S-duality and Boundary Conditions in $N=4$ SYM

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Outline

Motivations

Physics:

- Investigate $N = 4$ SYM
- New aspect of S-duality
- Engineer 3d SCFTs
  - Extend mirror symmetry
  - Includes high-SUSY Chern Simons

Mathematics (Geometric Langlands)

- Reduce $N = 4$ SYM to 2d $(4, 4)$ SCFTs
- Boundary conditions $\longrightarrow$ D-branes
- Domain walls $\longrightarrow$ Line operators

The $N = 4$ Gauge Theory

Elegant field theory

Dimensional reduction of 10d SUSY YM theory

- Gauge field, six adjoint scalars: $A_\mu^{10d} \to (A_\mu^{4d}, \Phi_i)$
- $SO(9,1) \to SO(3,1) \times SO(6)$

Exactly marginal coupling

- $\tau = \frac{2\pi i}{g_{YM}^2} + \frac{\theta_{YM}}{2\pi}$

$PSU(4|2,2)$ superconformal symmetry:

- $SU(2,2) \sim SO(4,2)$ conformal group in $3+1$
- $SU(4) \sim SO(6)$ R-symmetry

Superconformal Boundary Conditions

Boundary condition at $x_3 = 0$ breaks $\hat{P}_3$

- $\{Q_\epsilon, Q_{\epsilon'}\} = \bar{\epsilon}\gamma^3\epsilon'\hat{P}_3 + \cdots$
- Some SUSY must be broken
- 8 out of 16 SUSY can be preserved

3d superconformal symmetry can be preserved

- $OSp(4|4, R) \subset PSU(4|2, 2)$
- $SO(4) \sim SO(3)_X \times SO(3)_Y \subset SO(6)$
- Divide scalars in $X^{4,5,6}$ and $Y^{7,8,9}$.

Simple Examples of Boundary Conditions I

Neumann boundary conditions:

- $F_{3i}|_\partial = 0$
- $\partial_i Y^p|_\partial = 0$
- $X^a|_\partial = 0$
- Gauge symmetry survives at the boundary

Simple modifications

1. Couple 3d SCFT at the boundary
2. Add Chern Simons boundary action

Simple Examples of Boundary Conditions II

Dirichlet boundary conditions:

- $F_{ij}|_\partial = 0$
- $\partial_i X^a|_\partial = 0$
- $Y^a|_\partial = 0$
- Gauge symmetry trivial at the boundary

Moduli space of vacua

- $\frac{dX^a}{dx^3} = \epsilon^{abc}[X^b, X^c]$ preserve SUSY
- Well studied moduli space of solutions
- Family of "Nahm" modifications: $X^a \sim \frac{\rho^a}{x^3}$

S-duality and Boundary Conditions

S-duality symmetry

- $\tau \longrightarrow -\frac{1}{\tau}$
- $U(1)$ theory: electric-magnetic duality. $F \leftrightarrow *F$.

S-duality and Boundary Conditions

1. Pick a boundary condition $\mathcal{B}$
2. Make coupling strong $g_{YM} >> 1$
3. Describe bulk theory with weakly coupled dual fields.
4. Read $\mathcal{B}^{\vee}$ boundary condition for dual fields

Examples

$U(1)$ example:

Neumann and Dirichlet are dual to each other

- S-duality is just electric-magnetic duality.
- $*F_{3i} = \epsilon_{ijk} F^{jk}$

Non-Abelian examples?

- Dirichlet $\leftrightarrow$ Neumann $\oplus T(G)$ 3d SCFT
- Neumann $\leftrightarrow$ Dirichlet modified by Nahm pole

We need to consider large class of boundary conditions

Domain Walls and Boundary Conditions

We can study domain walls as well

- Domain walls for theory M are b.c for theory $M \times M^P$
- Same SUSY analysis as b.c

Domain wall $\oplus$ simple b.c. $\longrightarrow$ complicated b.c.

- Place boundary condition at $x_3{=}0$
- Place domain wall at $x_3 = L$
- Flow to IR to get new boundary condition

It is a very powerful tool if IR flow is under control!

Brane Realization: I

	0	1	2	3	4	5	6	7	8	9
D3	*	*	*	*						
D5	*	*	*		*	*	*			
NS5	*	*	*					*	*	*

Decoupling limit of brane system

- n D3-branes give $U(n)$ SYM in 4d
- Fivebranes worldvolume theory is irrelevant
- Each fivebrane intersects D3 at fixed x_3
- Codimenson one defects in 4d theory

Brane Realization: II

Symmetry

- System has 8 SUSY, 3d Poincare invariance
- $SO(3)_X \times SO(3)_Y \in SO(6)$ R-symmetry

D5 brane:

- $3-5$ strings do not decouple, live at intersection
- Domain wall with extra matter in fundamental of $U(n)$

NS5 brane:

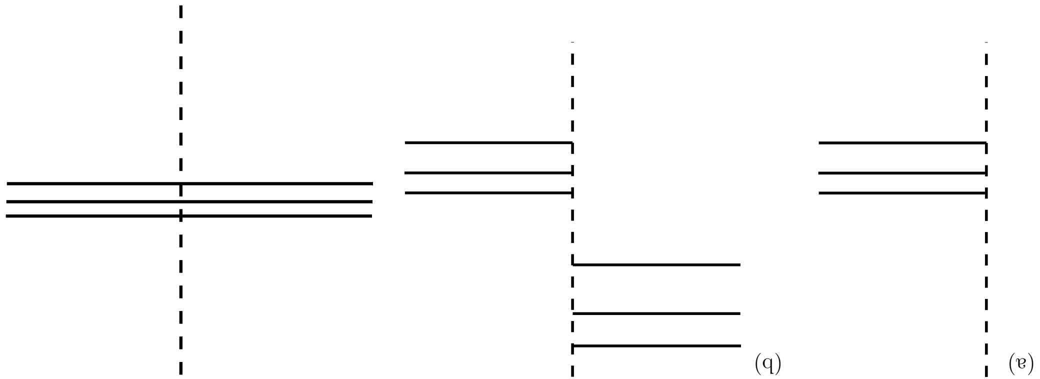
- It "splits" D3 branes: $U(n)$ gauge theories on half spaces
- Neumann b.c at the domain wall
- $3_L - 3_R$ strings give bifundamental matter at the domain wall

Superconformal domain walls in $N = 4$ $U(n)$

Brane Realization: III

From domain walls to boundary conditions

- D3 branes can break at fivebranes
- D3 branes can end on fivebranes



Half BPS boundary conditions for $U(n)$ SYM

Brane Realization: IV

n D3 branes ending on a single NS5-brane

- Neumann boundary conditions!
- $\partial_i Y^p|_\partial = 0$: D3 can move along NS5 (789)
- $X^a|_\partial = 0$: D3 are attached to NS5, cannot move along (456).

n D3 branes ending on D5 branes

A single D5 brane gets deformed a lot

- Situation intensely studied for $D1 - D3$ system
- Dirichlet bundary condition $\oplus$ "Maximal" Nahm pole for X^a

How to get Dirichlet boundary conditions?

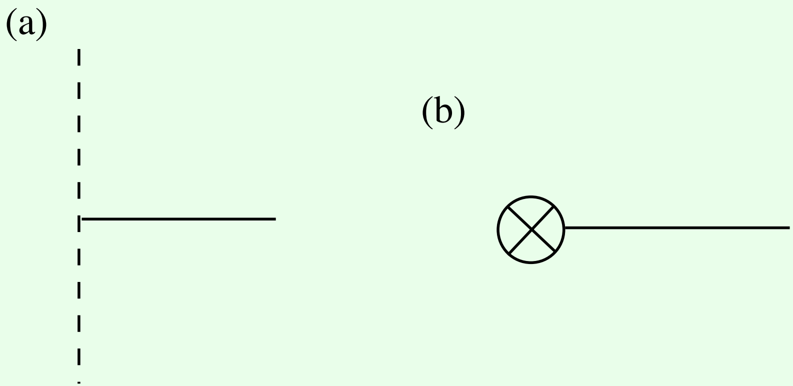
- n D3 branes ending on n D5-branes

Examples of S-dual pairs in $U(1)$ gauge theory

Neumann and Dirichlet are dual to each other

- S-duality is just electric-magnetic duality.
- $*F_{3i} = \epsilon_{ijk} F^{jk}$

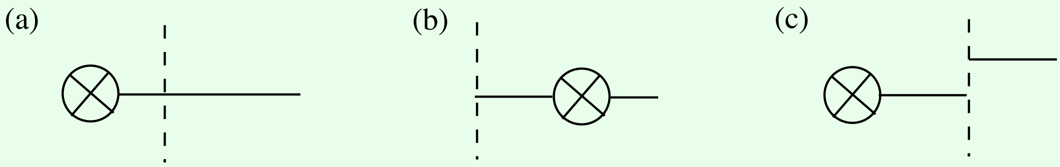
We can check with branes



- S-duality exchanges D5 and NS5

Neumann ⊕ charge 1 hypermultiplet

One D3 branes passing through D5, ending on NS5



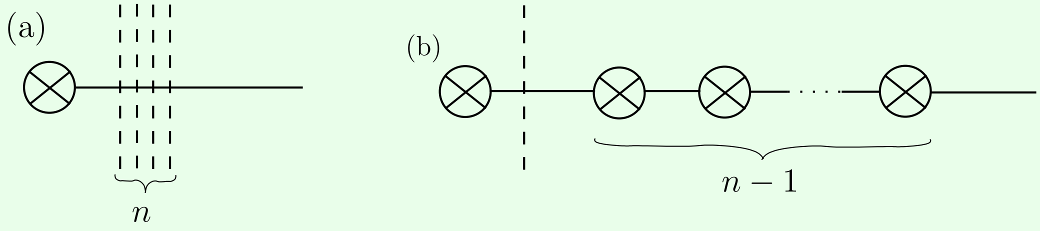
(a) Original. (b) S-dual. (c) Rearranged S-dual $\equiv$ original!

Self-dual boundary condition.

- Nontrivial field theory result
- Akin to particle-vortex duality in 3d
- Vortices in Higgs phase are free magnetic particles!

Neumann with many charge 1 hypermultiplets

One D3 branes passing through n D5, ending on NS5



(a) Original (b) Rearranged S-dual

Interesting field theory limit

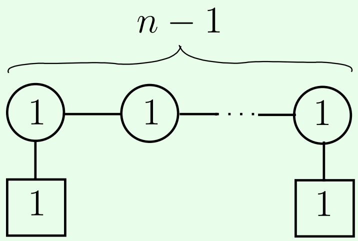
- 3d $U(1)^{n-1}$ gauge theory from segments
- Bifundamental matter across each NS5
- Last bifundamental couples 3d $U(1)$ and bulk $U(1)$

Neumann plus coupling to strongly interacting 3d theory

Neumann with many charge 1 hypermultiplets

A quiver description is useful

(c)



3d SCFT at the boundary is IR limit of A_{n-1} quiver

- Bulk theory couples to obvious $U(1)$ flavor symmetry
- Simple mirror realization: 3d $U(1)$ with n flavors
- $U(1)$ flavor symmetry mirror to shift of dual photon

S-duality for brane constructions

Four step process

- Engineer the boundary condition $\mathcal{B}$ from branes
  - Construct a configuration which will flow to $\mathcal{B}$ in IR
- S-duality in string theory: exchange D5 and NS5
  - New configuration is typically inconvenient!
- Move fivebranes around in x^3
  - Remember Hanani-Witten brane creation!
- Read the result: field theory limit, IR flow

Completely mechanical process, but not al b.c can be engineered

General Prescription

An S operation on $U(1)$ boundary conditions:

- Pick boundary condition Neumann $\oplus$ $\mathfrak{B}$ 3d SCFT
- Couple 3d $U(1)$ to $\mathfrak{B}$
- Flow to IR to produce $\mathfrak{B}^\vee$
- $\mathfrak{B}^\vee$ has $U(1)$ isometry shifting dual photon
- Make a dul boundary condition: Neuman $\oplus$ $\mathfrak{B}^\vee$ 3d SCFT

A bit of works checks $S^2 = 1$

- Can define a T operation as well
- T adds Chern Simons coupling at boundary
- $(ST)^3 = 1$!

How to Read the 3d SCFT

Ungauging trick

- Put the theory on $0 < x_3 < L$
- Set $\mathcal{B}^\vee$ at $x^3 = 0$
- Set Dirichlet at $x^3 = L$
- Dirichlet kills gauge theory in IR, leaves $\mathcal{B}^\vee$

Do S-duality:

- On one side original $\mathcal{B}$
- On the other side the dual of Dirichlet: $T(G)$
- Reproduces $U(1)$ prescription

Key point: identify $T(G)$ for general G!

General boundary conditions

A rich space of possibilities

We find various combinations of three basic ingredients:

- Nahm pole
- Gauging of a subgroup.
- Coupling to a 3d SCFT

This large class appears to be closed under S-Duality!

Future directions

Engineering of useful 3d SCFTs

- Novel Mirror symmetry relations?

AdS/CFT

- Study holographic dual of boundary conditions, domain walls.

Generalize to less SUSY

- Applications for new boundary conditions?