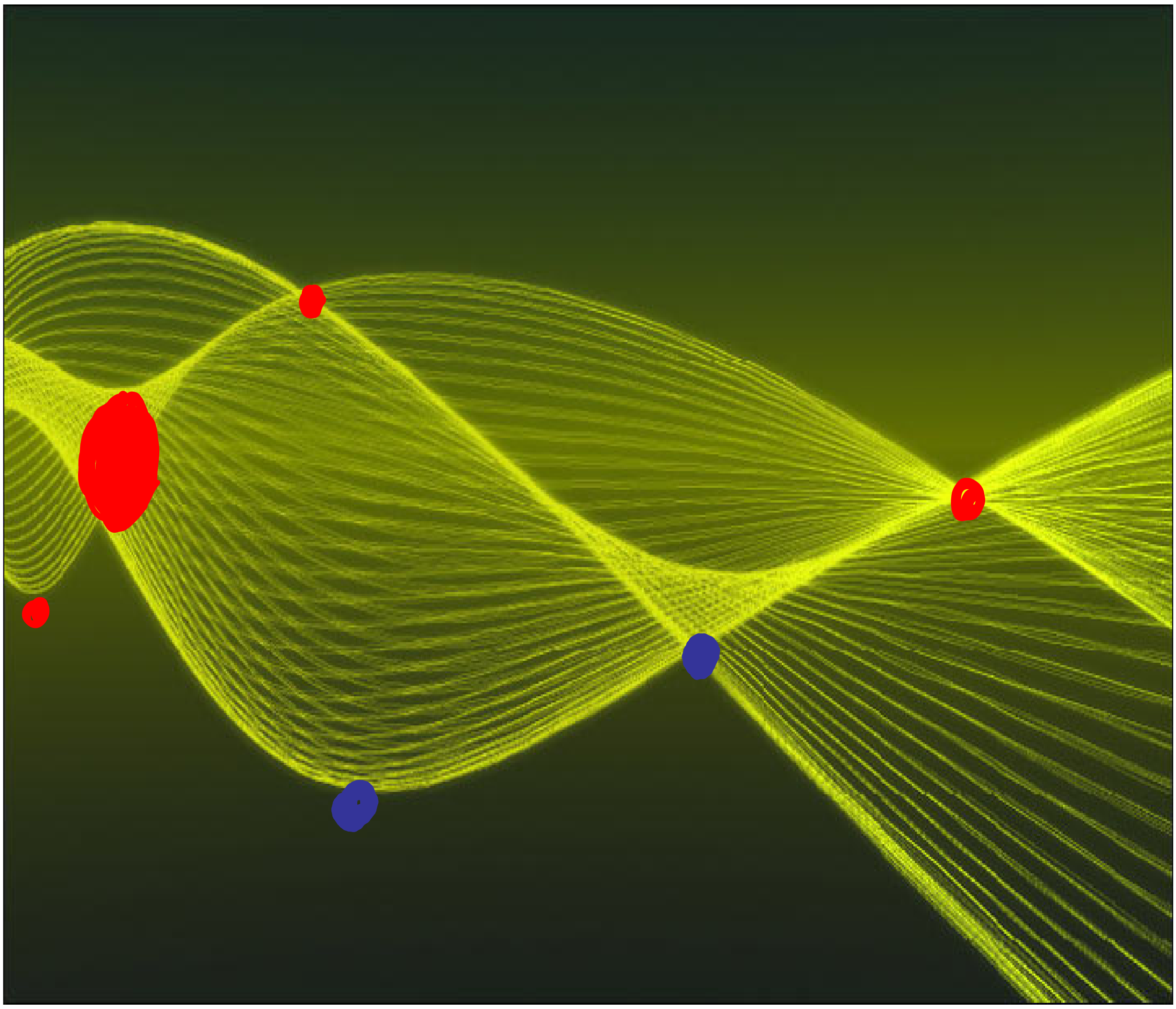
Geometric Metastability

Based on

M. Aganagic, C. Beem, J. Seo
arXiv:hep-th/0610249

J. Heckman, J. Seo + to appear
arXiv:hep-th/0702077



Breaking supersymmetry in string theory remains a challenge

recent Progress

- Construction of meta-stable vacua without susy
- Non-supersymmetric extremal black holes

focus of my talk

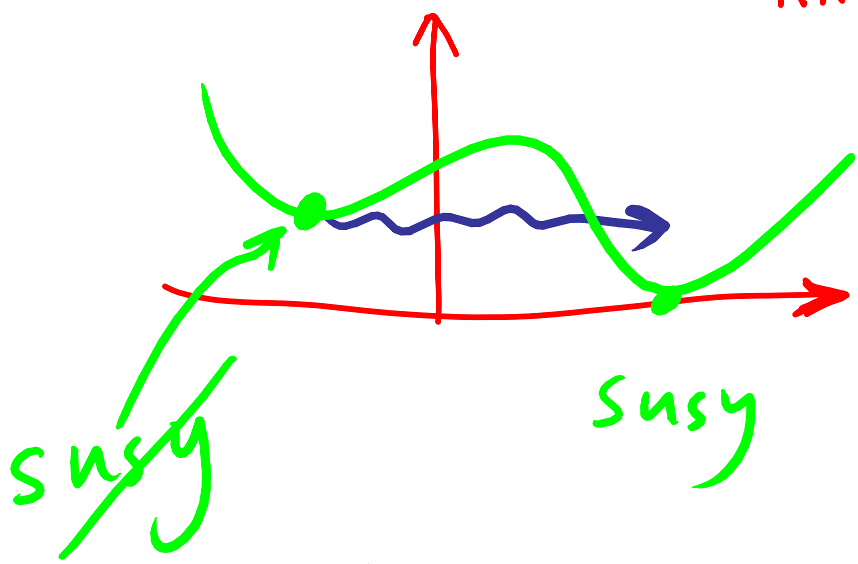
Geometry + Supersymmetry have played prominent role in the development of string theory. Here we study

Geometry → ~~Susy~~

+

implications of Large N duality

So far the only examples constructed in string theory to break susy involve the idea of meta-stable vacuum: KKLT, ...



* Abundant in field theory

Example 1: Ordinary non-supersymmetric $U(N)$ Yang-Mills in $d=4$ has $\mathcal{O}(N)$ vacua at large N.

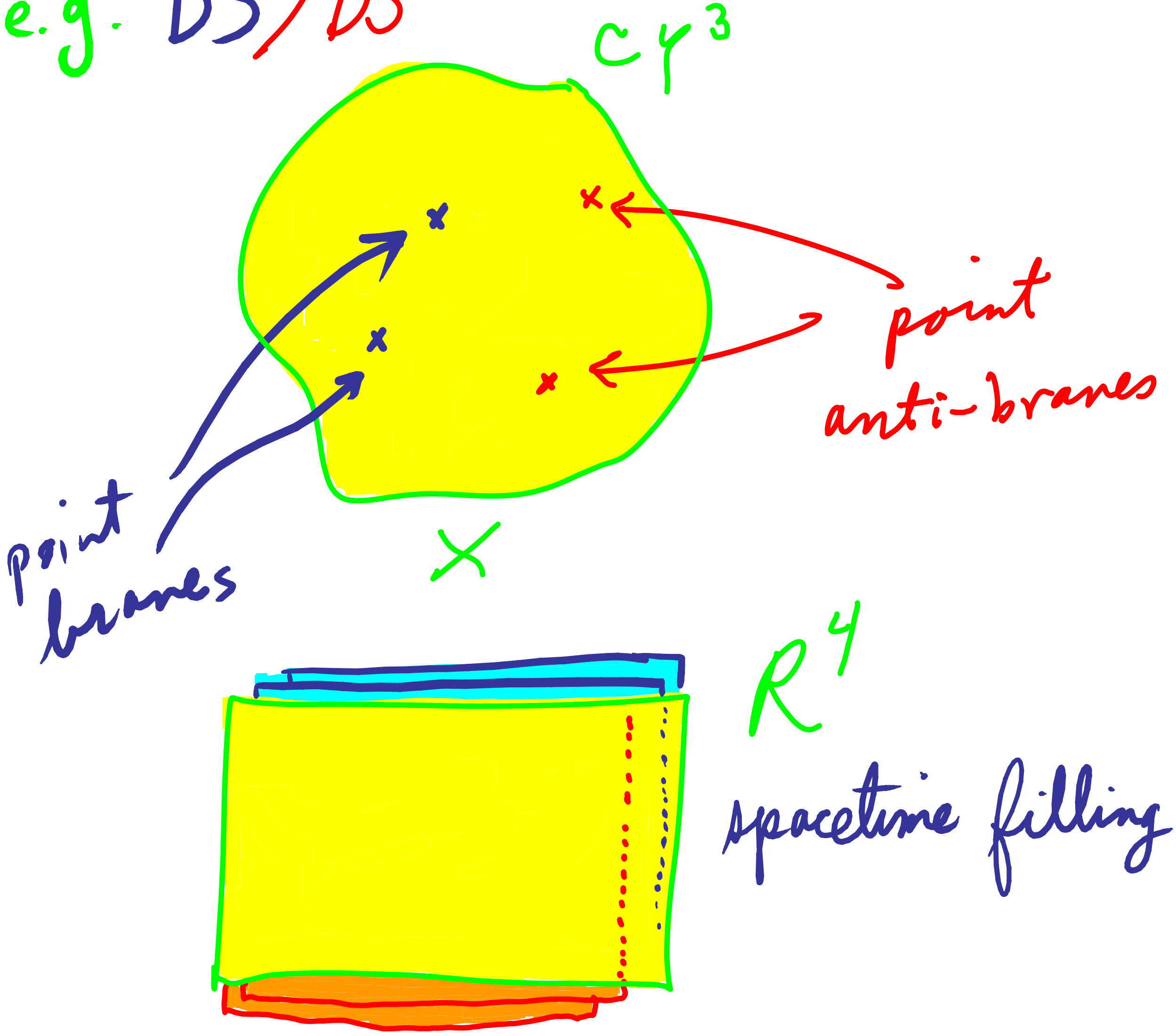
Witten '80

Example 2: Supersymmetric theories typically have many metastable vacua.

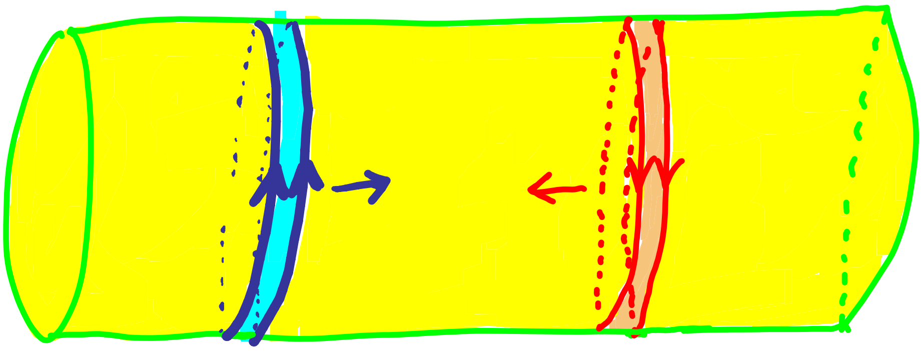
Intriligator, Seiberg, Shih

Perhaps the most natural stringy way to break susy is to use branes + anti-branes

e.g. $D3/\overline{D3}$



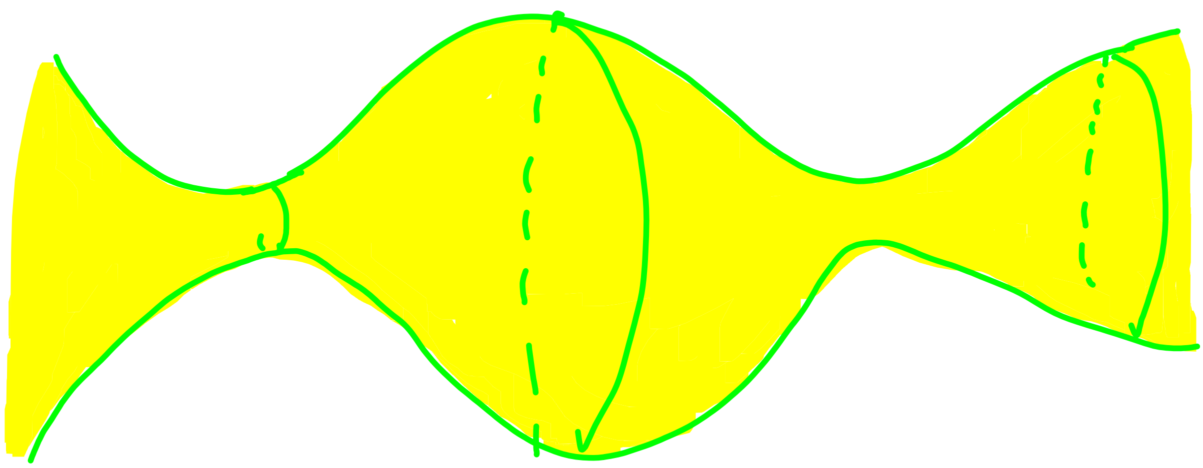
or higher dimensinal wrapped Brane/anti-branes



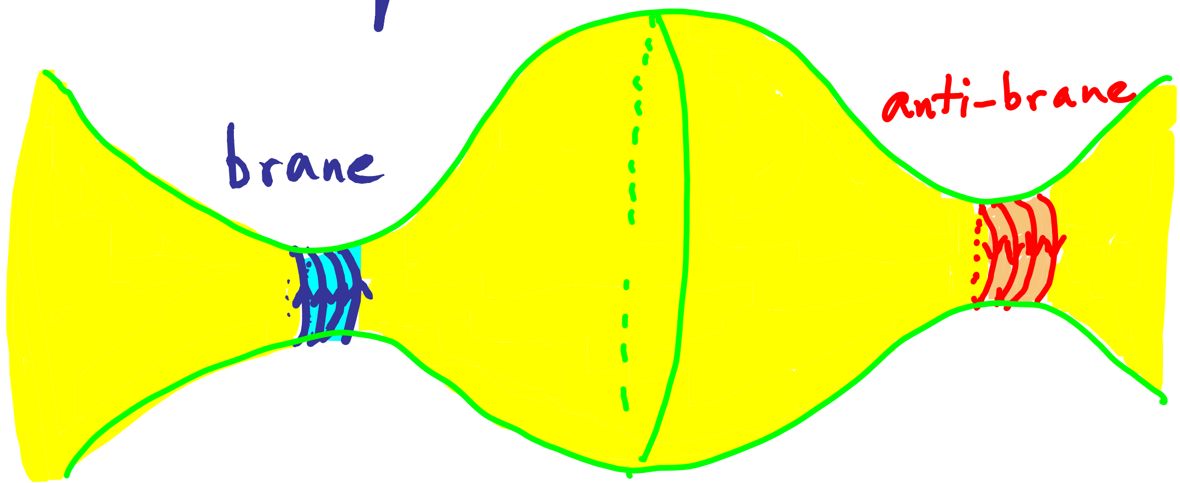
But these are unstable.

Instead,

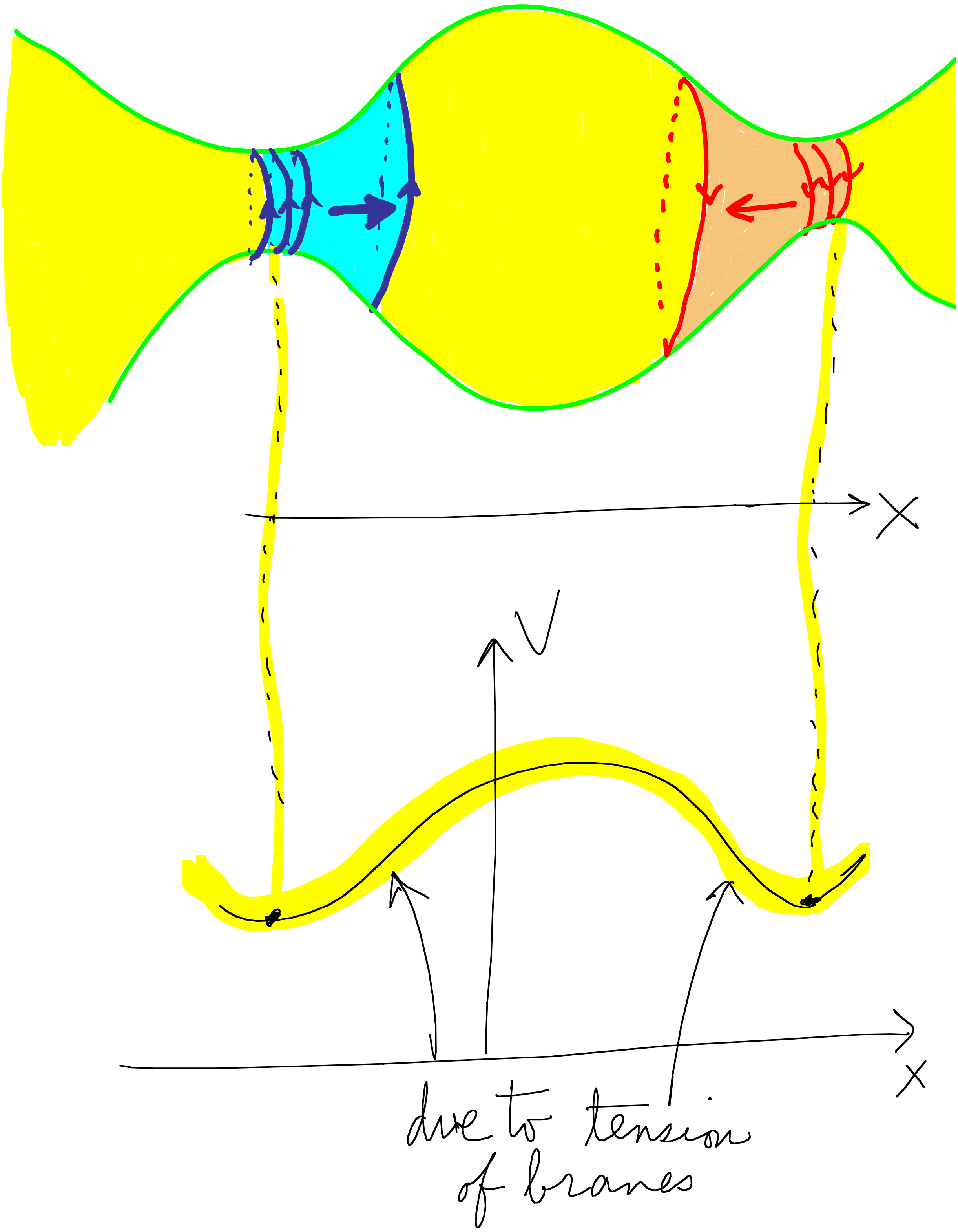
we look for geometries which look like:



and wrap branes + anti-branes



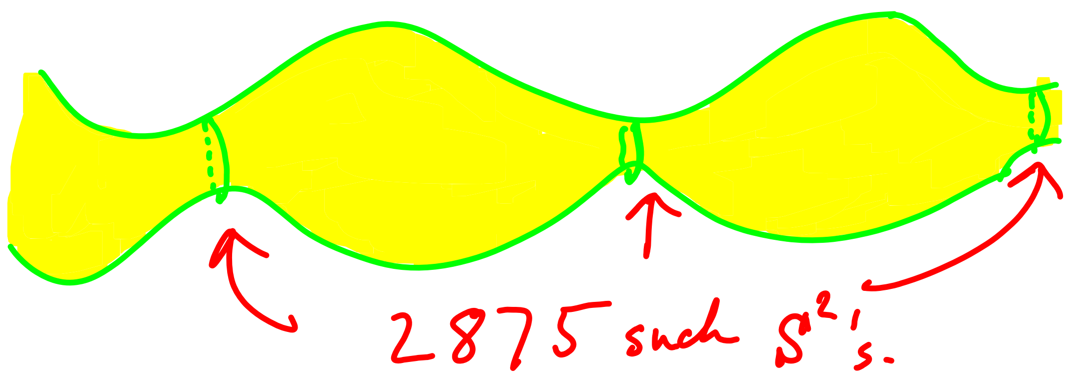
This will create a metastable configuration of branes and antibranes, where geometry plays the role of the potential barrier inducing this metastability.



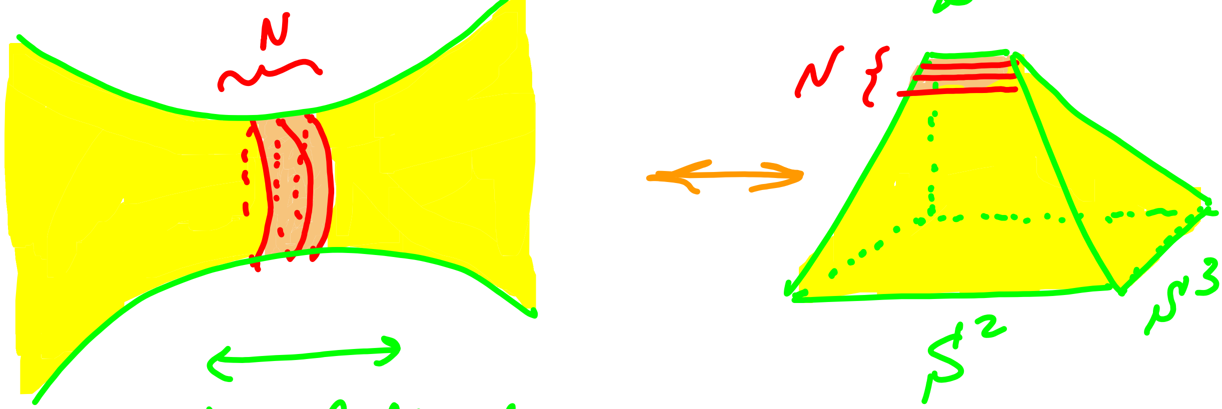
For this mechanism to work
there must be cycles which are

minimal + rigid (ie, isolated, with no continuous moduli), otherwise there will be flat/unstable directions. Luckily such isolated minimal cycles are generic for CY. For example the famous CY^3:

quintic $\longrightarrow$ 2875 isolated minimal S^2 in the same homology class



The simplest local (i.e. non-compact) geometry with rigid S^2 is the conifold geometry:

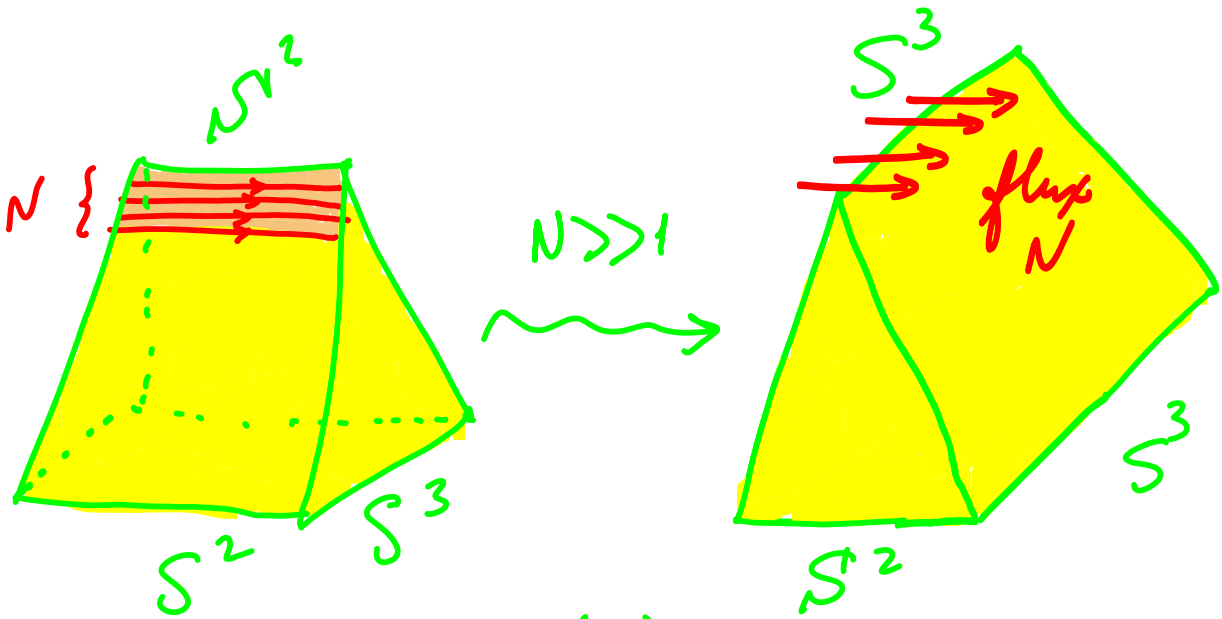


Normal direction to S^2

Type IIB D5-branes

Low energy Lagrangian involves a $U(N)$ gauge theory with $\mathcal{N}=1$ (CY breaks to $\mathcal{N}=2$, brane breaks $\frac{1}{2}$ more)

Large $N \gg 1$ dual description:



geometric transition
where branes replaced by flux

Klebanov-Strassler
Maldacena-Nunez
c.v.

There is a nice way to characterize the superpotential after the flux transition:

$$u^2+v^2+y^2+x^2=S$$

running coupling

$$W(S)=NS\left(\ln S/\Lambda_0^3-1\right)+\tau S$$

$$\left(= N \frac{\partial F_0(S)}{\partial S} + \tau S \right)$$

(special case of $n_i \Pi_i + m_i S_i$)

flux

periods of CY

$$\frac{\partial W(S)}{\partial S} = \frac{\partial \left(N S \left(\ln S/\Lambda^3 - 1 \right) + \tau S \right)}{\partial S}$$

$$= N \ln S/\Lambda^3 + \tau = 0$$

$$\Rightarrow \quad S = \Lambda^3 e^{-\tau/N}$$

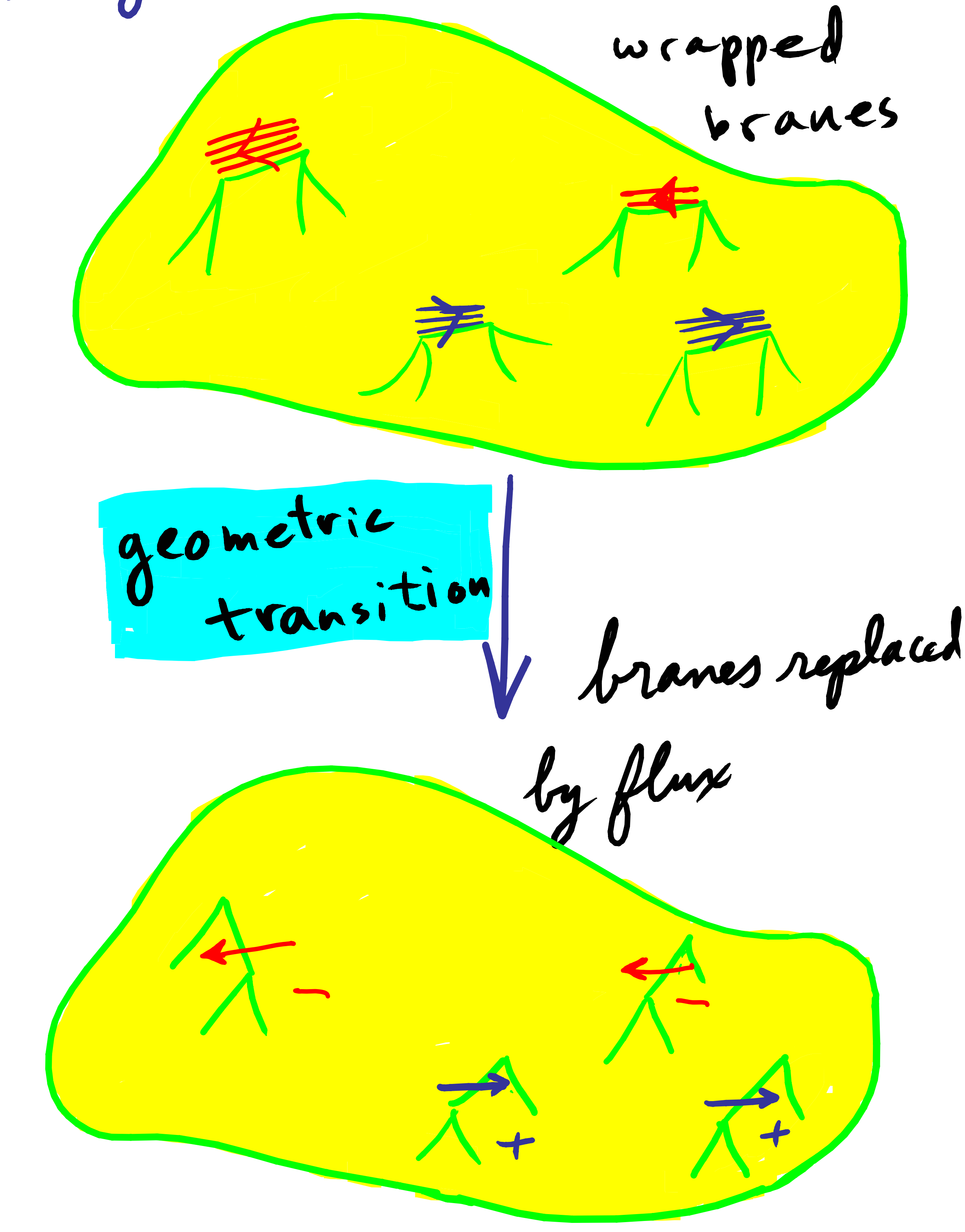
$\mathcal{N} = 1$ susy is preserved

($\partial W = 0$ exists)

$$S = \mathrm{Tr}\,\psi\psi \Rightarrow \langle S \rangle \neq 0$$

gluino condensation

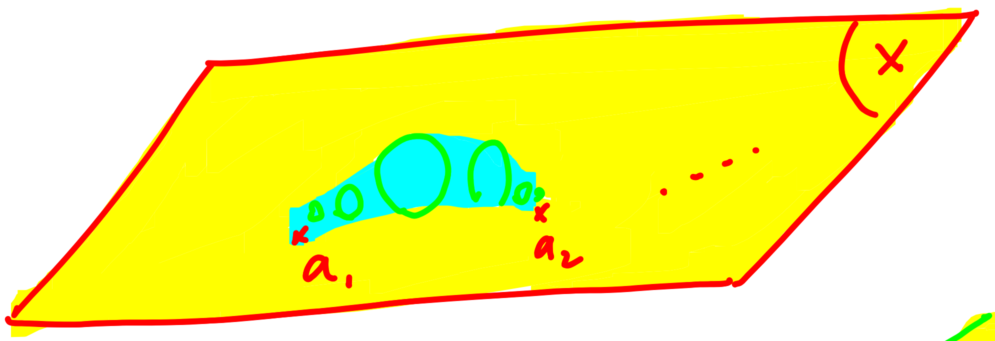
In our case we consider large N limit and conjecture

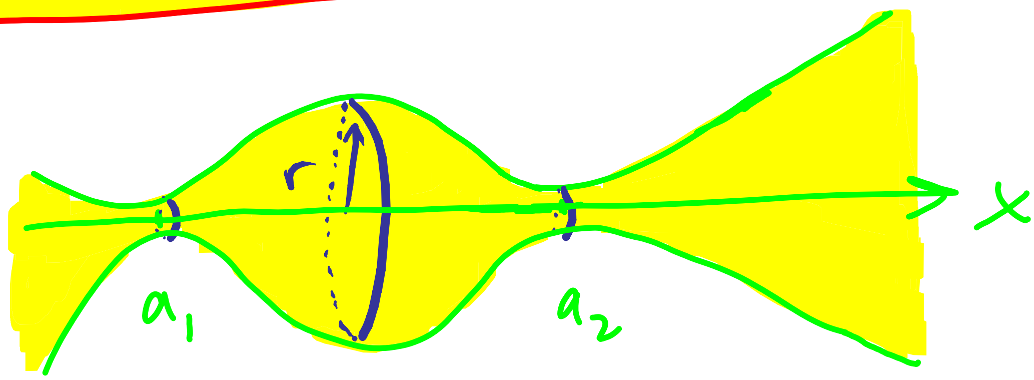


The geometries we study are non-compact CY given by hypersurface in $\mathbb{C}^4[u,v,y,x]$:

$$u^2+v^2+y^2=W'(x)^2$$

$$W'(x)=g\prod_{i=1}^{n}(x-a_i)$$





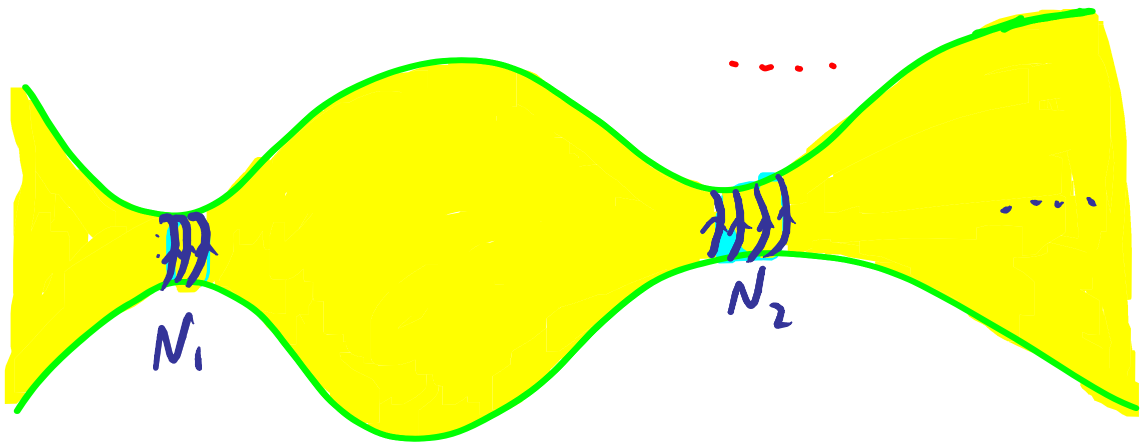
roughly $r(x)\sim|W'(x)|$

(At $x=a_i \to$ minimal S^2_i)

The case when only branes wrap cycles in this geometry has already been studied:

(Cachazo, Intriligator, V.)

(Dijkgraaf, V.)



$\Updownarrow$

$\mathcal{N}=1$ $U(N)$ gauge theory

Φ : adjoint field

$W(\Phi)$ = superpotential,

$$W'(\Phi = x) = W'(x)$$

$$u^2+v^2+y^2 = W'(x)^2$$

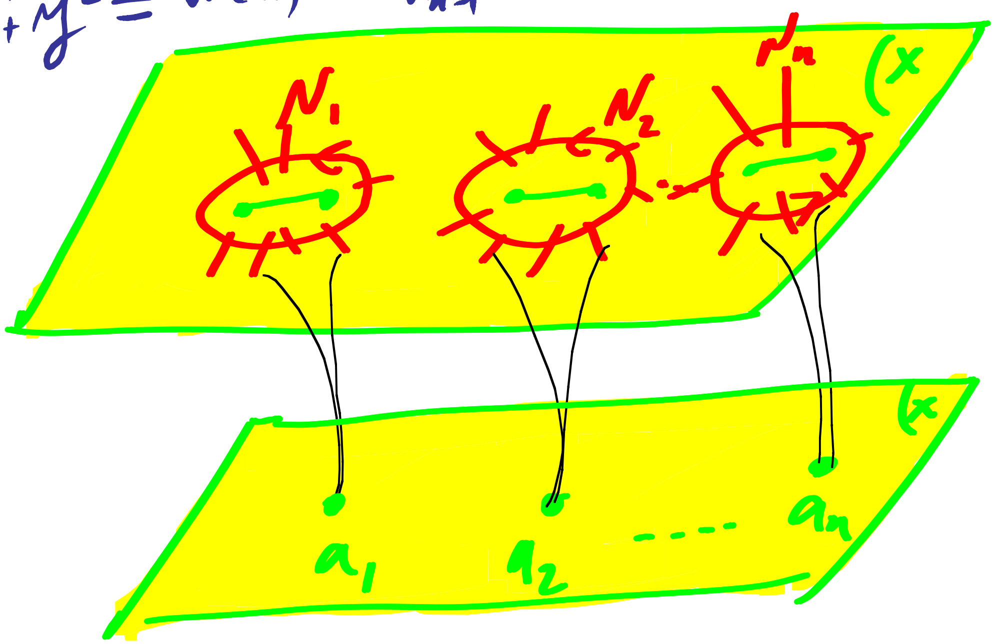
appears in CY defn

$$W'(\Phi) = 0 \longrightarrow \text{minima}$$

$$\Phi = \begin{pmatrix} \underbrace{a_1 \cdots a_1}_{N_1} & & & \\ & \underbrace{a_2 \cdots a_2}_{N_2} & & \\ & & \ddots & \\ & & & \underbrace{a_n \cdots a_n}_{N_n} \end{pmatrix}$$

Large N dual

$$u^2 + v^2 + y^2 = W'^2(x) + f_{n-1}(x)$$ ← deforms



$$W(S_i) = \sum_i N_i \Pi_i - \tau \sum_i S_i$$

$$\Pi_i = \frac{\partial F_0}{\partial S_i}$$ dual period S_i

$$S_i \equiv \langle \mathrm{Tr}_i \psi\psi \rangle = $$ size of S^3

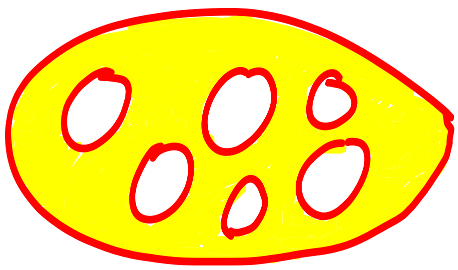
$$\frac{\partial W(S_i)}{\partial S_i} = 0 \longrightarrow \text{fixes } f_{n-1}(x)$$

the deformation

Moreover matrix integrals compute $F_0(S_i)$: Dijkgraaf, V.

$$\int d\Phi \; e^{\frac{\mathrm{Tr}\, W(\Phi)}{g_s}} = e^{\sum_\ell g_s^{2\ell-2} F_\ell(S_i)}$$

$$M_i g_s \longleftrightarrow S_i$$

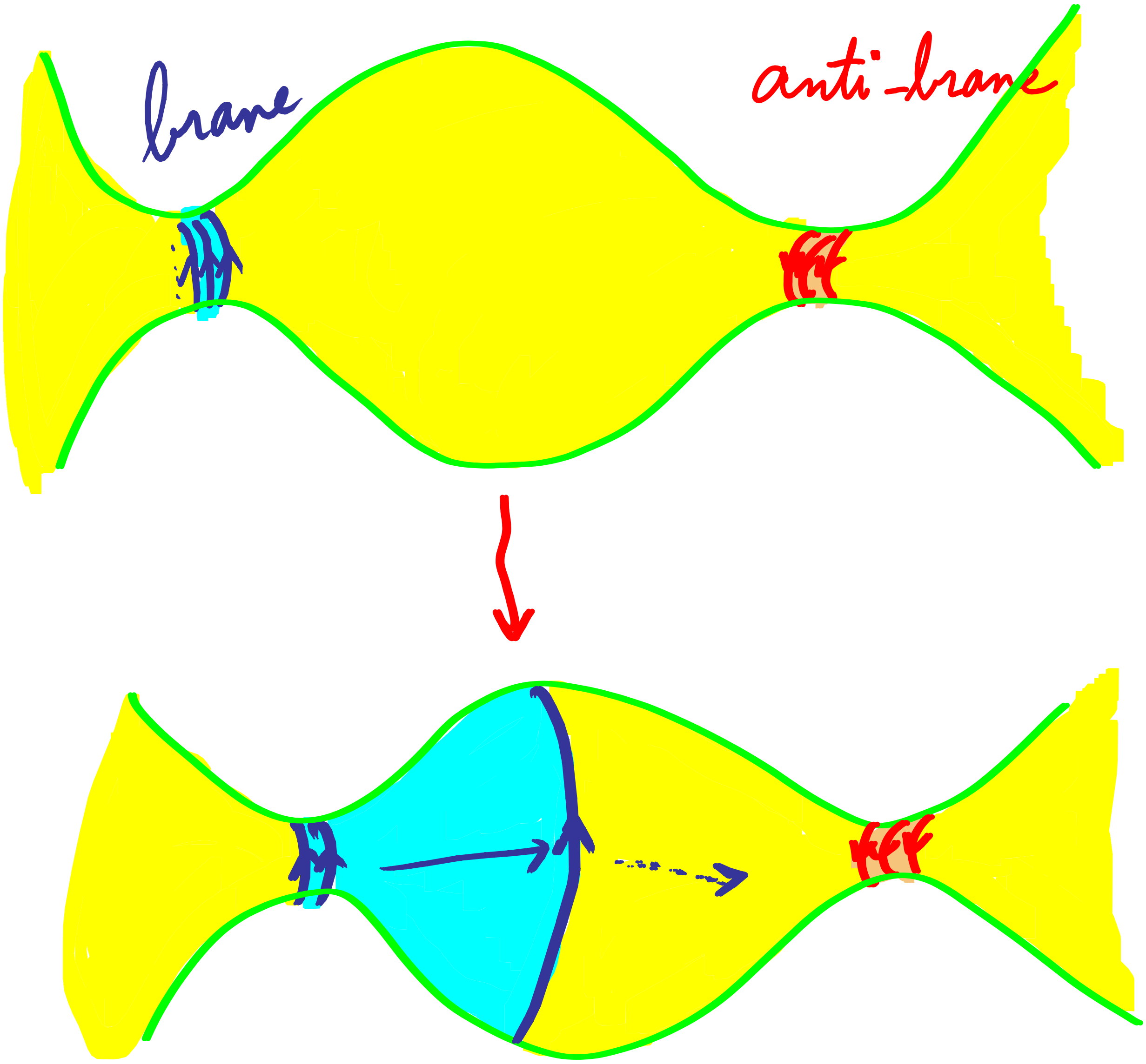


$\longleftrightarrow F_0(S_i)$

planar diagrams

$$\begin{tikzcd}
 & W(\Phi) \arrow[dl] \arrow[dr, "\text{large } N \text{ superpotential}"] & \\
\text{geometry of } CY \arrow[r, "\text{large } N"'] & \widetilde{CY} \arrow[r, leftrightarrow] & F_0(S_i)
\end{tikzcd}$$

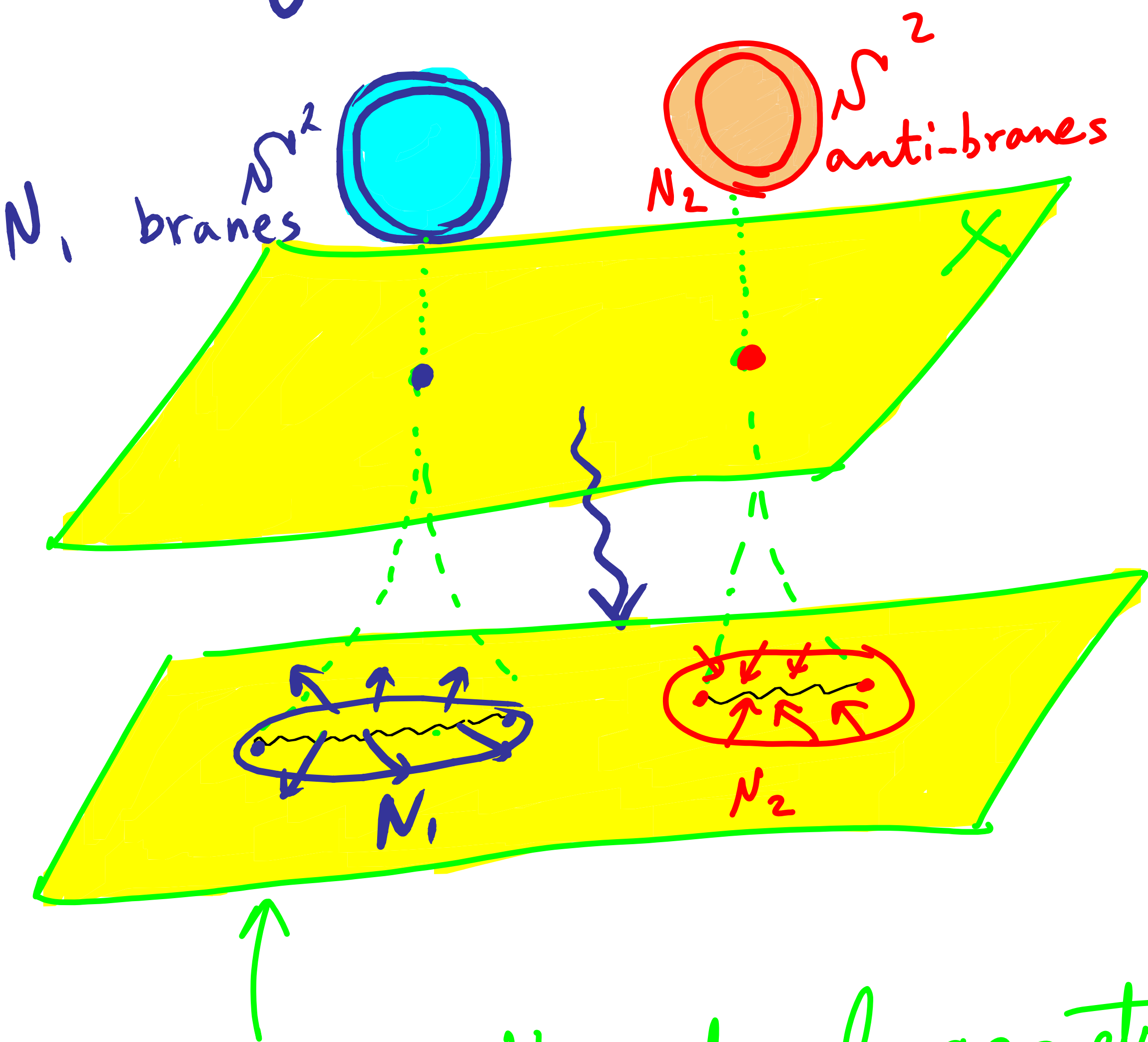
Now we consider the brane/anti-brane mix



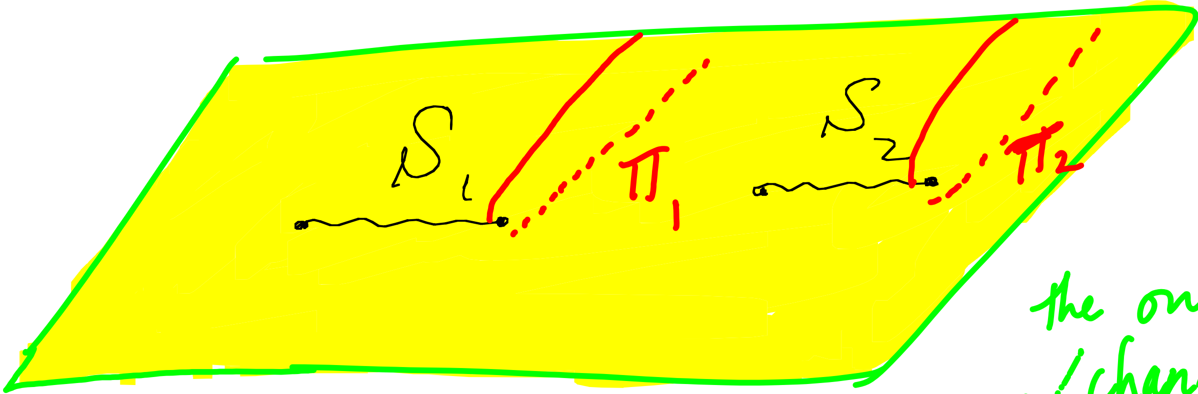
to annihilate it has to climb potential barrier

$\Longrightarrow$ Meta-stable non-supersymmetric vacuum

Large N dual:



holographically dual geometry where $S^2 \to 0 \to S^3$



the only change

$$W(S_i) = N_1 \Pi_1(S_i) - N_2 \Pi_2(S_i) + \alpha(S_1 + S_2)$$

$$V = g^{i\bar{j}}_{(S,\bar{S})} \partial_i W \overline{\partial_j W}$$

$$g_{i\bar{j}} = \operatorname{Im} \rho_{ij}$$

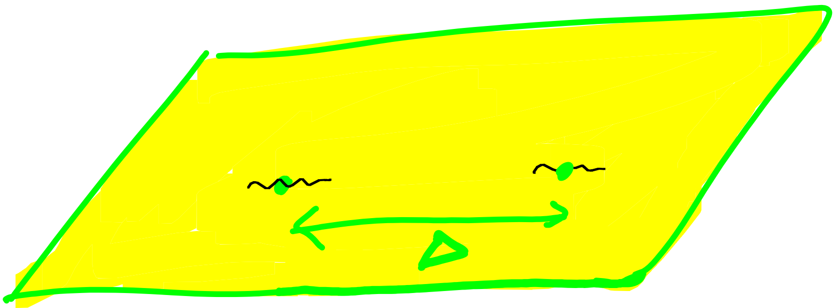
$$\rho_{ij} = \frac{\partial^2 F_0}{\partial S_i \partial S_j}$$

Note matrix model $\Rightarrow F_0 \Rightarrow (g_{i\bar{j}}, W)$

$\partial_i V = 0 \longrightarrow$ critical points

Example:

$$W'(\phi) = g\left(\phi - \frac{\Delta}{2}\right)\left(\phi + \frac{\Delta}{2}\right)$$



$$\left|S_i / g\Delta^3\right| \ll 1$$

$$F_0(X_i) \qquad X_i \sim \frac{S_i}{g\Delta^3}$$

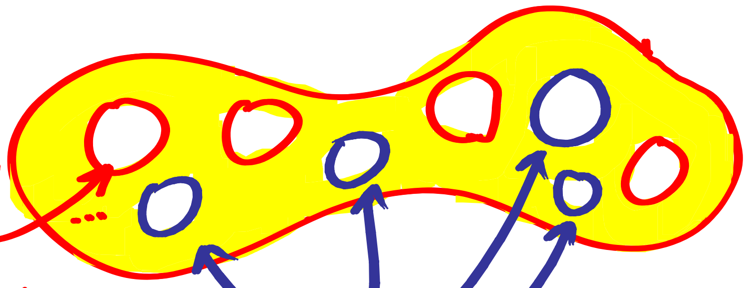
$$\approx X_i^2 \ln X_i + \text{power series in } X_i$$

$\Longrightarrow$ systematic expansion

The corresponding V sums up all planar diagrams.

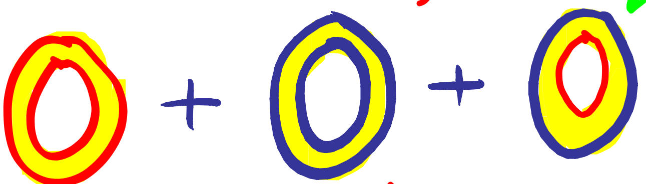
The term $X_1^{n_1} X_2^{n_2}$ is coming from

n_1 ending on brane



n_2 ending on antibrane

leading order $\partial_i V = 0$; $\partial W \neq 0$ Susy

(coming from )

$$\Longrightarrow \begin{cases} \left(\dfrac{S_1}{g\Delta^3}\right)^{N_1} = \left(\dfrac{\Lambda_0}{\Delta}\right)^{2N_1} \cdot \left(\dfrac{\bar{\Lambda}_0}{\bar{\Delta}}\right)^{2N_2} \\ \left(\dfrac{S_2}{g\Delta^3}\right)^{N_2} = \left(\dfrac{\Lambda_0}{\Delta}\right)^{2N_2} \cdot \left(\dfrac{\bar{\Lambda}_0}{\bar{\Delta}}\right)^{2N_1} \end{cases}$$

$N_1 \cdot N_2$ Sol'ns

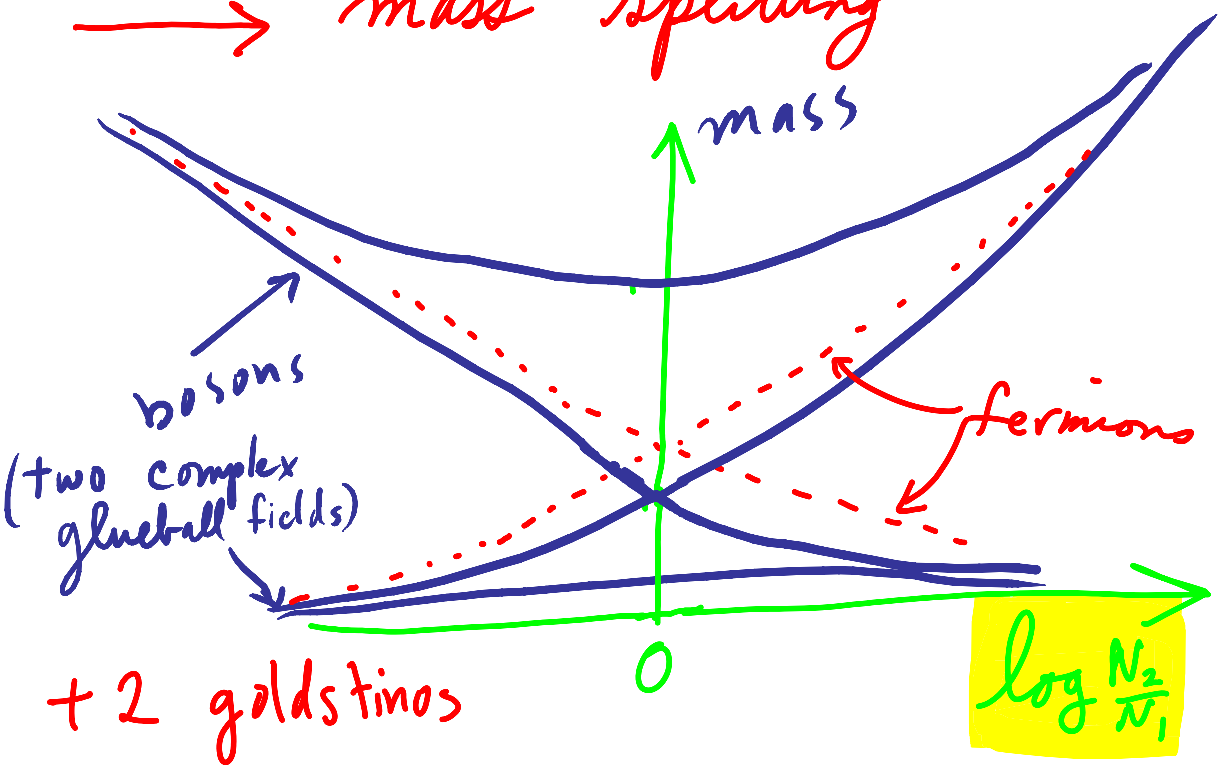
Similar to the susy case:

$$\left(\frac{S_1}{g\Delta^3}\right)^{N_1} = \left(\frac{\Lambda_0}{\Delta}\right)^{2N_1} \cdot \left(\frac{\Lambda_0}{\Delta}\right)^{2N_2}$$

$$\left(\frac{S_2}{g\Delta^3}\right)^{N_2} = \left(\frac{\Lambda_0}{\Delta}\right)^{2N_2} \cdot \left(\frac{\Lambda_0}{\Delta}\right)^{2N_1}$$

How to check susy is broken?

→ mass splitting

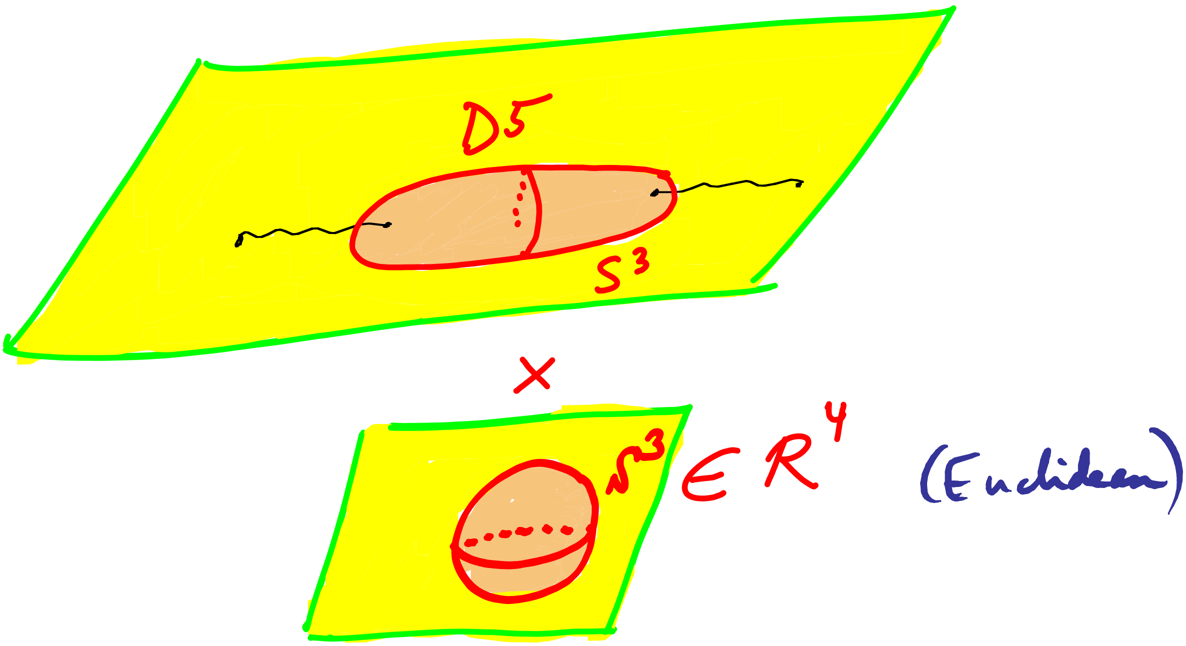


+ 2 goldstinos

Note: maximal splitting $N_1 = N_2$

Also: $\Delta \to \infty \Rightarrow$ splitting $\to 0$ ✓

Decay Mechanism



$$\Gamma \sim \exp\left(-\# \frac{|(g\Delta^3)^4|}{g_s}\right)$$

note $\Gamma \to 0$ as $\Delta \to \infty$.

Phase Structure at Large N

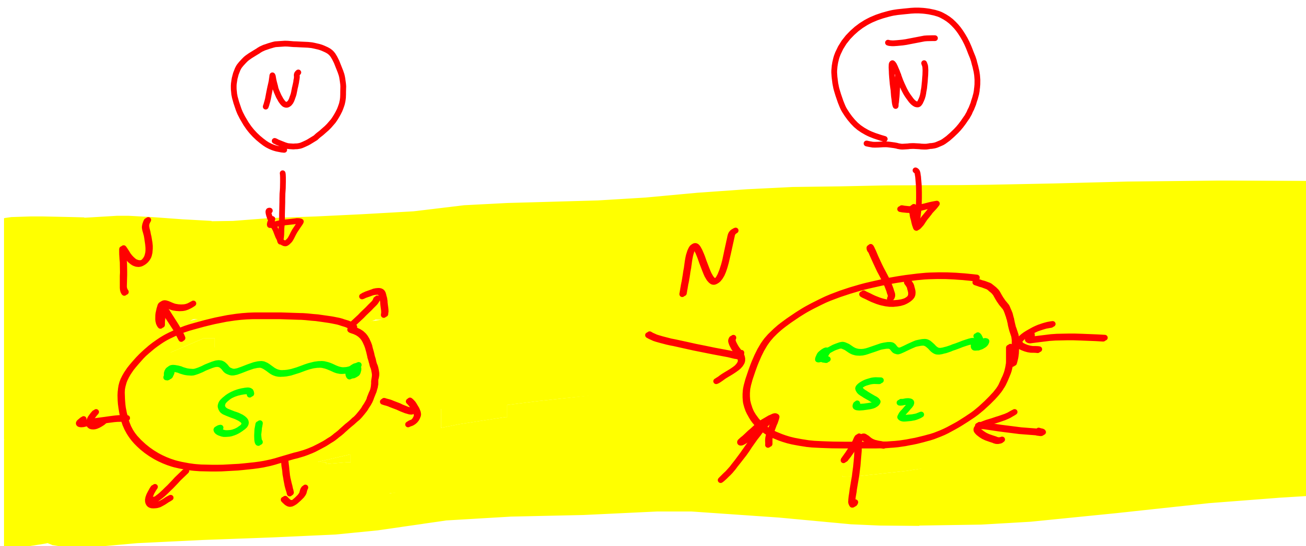
It is important to find out whether large N description modifies the weak coupling intuition. This could happen for N large enough. We have found a rich phase structure:

For small enough $|N_i|$ solution

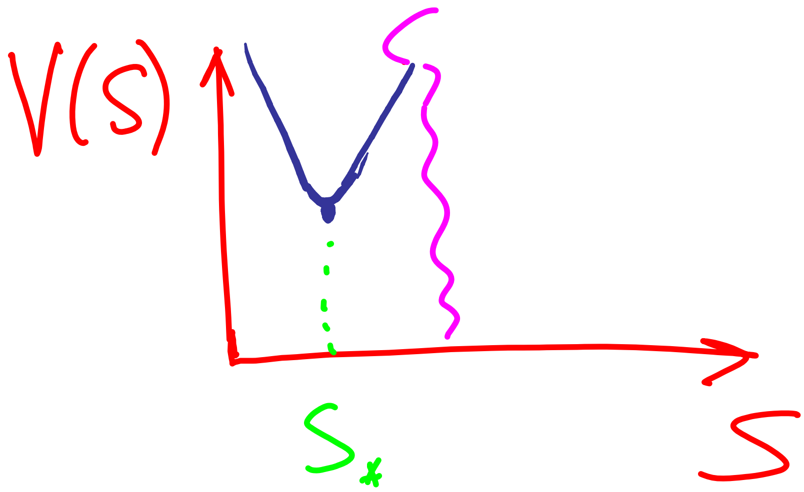
exists at least in a finite region near $|S_i| \sim 0$.

However, if we increase $|N_i|$ critical point near $|S_i| \sim 0$ disappears!

Consider the 2-cut case.



$S = |S_1| = |S_2|$ by symmetry



small enough N → S_* small

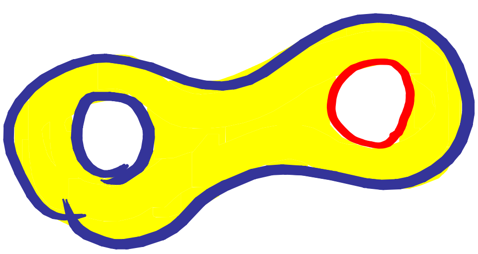
(dominated by V-Y potential

and one-loop annulus diagram)

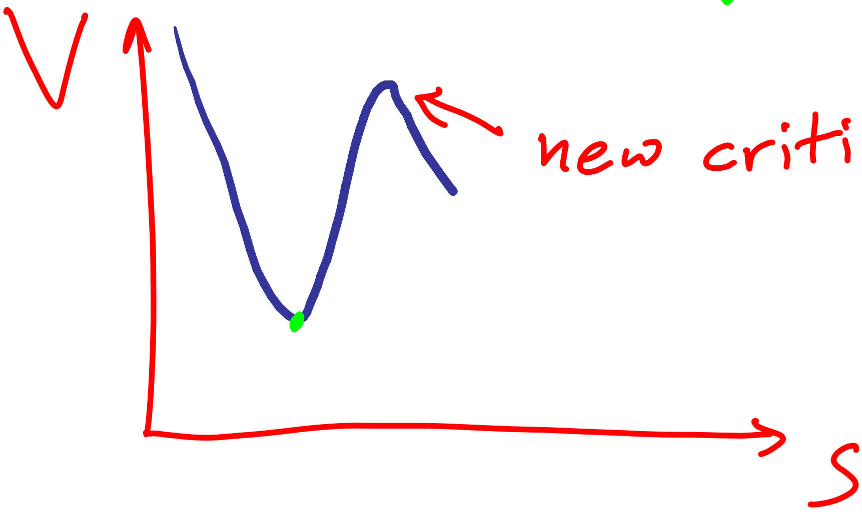
This is what we have already discussed. But already taking into account the next term in

W coming from 2-loop planar

diagram:

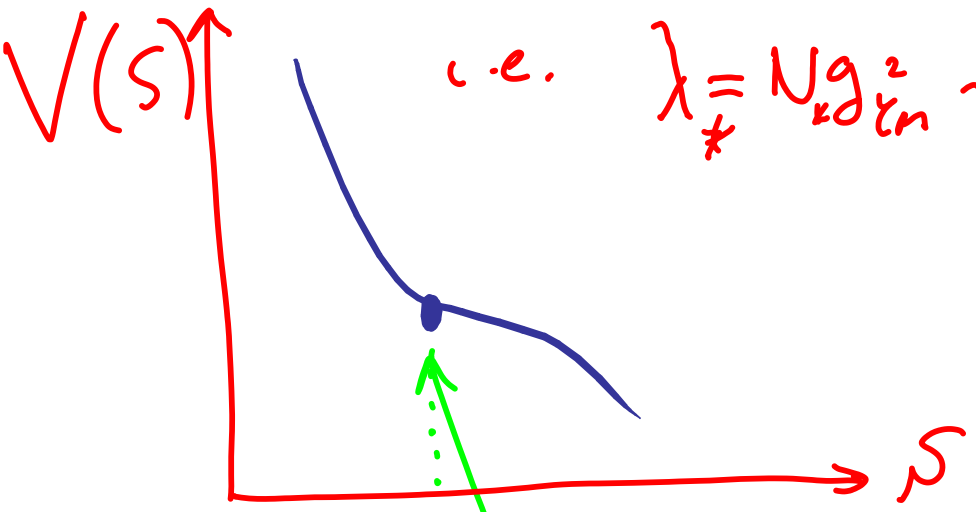


Also this 2-loop correction leads to



If $N > N_* \sim \dfrac{1}{g_{YM}^2 \log \frac{\Lambda_0}{\Delta}}$

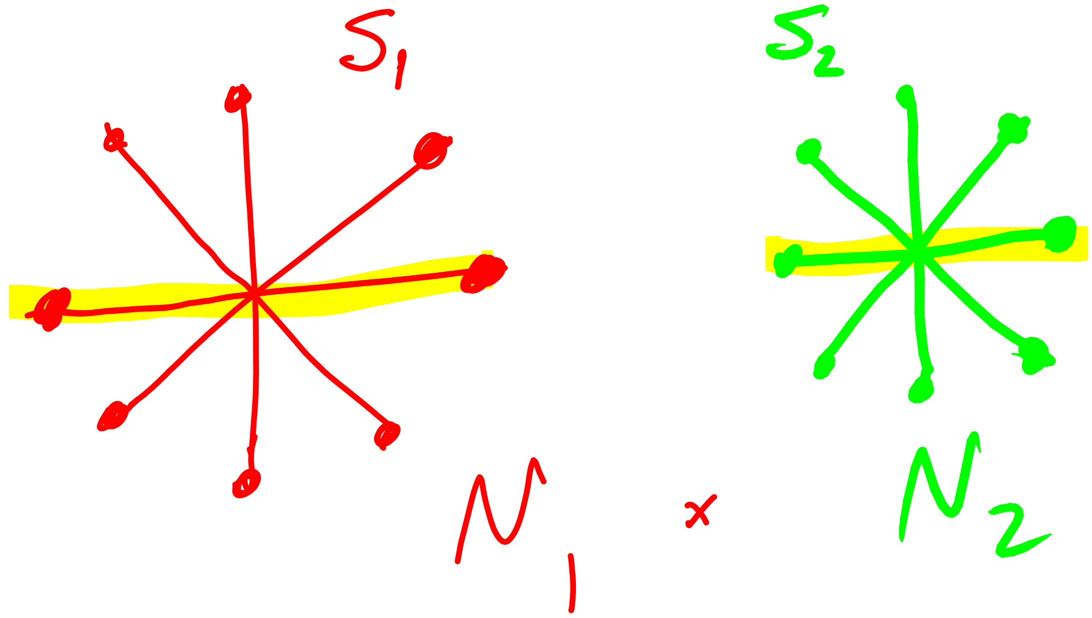
i.e. $\lambda_* = N_* g_{YM}^2 \sim \dfrac{1}{\log \frac{\Lambda_0}{\Delta}}$



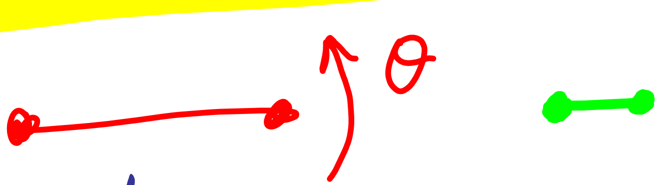
critical points meet →

local minimum is lost ⟶ **metastability gets lost at large N**

Also $N_1 \cdot N_2$ degeneracy lifted:



At 1-loop the energy is degenerate but already at 2-loop the cuts lower energy by alignment:

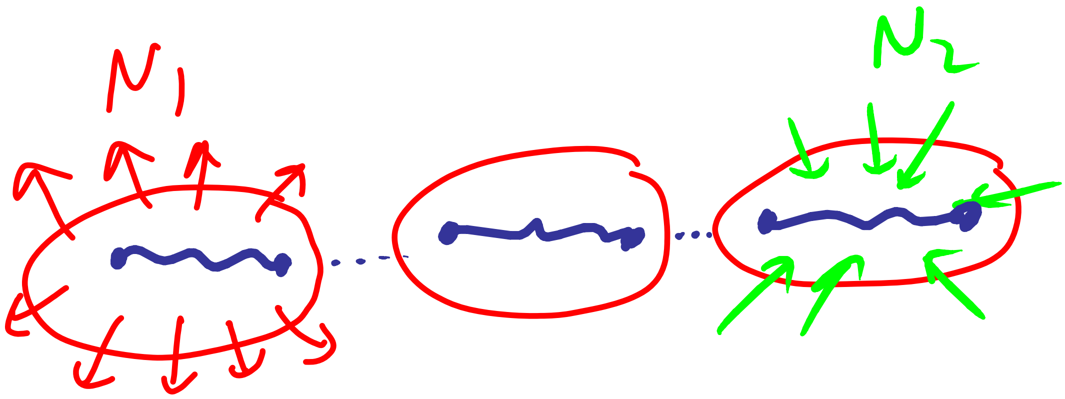


e.g. $N_1 = N_2 = N$

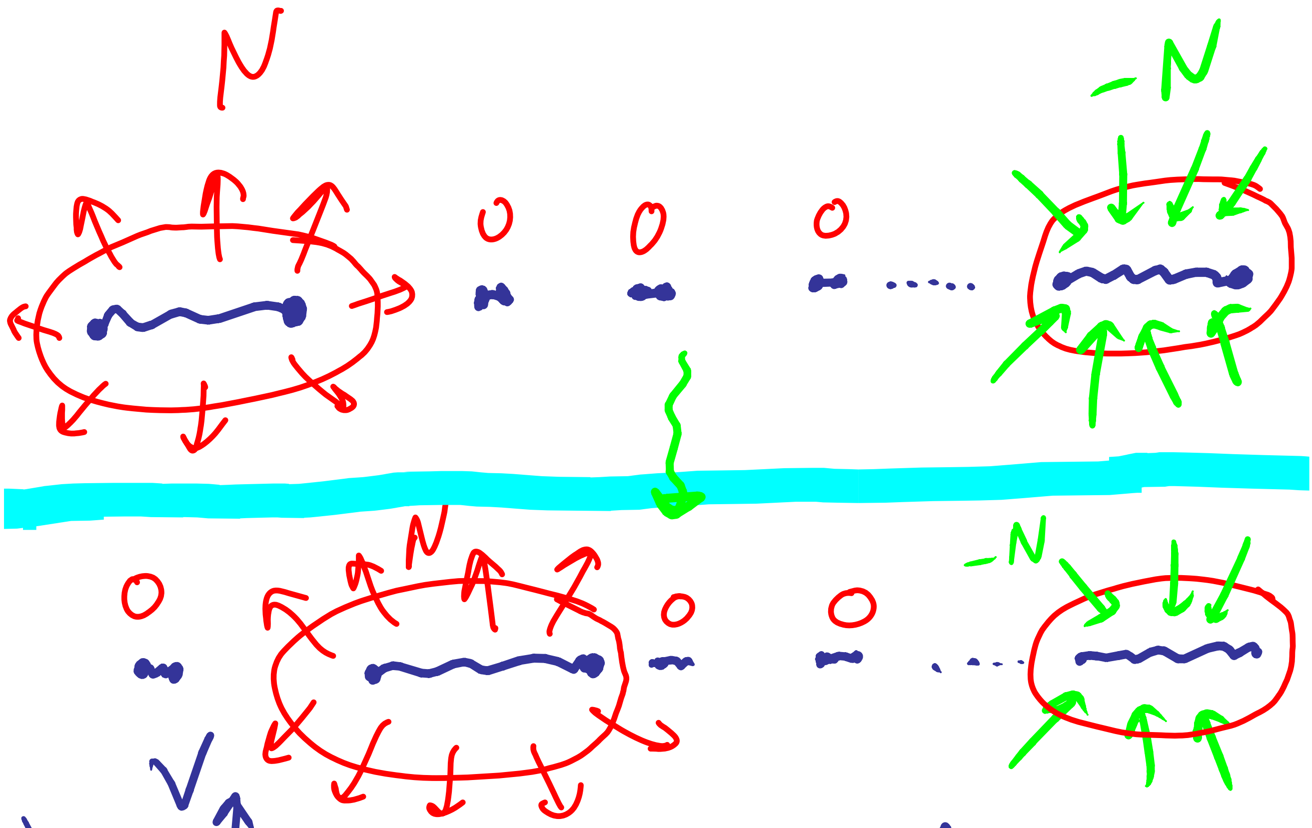
$$E \sim C_0 - C_1 N^2 \cos\frac{\theta}{N} + \dots$$

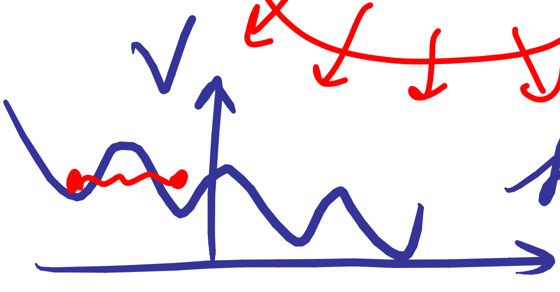
($\theta \approx$ axion); Note $N_1 N_2$ additional metastable vacua!

Amusing Multi-cut Geometries



Consider special example:





hops one step at a time

Concluding Remarks

- This construction has been lifted to M-theory by Marsano, Papadodimas, Shigemori

M5 brane: $\boxed{\underset{\Sigma}{\overset{\text{susy}}{\text{holomorphic}}}} \rightarrow \boxed{\underset{\Sigma}{\overset{\cancel{\text{susy}}}{\text{harmonic}}}}$

$(R^4 \times \Sigma \subset R^{11})$

- Open questions:
  - extend to compact case? (corrections to K, Douglas)
  - other geometries? (Aganagic et al)
  - fate of lost metastability?
  - Connection to open string tachyons?
  - relations with non-susy B.H.?