Heterotic Standard Models

Ron Donagi (Penn)

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Strings 08 @ CERN
Heterotic Standard Models
The High Country region of the string landscape

- **Goal:** Study string vacua which reproduce the MSSM (or close cousins thereof) at low energies
  - String landscape is huge, but High Country region may be much smaller
- **Questions:**
  - How many such vacua?
  - Do they have common properties (predictions)?
  - Constraints coming from string UV completion?
- **Crucial:** Must require global consistency of the string vacuum

A particular corner of the string landscape:

$E_8 \times E_8$ heterotic string on $\mathbb{R}^{3,1} \times X$ with gauge instanton $V$, where $X$ is a smooth compact Calabi-Yau threefold
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$$E_8 \times E_8$$ heterotic string on $$\mathbb{R}^{3,1} \times X$$ with gauge instanton $$V$$, where $$X$$ is a smooth compact Calabi-Yau threefold
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The High Country region of the string landscape

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String landscape is huge but the high country region may be much smaller

Questions: how many such vacua are there? Do they have common properties? Do predictions and constraints coming from string UV completion require global consistency of the string vacuum in particular corner of the string landscape?

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SUSY heterotic vacua

Data:
- $X$: smooth compact Calabi-Yau threefold
- $V \rightarrow X$: hol. vector bundle with structure group $G \subset E_8$

Consistency constraints:
- $V$ is polystable w.r.t a Kähler class [DUY: connection soln to HYM]
- $c_2(X) - c_2(V) = [M5]$ [anomaly cancellation with $M5$-branes]

Phenomenological requirements:
- Commutant $H$ of $G$ in $E_8$ is low-energy GUT group
- $\pi_1(X) = F$ to break $H$ to MSSM gauge group with discrete Wilson line
- Various extra phenomenological constraints:
  - $c_1(V) = 0$ [$G = SU(n)$]
  - $c_3(V) = \pm 6$ [3 generations]
  - $H^1(V), H^2(V), H^1(V^\wedge 2), H^2(V^\wedge 2), \ldots$ [Particle spectrum]
  - Triple products of cohomology groups, \ldots [Tri-linear couplings]

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Summary and examples

Heterotic vacuum:

1. Non-simply connected Calabi-Yau threefold $X$
2. Polystable bundle $V \to X$ satisfying a lot of constraints

Examples:

- $\pi_1(X) = \mathbb{Z}_2$, $G = SU(5)$
  - $SU(5)$ GUT
  - $SU(5) \xrightarrow{\mathbb{Z}_2} SU(3) \times SU(2) \times U(1)$

- $\pi_1(X) = \mathbb{Z}_6$ or $(\mathbb{Z}_3)^2$, $G = SU(4)$
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Heterotic Standard Models

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Consider a smooth simply connected Calabi-Yau threefold $\tilde{X}$ admitting a group $F$ of automorphisms acting freely on $\tilde{X}$

$\rightarrow X = \tilde{X}/F$ is a smooth Calabi-Yau threefold with $\pi_1(X) = F$

- $\tilde{X}$ is a smooth fiber product of two rational elliptic surfaces [Schoen]
- We classified all possible finite groups $F$ acting freely on $\tilde{X}$ [BD2]
- $\tilde{X}$ is the small resolution of a particular complete intersection of four quadrics in $\mathbb{P}^7$ [Gross]
  - free $(\mathbb{Z}_8)^2$ action [Gross]
  - 2 non-Abelian groups of order 64 act freely [Borisov-Hua]
- Hypersurfaces/complete intersections in toric threefolds, …
1st step: Constructing non-simply connected CY 3-folds $X$

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Let $B$ and $B'$ be RES, and $\tilde{X} = B \times_{\mathbb{P}^1} B'$ a smooth fiber product:

\begin{center}
\begin{tikzcd}
\tilde{X} \dar{\pi'} \rar{\pi} &
\arrow[loop left]{d}{\beta} \ar[loop right]{d}[swap]{\beta'}
B \aar{\pi'} \rar{\pi} &
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**Idea:** Consider special $B$ and $B'$ s.t. $\tilde{X}$ admits a free group of automorphisms $F_{\tilde{X}}$.
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\tilde{X} & \xrightarrow{\pi} & B \\
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Idea: Consider special $B$ and $B'$ s.t. $\tilde{X}$ admits a free group of automorphisms $F_{\tilde{X}}$. 
Free quotients of Schoen’s threefolds

- Automorphisms $\tau_{\tilde{X}} : \tilde{X} \to \tilde{X}$ have the form $\tau_{\tilde{X}} = \tau_B \times \mathbb{P}^1 \tau_{B'}$

- Classification of $(\tilde{X}, F_{\tilde{X}})$ reduces to classification of $(B, F_B)$, for suitable groups of automorphisms $F_B$

We produced such a classification, and we obtained a large class of $\tilde{X}$ with $F_{\tilde{X}}$ one of the following: $[BD2]B$

\[
(\mathbb{Z}_3)^2, \quad \mathbb{Z}_4 \times \mathbb{Z}_2, \quad \mathbb{Z}_6, \quad \mathbb{Z}_5,
\]
\[
\mathbb{Z}_4, \quad (\mathbb{Z}_2)^2, \quad \mathbb{Z}_3, \quad \mathbb{Z}_2
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2nd step: Constructing stable vector bundles $V \rightarrow X$

- Fourier-Mukai transform [FMW, D]B
  - Use dual Fourier-Mukai data to construct the bundle
  - Needs $X$ to be fibered (usually torus-fibered, but can be generalized)
  - **Pros:** Easy to prove stability from FM data [FMW]B
  - **Cons:** If start with $\tilde{V} \rightarrow \tilde{X}$, invariance under $F_{\tilde{X}}$ hard to prove

- Serre construction by extension

  $$0 \rightarrow V_1 \rightarrow V \rightarrow V_2 \rightarrow 0$$

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- To satisfy phenomenological constraints, may need combination of both methods [DOPW]B

- Other methods: monads [AHL]B, Hecke transforms, …
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Best (and only) model so far \cite[BD1, BCD]B

- **The manifold:** \( \tilde{X} = B \times_{\mathbb{P}^1} B' \), with special \( B \) and \( B' \) such that \( F_{\tilde{X}} \simeq \mathbb{Z}_2 \) acts freely on \( \tilde{X} \)

- **The bundle:** \( SU(5) \), \( \mathbb{Z}_2 \)-invariant, stable bundle \( \tilde{V} \rightarrow \tilde{X} \) constructed by

\[
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where \( V_3 \) and \( V_2 \) are rank 3 and 2 bundles on \( \tilde{X} \) constructed using Fourier-Mukai transform

- **Anomaly is cancelled**, either with \( M5 \)-branes, or without \( M5 \)-branes but with a non-trivial hidden bundle
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MSSM gauge group $SU(3) \times SU(2) \times U(1)$ with no extra $U(1)$’s

Precisely the MSSM massless spectrum with no exotic particles, up to moduli fields

Semi-realistic tri-linear couplings at tree level

R-parity is conserved at tree level (proton is stable)

Higgs $\mu$-terms and (possible) neutrino mass terms

To be addressed:

- SUSY breaking? (hidden sector)
- Moduli stabilization?
- Higher order corrections?
- More phenomenology needed
Phenomenology of this model

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$SU_N \times SU_N \times U_{N\omega}$ with no extra $U_{N\omega}$'s

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Other models

- Buchmuller, Hamaguchi, Lebedev, Ratz (and many others) \( \mathbb{Z}/6 \)-orbifolds
- Braun, He, Ovrut, Pantev
  \( \pi_1(X) = \mathbb{Z}/3 \times \mathbb{Z}/3 \), \( V \) unstable
- Faraggi, NAHE: Free Fermionic models
  \( (\mathbb{Z}/2)^n \) orbifolds “non-geometric”
- D, Faraggi: not within a particular class of geometric orbifolds
- DW: not a geometric orbifold
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Constructing other realistic models

- Using methods similar to [BD1], we tried to construct realistic bundles on fiber product $\tilde{X}$ with $F_{\tilde{X}} \sim \mathbb{Z}_6$
  $\rightarrow$ **No realistic bundle** [BD3]B
  - Main insight: strong tension between inequalities coming from anomaly cancellation and stability

- Work in progress: physical bundles on Gross’ threefold with $\pi_1(X) = (\mathbb{Z}_3)^2$ [BBDG]B
  - We constructed bundles phenomenologically viable at the topological level (up to a few subtleties that remain to be checked), using Fourier-Mukai transform on Abelian surface fibrations, and Hecke transforms
  - Next step: cohomology computation
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Recall: strong tension between anomaly cancellation and stability

- In principle, one can “forget” about anomaly cancellation
  - non-SUSY vacua with $M5$- and anti-$M5$-branes \([B,BBO]B\)
  - SUSY broken at the compactification scale :- ( \\

- We get infinite families of such non-SUSY vacua with exactly the phenomenological properties above \([BD3]B\)
- Also get infinite families of models on $\tilde{X}$ with $\pi_1(\tilde{X}) = \mathbb{Z}_6$
- One such model on $\tilde{X}$ with $\pi_1(\tilde{X}) = (\mathbb{Z}_3)^2$ \([BHOP]B\), perhaps more

- Such infinite families considered by Acharya-Douglas \(B\) in landscape study

→ phenomenological cutoff on scale of SUSY breaking
Relaxing the constraints (...)

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- In principle, one can “forget” about anomaly cancellation
  - non-SUSY vacua with $M5$- and anti-$M5$-branes $[B,BBO]B$
  - SUSY broken at the compactification scale :-(

- We get infinite families of such non-SUSY vacua with exactly the phenomenological properties above $[BD3]B$
- Also get infinite families of models on $\tilde{X}$ with $\pi_1(\tilde{X}) = \mathbb{Z}_6$
- One such model on $\tilde{X}$ with $\pi_1(\tilde{X}) = (\mathbb{Z}_3)^2$ $[BHOP]B$, perhaps more

- Such infinite families considered by Acharya-Douglas $B$ in landscape study
  $\rightarrow$ phenomenological cutoff on scale of SUSY breaking
$G_s \rightarrow G \rightarrow G_l$, where $G_s$ is shifts and $G_l$ twists $T\mathbb{Z}/\mathbb{Z} \times \mathbb{Z}/\mathbb{Z}$ acts $N \times y \times z \mathbb{O} \implies N \pm x, \pm y, \pm z \mathbb{O}$ with even number of sign changes after some reduction.

$\exists G_T \subset G_R$ $G_T \sim \rightarrow G_l$, four inequivalent types of $G_T$. Use reduction procedure to classify. For each model we calculate to compute $N$ via orbifold cohomology, fundamental groups, and Wilson lines. Some geometry of discrete torsion, no new tope numbers except mirror symmetry like interchange.

$H_m, m \iff H_n, m$ vs compare, Ramos, Sanchez, Ratz, Vaudrevange.
of orbifolds $T^6/G$

$$T^6 = E_1 \times E_2 \times E_3$$

0 → $G_S$ → $G$ → $G_T^0$ → 0 where $G_S$ is shifts and $G_T^0$ twists.

$G_T^0 = \mathbb{Z}/2 \times \mathbb{Z}/2$ acts $(x, y, z) \mapsto (\pm x, \pm y, \pm z)$ with even number of sign changes.

After some reduction: $\exists G_T \subset G$, $G_T \sim G_T^0$

Four inequivalent types of $G_T$.

Use reduction procedure to classify. For each model we calculate:

- Hodge Numbers (via orbifold cohomology)
- Fundamental groups (Wilson lines)
- Some geometry

Effect of discrete torsion, no new Hodge numbers, except mirror-symmetry like interchange $H^{1,1} \iff H^{2,1}$. (Compare: Mirage torsion: Ploger, Ramos-Sanchez, Ratz, Vaudrevange.)
of orbifolds $T^6/G$

$$T^6 = E_1 \times E_2 \times E_3$$

$0 \rightarrow G_S \rightarrow G \rightarrow G^0_T \rightarrow 0$ where $G_S$ is shifts and $G^0_T$ twists.

$G^0_T = \mathbb{Z}/2 \times \mathbb{Z}/2$ acts $(x, y, z) \mapsto (\pm x, \pm y, \pm z)$ with even number of sign changes.

After some reduction: $\exists G_T \subset G$, $G_T \sim G^0_T$

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$$T^6 = E_1 \times E_2 \times E_3$$

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where $G_S$ is shifts and $G^0_T$ twists.

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After some reduction: $\exists G_T \subset G$, $G_T \sim G^0_T$

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- Some geometry

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of orbifolds $T^6/G$

$T^6 = E_1 \times E_2 \times E_3$

$0 \to G_S \to G \to G^0_T \to 0$ where $G_S$ is shifts and $G^0_T$ twists.

$G^0_T = \mathbb{Z}/2 \times \mathbb{Z}/2$ acts $(x, y, z) \mapsto (\pm x, \pm y, \pm z)$ with even number of sign changes

After some reduction: $\exists G_T \subset G, \ G_T \sim G^0_T$

Four inequivalent types of $G_T$.

Use reduction procedure to classify. For each model we calculate:

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Effect of discrete torsion, no new Hodge numbers, except mirror-symmetry like interchange $H^{1,1} \iff H^{2,1}$. (Compare: Mirage torsion: Ploger, Ramos-Sanchez, Ratz, Vaudrevange.)
of orbifolds $T^6/G$

\[ T^6 = E_1 \times E_2 \times E_3 \]
\[ 0 \to G_S \to G \to G_T^0 \to 0 \] where $G_S$ is shifts and $G_T^0$ twists.
\[ G_T^0 = \mathbb{Z}/2 \times \mathbb{Z}/2 \] acts $(x, y, z) \mapsto (\pm x, \pm y, \pm z)$ with even number of sign changes
After some reduction: $\exists G_T \subset G$, $G_T \sim G_T^0$

Four inequivalent types of $G_T$.
Use reduction procedure to classify. For each model we calculate:
- Hodge Numbers (via orbifold cohomology)
- Fundamental groups (Wilson lines)
- Some geometry

Effect of discrete torsion, no new Hodge numbers, except mirror-symmetry like interchange $H^{1,1} \iff H^{2,1}$. (Compare: Mirage torsion: Ploger, Ramos-Sanchez, Ratz, Vaudrevange.)
We list the automorphism groups by rank. For each group $G$ we list its twist group $G_T$, its shift part $G_S$ (if non-empty), the Hodge numbers $h^{1,1}, h^{2,1}$ of a small resolution of $X/G$, the fundamental group $\pi_1(X/G)$, and the list of contributing sectors and their contribution. For the fundamental groups we use the abbreviations:

- **A**: the extension of $\mathbb{Z}_2 \times \mathbb{Z}$ by $\mathbb{Z}^2$ (so $H_1(X) = (\mathbb{Z}_2)^3$)
- **B**: any extension of $(\mathbb{Z}_2)^2$ by $\mathbb{Z}^6$ (with various possible $H_1(X)$)
- **C**: $\mathbb{Z}_2$
- **D**: $(\mathbb{Z}_2)^2$

A shift element is denoted by a triple $(\epsilon_1, \epsilon_2, \epsilon_3)$, where $\epsilon_i \in E_i$ is a point of order 2, abbreviated as one of $0, 1, \tau, \tau 1 := 1 + \tau$. A twist element is denoted by a triple $(\epsilon_1 \delta_1, \epsilon_2 \delta_2, \epsilon_3 \delta_3)$, where $\epsilon_i \in E_i$ is as above and $\delta_i \in \{\pm\}$ indicates the pure twist part. A two-entry contribution $(a, b)$ adds $a$ units to $h^{1,1}$ and $b$ units to $h^{2,1}$. When $b = 0$ we abbreviate $(a, b)$ to the single entry contribution $a$. 

Ron Donagi (Penn)  Heterotic Standard Models
We list the automorphism groups by rank for each group $G$ we list its twist group $G_T$ its shift part $G_S$ if non-empty or the toddge numbers $h_m, m, h_n, m$ of a small resolution of $X/G$ the fundamental group $\pi_m N/X/G$ or and the list of contributing sectors and their contribution for the fundamental groups we use the abbreviations $A$ the extension of $\mathbb{Z}_n$ by $\mathbb{Z}_n N$ so $H_m N X/O$ $\mathbb{Z}_n O$ $O$ any extension of $N$ $\mathbb{Z}_n O$ by $\mathbb{Z}_s N$ with various possible $H_m N X/O$ $O$.

<table>
<thead>
<tr>
<th>Rank 0:</th>
<th>$G_T$</th>
<th>sectors</th>
<th>contribution</th>
<th>$(h^{1,1}, h^{2,1})$</th>
<th>$\pi_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0 − 1)</td>
<td>$(0+, 0-, 0-), (0-, 0+, 0-)$</td>
<td>$0+, 0-, 0-$</td>
<td>16</td>
<td>(51, 3)</td>
<td>0</td>
</tr>
<tr>
<td>(0 − 2)</td>
<td>$(0+, 0-, 0-), (0-, 0+, 1-)$</td>
<td>$0+, 0-, 0-$</td>
<td>16</td>
<td>(19, 19)</td>
<td>0</td>
</tr>
<tr>
<td>(0 − 3)</td>
<td>$(0+, 0-, 0-), (0-, 1+, 1-)$</td>
<td>$0+, 0-, 0-$</td>
<td>8, 8</td>
<td>(11, 11)</td>
<td>$A$</td>
</tr>
<tr>
<td>(0 − 4)</td>
<td>$(1+, 0-, 0-), (0-, 1+, 1-)$</td>
<td>$0+, 0-, 0-$</td>
<td>8, 8</td>
<td>(3, 3)</td>
<td>$B$</td>
</tr>
</tbody>
</table>

Note: These are the four types of groups $G_T$. 
<table>
<thead>
<tr>
<th>Rank</th>
<th>$G_T$</th>
<th>$G_S$</th>
<th>sectors</th>
<th>contribution</th>
<th>$(h^{1,1}, h^{2,1})$</th>
<th>$\pi_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1−1</td>
<td>$0+, 0-, 0-$, $(0-, 0+, 0-)$</td>
<td>$(\tau, \tau, \tau)$</td>
<td>0+, 0-, 0-</td>
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<tr>
<td>1−2</td>
<td>$0+, 0-, 0-$, $(0-, 0+, \tau-)$</td>
<td>$(\tau, \tau, \tau)$</td>
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<td>(15, 15)</td>
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</tr>
<tr>
<td>1−3</td>
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<td>$(\tau, \tau, \tau)$</td>
<td>0+, 0-, 0-</td>
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<td>(11, 11)</td>
<td>C</td>
</tr>
<tr>
<td>1−4</td>
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<td>0+, 0-, 0-</td>
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<td>(7, 7)</td>
<td>A</td>
</tr>
<tr>
<td>1−5</td>
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<td>(3, 3)</td>
<td>B</td>
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<tr>
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<td>(11, 11)</td>
<td>C</td>
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<td>A</td>
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<tr>
<td>1−11</td>
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<td>(3, 3)</td>
<td>B</td>
</tr>
<tr>
<td>Rank 2:</td>
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<td>$G_S$</td>
<td>sectors</td>
<td>contribution</td>
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<td>$\pi_1$</td>
</tr>
<tr>
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<td>4</td>
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<td>$D$</td>
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<td>(9, 9)</td>
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Heterotic Standard Models
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<td>&amp;G_7</td>
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<td>\text{sectors}</td>
<td>(h^{1,1}, h^{2,1}) \text{ contribution}</td>
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**Rank 3:**

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<tr>
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<td>τ−, 1−, 0+</td>
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<tr>
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<td>τ−, τ1−, 0+</td>
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<td>(G_S) sectors contribution</td>
<td>((h^{1,1}, h^{2,1}))</td>
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<td>(0^-, 0^+, 0^-)</td>
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<td>(0^+, \tau 1^-, 1^-)</td>
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<td>(\tau 1^-, 0^+, 1^-)</td>
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<td>(1^-, 1^-, 0^+)</td>
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<td>(\tau^-, \tau 1^-, 0^+)</td>
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<td>(\tau 1^-, \tau 1^-, 0^+)</td>
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<tr>
<td>((3 - 4))</td>
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<td>((1, 1, 0), (\tau, \tau, 0), (1, \tau, 1))</td>
<td>((7, 7))</td>
<td>(C)</td>
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<td>(0^+, 0^-, 0^-)</td>
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<td>(0^-, 0^+, \tau^-)</td>
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<td>(0^+, \tau 1^-, 1^-)</td>
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<td></td>
<td>(\tau 1^-, 0^+, \tau 1^-)</td>
<td>1, 1</td>
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<tr>
<td>((3 - 5))</td>
<td>((0^+, 0^-, 0^-), (0^-, 0^+, 0^-))</td>
<td>((0, 1, 1), (1, 0, 1), (\tau, \tau, \tau))</td>
<td>((15, 3))</td>
<td>(C)</td>
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<td>(0^+, 0^-, 0^-)</td>
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<td>((3 - 6))</td>
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<td>((9, 9))</td>
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<td>(\tau^- ,\tau 1^-, 0^+)</td>
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<td>(0^+, 1^-, 1^-)</td>
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<td>(1^-, 0^+, \tau 1^-)</td>
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<td>(\tau 1^-, \tau 1^-, 0^+)</td>
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<tr>
<td>$G_T$</td>
<td>$G_S$</td>
<td>sectors</td>
<td>contribution</td>
<td>$(h^{1,1}, h^{2,1})$</td>
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<td>Rank 4:</td>
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<tr>
<td>$(4 - 1)$</td>
<td>$(0+, 0-, 0-), (0-, 0+, 0-)$</td>
<td>$(0, \tau, 1), (\tau, 1, 0), (1, 0, \tau), (1, 1, 1)$</td>
<td>$0+, 0-, 0-$</td>
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<td>$0+, \tau-, 1-$</td>
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</tbody>
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- (1-1): (27,3) $\pi_1 = \mathbb{Z}/2$
  (2-9): (27,3) $\pi_1 = 0$
  (2-9) is the NAHE$^+$ model=$(\mathbb{Z}/2)^2$-orbifold of $SO(12)$-torus
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- (0-2): Schoen (19,19)
- (1-3): DOPW, BD1 (11,11)
  $\Rightarrow$ FFM$s$ may produce Het Standard Model!
- (1-7), (2-5), (2-14): other free $\mathbb{Z}/2$ and $(\mathbb{Z}/2)^2$ Schoen quotients of [BD2].
- Seven cases of Borcea-Voisin threefolds
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Outline

Ron Donagi (Penn)  Heterotic Standard Models
World record for $\pi_1(X)$

$X=$Gross threefold [Gross-Popescu, Gross-Pavanelli]

$X \to \mathbb{P}^1$ simply-connected, fibered

Fibers=abelian surfaces ($T^4$) with polarization of type $(1,8)$.

Dual fibration: $X^\vee \to \mathbb{P}^1$

$X^\vee \cong X/(\mathbb{Z}/8)^2$, $\pi_1(X^\vee) = (\mathbb{Z}/8)^2$.

Explicitly: $X \to X' \subset \mathbb{P}^7$, $X'$ intersection of 4 quadrics has 64 nodes. $X \to X'$ small resolution.

Advantages:

- Huge $\pi_1 \Rightarrow$ greater phenomenological flexibility
- $V$ on $X^\vee \iff$ spectral data on $V$
- so don’t need invariance

Difficulties:

- Hard to find spectral curves (codim 2)
- Spectral construction needs to combine with Hecke transforms
  $\Rightarrow$ need to check stability

We have: one new example. It seems: many examples.

Construction of spectral curve uses: GW invariants!

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Following SUSY breaking at the compactification scale leads to infinite families of models, probably not phenomenologically viable.
Summary

- Within the “$E_8 \times E_8$ heterotic on smooth threefolds” corner of the landscape, the High Country region is very small (only one model so far! :-)

- Perhaps other models in the class of threefolds constructed as quotients of Schoen’s threefolds, but not on the $\mathbb{Z}_6$ one, at least with current bundle construction methods.

- Hope to get more physical models on Gross’ threefold (more to come soon :-)

- Allowing SUSY breaking at the compactification scale leads to infinite families of models, probably not phenomenologically viable.
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  • Necessary if we want to study common properties etc.


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Ron Donagi (Penn)  Heterotic Standard Models
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