The gauge-string duality and QCD at finite temperature

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Contents

1. Near-extremal D3-branes

The near-extremal D3-brane metric describes $\mathcal{N} = 4$ gauge theory at finite temperature [Gubser et al. 1996] (also unpublished work of Strominger):

$$\begin{aligned} ds^2 &= H^{-1/2}\left(-h dt^2 + d\vec{x}^2\right) + H^{1/2}\left(\frac{dr^2}{h} + r^2 d\Omega_5^2\right) \\ H &= 1 + \frac{L^4}{r^4} \qquad h = 1 - \frac{r_0^4}{r^4}\,. \end{aligned} \tag{1}$$

In the now-familiar strong coupling limit of AdS/CFT [Maldacena 1998; Gubser et al. 1998a; Witten 1998]

$$\frac{L^8}{G_{10}} = \frac{2N^2}{\pi^4} \gg 1 \qquad \frac{L^4}{\alpha'^2} = \lambda \equiv g_{YM}^2 N \gg 1 \tag{2}$$

One finds free energy density [Gubser et al. 1998b]

$$f(\lambda) = \frac{F}{V} = \left(\frac{3}{4} + \frac{15\zeta(3)}{8\lambda^{3/2}} + \ldots\right) f_{\text{free}} \tag{3}$$

where $f_{\text{free}} = -\frac{\pi^2}{6}(N^2-1)T^4$ for $SU(N)$ super-Yang-Mills.

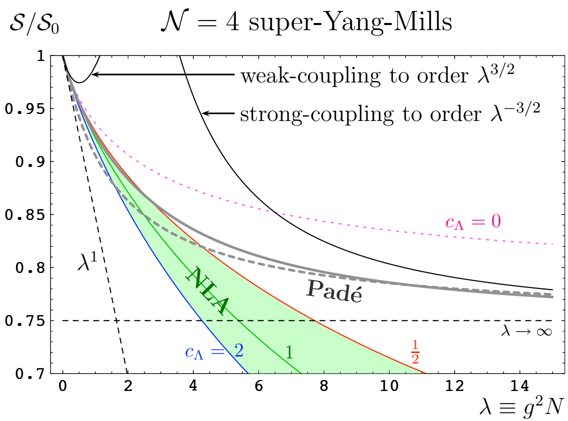
At weak coupling [Fotopoulos and Taylor 1999; Vazquez-Mozo 1999; Kim and Rey 2000; Nieto and Tytgat 1999],

$$f(\lambda) = \left(1 - \frac{3}{2\pi^2}\lambda + \frac{\sqrt{2}+3}{\pi^3}\lambda^{3/2} + \ldots \right) f_{\text{free}} \tag{4}$$

The most modern treatment I know of is by [Blaizot et al. 2006]: (3) and (4) uniquely fix a (4,4) Padé estimate,

$$\frac{f}{f_{\text{free}}} = \frac{1 + \alpha\lambda^{1/2} + \beta\lambda + \gamma\lambda^{3/2}}{1 + \bar{\alpha}\lambda^{1/2} + \bar{\beta}\lambda + \bar{\gamma}\lambda^{3/2}} \tag{5}$$

Comparison with a hard thermal loop calculation of s/s_{free} (roughly, two-loop perturbation theory supplemented by a self-consistent gap equation for thermal masses) does pretty well out to $\lambda \sim 4$.



HTL (green) calculations of entropy in $\mathcal{N} = 4$ [Blaizot et al. 2006].

2. Shear viscosity

Neglecting loop and stringy corrections to two-derivative gravity, a broad set of black branes have [Policastro et al. 2001; Buchel and Liu 2004; Kovtun et al. 2005]

$$\frac{\eta}{s} = \frac{1}{4\pi}\,; \tag{6}$$

and D3-branes in particular have [Buchel et al. 2005]

$$\frac{\eta}{s} = \frac{1}{4\pi}\left(1 + \frac{135\zeta(3)}{8\lambda^{3/2}} + \ldots\right)\,. \tag{7}$$

Loop corrections may lead to violations [Kats and Petrov 2007; Brigante et al. 2008] of the conjectured bound $\eta/s \geq 1/4\pi$.

η is a key input for relativistic hydrodynamics:

$$\begin{aligned} T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu + pg^{\mu\nu} - P^{\mu\alpha}P^{\nu\beta}\Bigg[\eta\left(\nabla_\alpha u_\beta + \nabla_\beta u_\alpha - \frac{2}{3}g_{\alpha\beta}\nabla_\lambda u^\lambda\right) \\ + \zeta g_{\alpha\beta}\nabla_\lambda u^\lambda\Bigg] \qquad \text{where} \qquad P^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu\,. \end{aligned} \tag{8}$$

Lattice simulations of pure glue [Meyer 2007] indicate

$$\left[\frac{\eta}{s}\right]_{\text{best}} = 0.134 \approx \frac{5/3}{4\pi} \qquad\qquad \frac{\eta}{s} \lesssim 1 \ \ \text{@ } 90\% \ \textbf{CL} \tag{9}$$

This is hard work for the lattice because viscosities arise from real-time correlators:

$$\begin{gathered}\eta\left(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk} - \frac{2}{3}\delta_{ij}\delta_{kl}\right) + \zeta\delta_{ij}\delta_{kl} = -\lim_{\omega\to 0}\frac{1}{\omega}\operatorname{Im} G^R_{ij,kl}(\omega) \\ G^R_{ij,kl}(\omega) \equiv -i\int d^3x\, dt\, e^{i\omega t}\theta(t)\langle[T_{ij}(t,\vec{x}), T_{kl}(0,0)]\rangle\,,\end{gathered} \tag{10}$$

whereas lattice provides direct access only to Euclidean correlators:

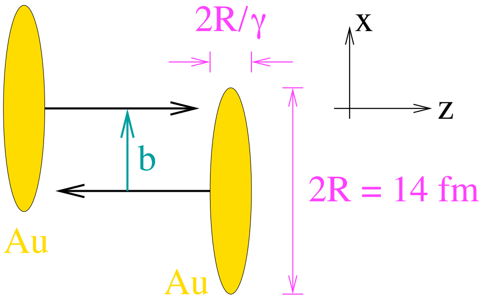
$$\begin{aligned}G^E(\omega_n) &= \int_0^\beta d\tau \int d^3x\, e^{i\omega_n\tau}\left\langle T_E\left\{\mathcal{O}(\tau,\vec{x})\mathcal{O}(0)\right\}\right\rangle \qquad \omega_n = \frac{2\pi n}{\beta} \\ &= -G^R(i\omega_n) = \int_{-\infty}^{\infty} d\omega \frac{\rho(\omega)}{\omega - i\omega_n} \qquad \text{for } n > 0.\end{aligned} \tag{11}$$

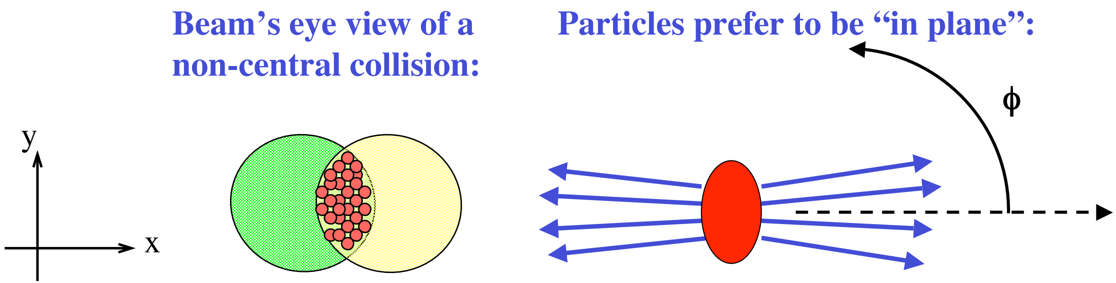
To get $G^R(\omega)$ for real ω starting from lattice data, some assumptions about spectral density $\rho(\omega)$ have to be made.

Elliptic flow in heavy ion collisions puts bounds on η. Here's the relevant geometry:

Side view of an off-center gold-gold collision. The reaction plane is the plane of the page b as a vector is approximately determined for each event.

$\gamma \approx 100$ at RHIC, 2800 at LHC.

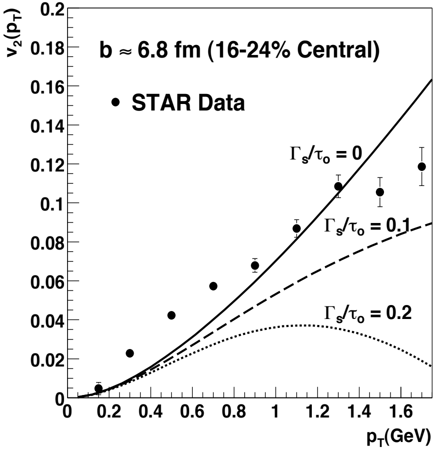




Cartoon of elliptic flow. From [Baker 2001]. Uneven pressure gradients lead to anisotropic expansion.

Experimental measure of elliptic flow is d-wave coefficient in an expansion of azimuthal distribution of particles (here $y = \tanh^{-1} p_z/E$ is rapidity):

$$\frac{dN}{p_T dp_T dy d\phi} = \frac{dN}{p_T dp_T dy} [1 + 2v_2 \cos 2\phi + \ldots] \tag{12}$$



Effect of shear viscosity on predictions of $v_2(p_T)$. From [Teaney 2003]. Data points are pions, from STAR [Adler et al. 2002].

Viscosity dependence of v_2 was studied e.g. in [Teaney 2003] in terms of Γ_s/τ_o, where

$$\begin{aligned} \Gamma_s &= \frac{4}{3T}\frac{\eta}{s} && \text{sound attenuation length} \\ \tau_o T &\approx 1 && \text{characteristic expansion} \\ \frac{\Gamma_s}{\tau_0} = 0.1 &\longleftrightarrow \frac{\eta}{s} \approx \frac{1}{4\pi} \end{aligned} \tag{13}$$

But... Ideal hydro, $\Gamma_s = 0$, was "designed" to agree with data in this study.

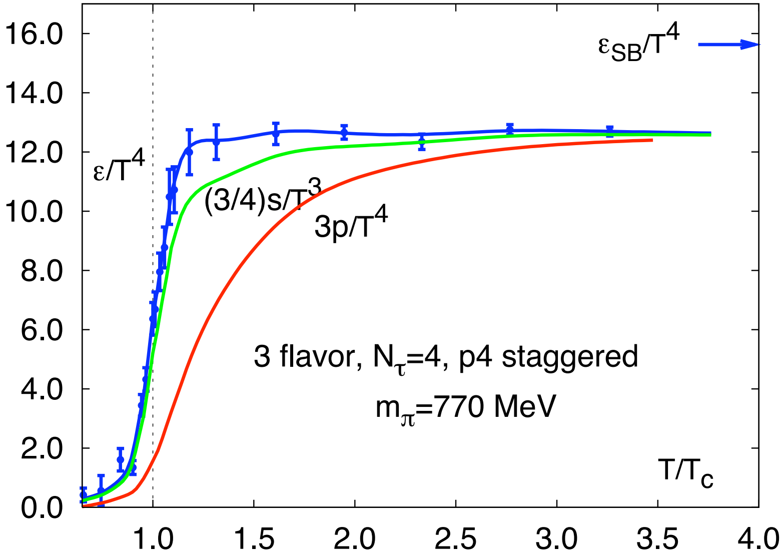
Upshot: data favors the range

$$0 \leq \frac{\eta}{s} \lesssim 0.2 \approx \frac{5/2}{4\pi} \,. \tag{14}$$

3. Equation of state and bulk viscosity

QCD is significantly non-conformal near T_c, and confinement is a smooth cross-over, not a phase transition.

Lattice results for the equation of state of QCD. From [Karsch 2002]. $\epsilon_{\rm SB}$ *is the energy density for free quarks and gluons. The* 20% *deficit in* $\epsilon/\epsilon_{\rm SB}$ *is suggestive of strong coupling.*



- $T_c \approx 170$ MeV.
- RHIC operates at $T \approx 280$ MeV.
- LHC will operate at $T \approx\approx 600$ MeV.

In a bottom-up approach [Gubser and Nellore 2008], we can reproduce the lattice eos using

$$\mathcal{L} = \frac{1}{2\kappa_5^2}\left[R - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right] . \qquad (15)$$

$V(\phi)$ can be adjusted to match dependence of

$$\text{speed of sound:} \quad c_s^2 \equiv \frac{dp}{d\epsilon} \qquad (16)$$

on T. Then adjust κ_5^2 to get desired ϵ/T^4 at some high scale (say $3\,\text{GeV}$). Here's a quasi-realistic choice:

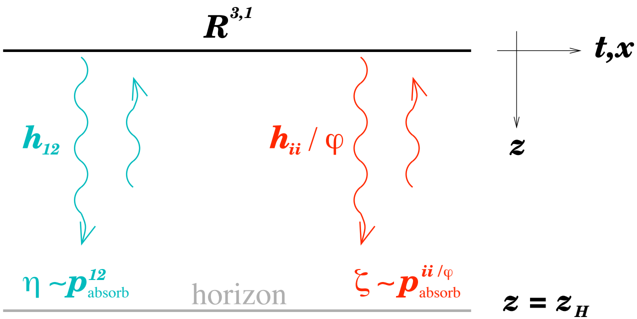
$$V(\phi) = \frac{-12\cosh\gamma\phi + b\phi^2}{L^2} \qquad \gamma = 0.606\,, \quad b = 2.057\,. \qquad (17)$$

Authors of [Gursoy and Kiritsis 2008; Gursoy et al. 2008ab] took same starting point (15) further: an appropriate $V(\phi)$, with $V \sim -\phi^2 e^{\sqrt{\frac{2}{3}}\phi}$, gives a Hawking-Page transition to confinement, logarithmic RG in UV, and glueball with $m^2 \sim n$, as in linear confinement.

Once conformal invariance is broken, we can investigate bulk viscosity [Gubser et al. 2008cb], following a number of earlier works, e.g. [Parnachev and Starinets 2005; Buchel 2005 2007]:

$$\zeta = \frac{1}{9} \lim_{\omega \to 0} \frac{1}{\omega} \operatorname{Im} \int d^3x\, dt\, e^{i\omega t} \theta(t) \langle [T^\mu{}_\mu(t, \vec{x}), T^\nu{}_\nu(0,0)] \rangle \,. \tag{18}$$

Shear viscosity relates to absorption probability for an h_{12} graviton. Bulk viscosity relates to absorption of a mixture of the h_{ii} graviton and the scalar ϕ.

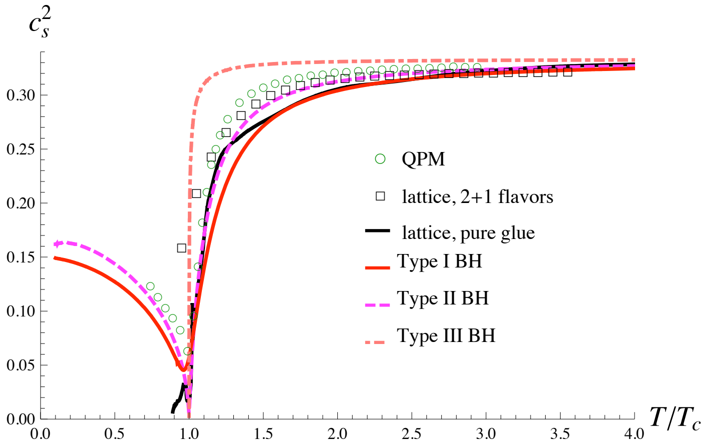


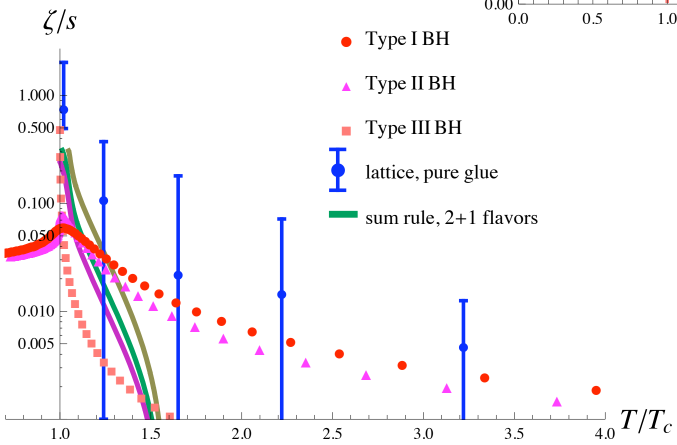
$$ds^2 = e^{2A(r)} \left(-h(r) dt^2 + d\vec{x}^2 \right) + e^{2B(r)} \frac{dr^2}{h(r)} \qquad \phi = \phi(r) \,. \tag{19}$$

In a gauge where $\delta\phi = 0$, let's set $h_{11} = e^{-2A} \delta g_{11} = e^{-2A} \delta g_{22} = e^{-2A} \delta g_{33}$. Then

$$h_{11}'' = \left(-\frac{1}{3A'} - 4A' + 3B' - \frac{h'}{h} \right) h_{11}' + \left(-\frac{e^{-2A+2B}}{h^2} \omega^2 + \frac{h'}{6hA'} - \frac{h'B'}{h} \right) h_{11} \tag{20}$$

- Type I: smooth cross-over, like (17).
- Type II: nearly second order, $c_s^2 \to 0$ at T_c.
- Type III: No BH below T_c, like [Gursoy et al. 2008b].

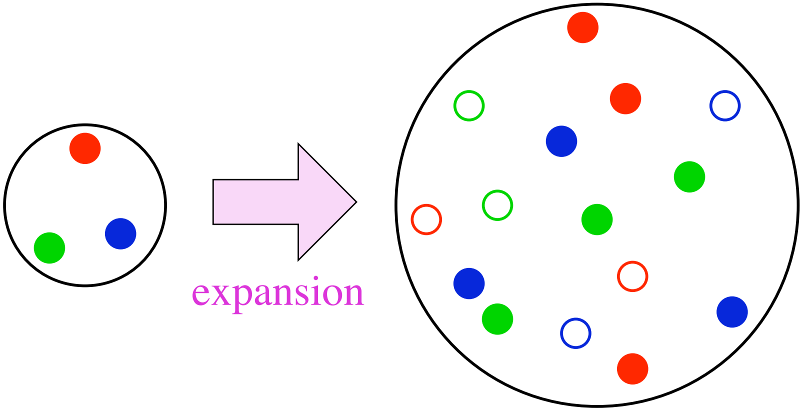




- Sharper behavior of c_s^2 gives sharper ζ/s.
- Large ζ at T_c is hard to arrange with a reasonably realistic EOS.
- Poses a challenge for "soft statistical hadronization" proposal of [Karsch et al. 2007].

Is bulk viscosity experimentally relevant?

Interesting proposal of Kharzeev and collaborators [Kharzeev and Tuchin 2007; Karsch et al. 2007]: bulk viscosity is a strong correction to hydro at $T = T_c$ leading to last-instant entropy production accompanying freezeout:



If ζ is large, much entropy / many soft particles are produced as thermal medium expands. This depiction is in imitation of a figure in [Kharzeev].

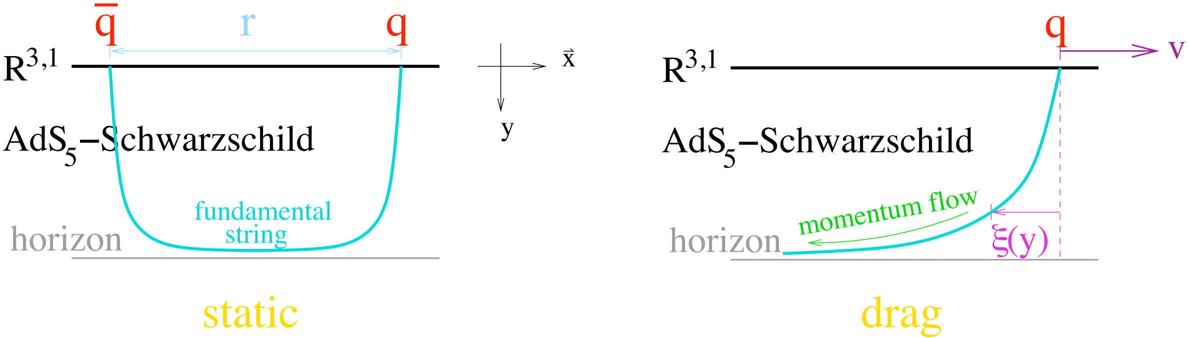
Bottom-up calculations in AdS suggest that it's hard to get $\zeta/s > 0.1$ with quasi-realistic eos. If that's right, then expansion-induced entropy is probably not so significant.

4. The trailing string

A heavy external quark moving at speed v experiences a drag force [Herzog et al. 2006; Gubser 2006a] (see also [Casalderrey-Solana and Teaney 2006]):

$$\frac{dp}{dt} = -\frac{\pi\sqrt{\lambda}}{2} T^2 \frac{v}{\sqrt{1-v^2}} \,. \tag{21}$$

(21) arises in a simple way: a fundamental string trails out behind the quark into AdS_5-Schwarzschild, pulling back upon it.



Static force versus drag force. In both cases, the classical shape of the string is known analytically.

Mass is formally infinite, but if we use instead a finite heavy quark mass M, find

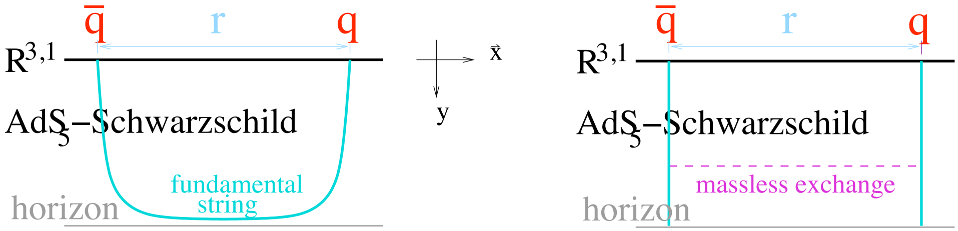
$$\frac{dp}{dt} = -\frac{p}{\tau_Q} \qquad \text{where} \quad \tau_Q = \frac{2}{\pi\sqrt{\lambda}}\frac{M}{T^2}\,, \tag{22}$$

So characteristic stopping length / time is τ_Q.

To get a numerical value for τ_Q, I favor comparing $\mathcal{N} = 4$ SYM to QCD at *fixed energy density* rather than temperature. $SU(3)$ SYM has about $3\times$ the number of degrees of freedom as QCD, and I expect τ_Q to decrease with number of dof's.

To fix λ, I favor [Gubser 2006c] using the following effective measure of α_s:

$$\alpha_{q\bar{q}}(r,T) \equiv \frac{3}{4} r^2 \frac{\partial F_{q\bar{q}}}{\partial r} \qquad F_{q\bar{q}} \text{ is excess free energy from heavy } q\text{-}\bar{q} \text{ pair.} \tag{23}$$

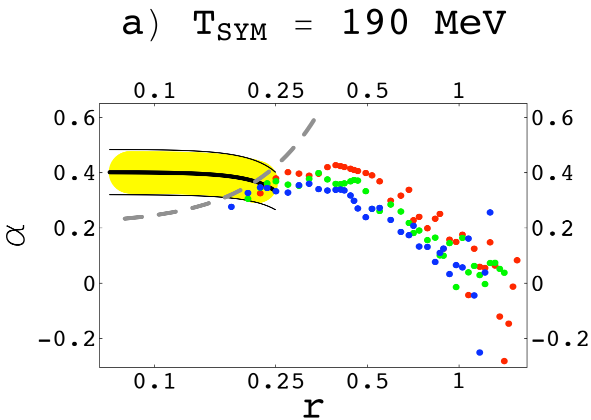


Two string theory configurations contributing to $F_{q\bar{q}}$. Only U-shape is fully understood. But see [Bak et al. 2007] for recent work on exchange diagram.

Simplest approximation to U-curve contribution is zero temperature result:

$$\alpha_{\rm SYM}(T=0) \equiv \frac{3}{4} r^2 \frac{\partial V_{q\bar{q}}}{\partial r} = \sqrt{\lambda} \frac{3\pi^2}{\Gamma(1/4)^4} \,. \tag{24}$$

To fix $\lambda \approx 5.5$, compare to lattice at largest r where U-shape dominates.



Static quark force for $\mathcal{N}=4$ SYM (yellow band) versus $N_f=2$ lattice results from [Kaczmarek and Zantow 2005].

- $\epsilon_{\rm SYM} = \epsilon_{\rm QCD}$ means $T_{\rm SYM} = T_{\rm QCD}/3^{1/4}$. I took $T_{\rm QCD} \approx 250\,{\rm MeV}$ here.
- An alternative perspective can be found in [Sin and Zahed 2007].

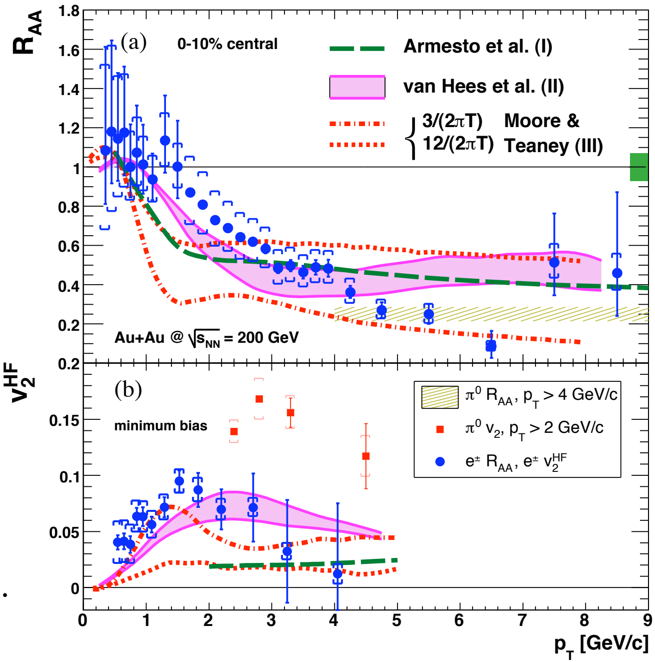
The match is conspicuously imperfect! At least we fix λ from a leading-order effect. Matching Debye length in large r tail gives even smaller λ [Bak et al. 2007].

A sensible alternative is $T_{\rm QCD} = T_{\rm SYM}$ with $\lambda \approx 6\pi$ from setting $\frac{g_{\rm YM}^2}{4\pi} = \alpha_s \approx 0.5$. Always, $N=3$.

Using my preferred comparison scheme, $\tau_c \approx 2\,\mathrm{fm}/c$ for charm at RHIC; also $\tau_b/\tau_c = m_b/m_c$. So charm equilibrates, and b does so only partially.

R_{AA} and v_2 for heavy quarks. p_T is for a non-photonic electron. From [Adare et al. 2006].

- Crudely, $R_{AA}(p_T)$ is the $\%$ of charm quarks escaping at a given transverse momentum.
- But p_T shown is for $e^{\pm}$ decay product, so roughly double it to get p_T of c.
- Smaller R_{AA} and bigger v_2 go together.
- van Hees curves have $\tau_c \approx 4.5\,\mathrm{fm}$.



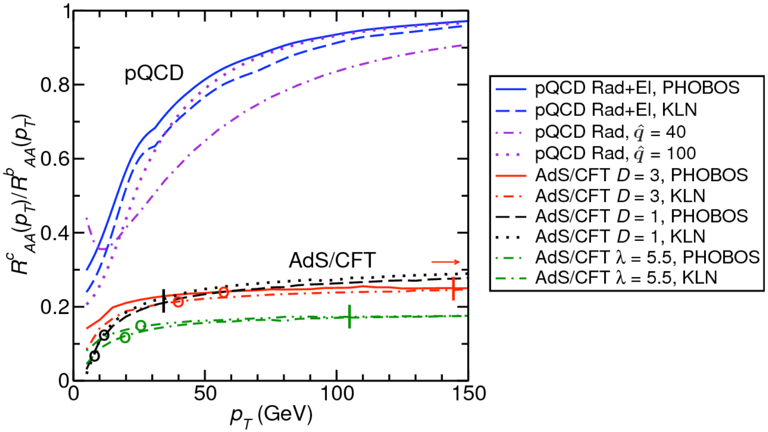
Upshot: Data favors larger τ_c, but not much larger, than string theory analysis. For an alternative viewpoint, see e.g. [Teaney 2008]; also, beware b contribution.

Tagging b's and c's should be possible after detector upgrades at RHIC, and at LHC.

A distinctive difference [Horowitz and Gyulassy 2007] between pQCD and AdS/CFT predictions from RHIC to LHC energies may come from

$$R_{AA}^{cb} \equiv \frac{R_{AA}^b}{R_{AA}^c} \sim \begin{cases} \dfrac{t_{\rm bottom}}{t_{\rm charm}} \approx \dfrac{m_{\rm charm}}{m_{\rm bottom}} & \text{for AdS/CFT} \\ 1 - p_{cb}/p_T & \text{for pQCD, } p_{cb} \propto \hat{q} L^2 \end{cases} \tag{25}$$

pQCD predictions for R_{AA}^{cb} separate cleanly from AdS/CFT because assumptions about initial conditions cancel out. But beware uncertainty on the limits of validity of AdS/CFT.



Related studies by Brasoveanu and d'Enterria are in progress.

4.1. Stochastic forces on heavy quarks

Drag force is not the whole story: in a Langevin description [Casalderrey-Solana and Teaney 2006; Gubser 2006b; Casalderrey-Solana and Teaney 2007]

$$\frac{d\vec{p}}{dt} = -\eta\vec{p} + \vec{F}(t) \qquad \eta = \frac{\pi\sqrt{\lambda}T^2}{2m} \tag{26}$$

where $\vec{F}$ is a *stochastic* force: if $\vec{p}$ is in the $\hat{1}$ direction, then

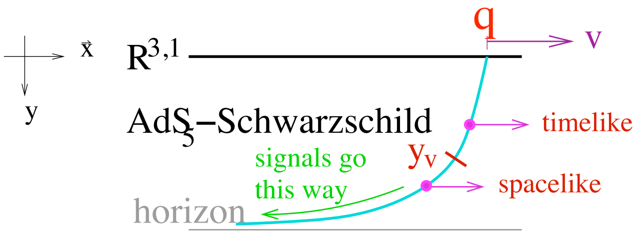
$$\begin{aligned} \langle F_1(t_1)F_1(t_2)\rangle &\approx \kappa_L \delta(t_1 - t_2)\,, \qquad & \kappa_L &= \pi\sqrt{\lambda}\frac{T^3}{(1-v^2)^{5/4}} \\ \langle F_i(t_1)F_j(t_2)\rangle &\approx \kappa_T \delta_{ij}\delta(t_1 - t_2)\,, \qquad & \kappa_T &= \pi\sqrt{\lambda}\frac{T^3}{\sqrt[4]{1-v^2}} \end{aligned} \tag{27}$$

String theory value for κ_L exceeds Einstein relation except near $v = 0$:

$$\kappa_L = \frac{1}{(1-v^2)^{3/4}} 2TE\eta\,, \tag{28}$$

hinting that Langevin description doesn't capture all the physics.

Also: correlation time in $\vec{F}(t)$ diverges as $1/\sqrt[4]{1-v^2}$.



The horizon on the worldsheet is at $y = y_v$.

Stochastic fluctuations are controlled by *causal horizon on the worldsheet.*

AdS_5-Schwarzschild geometry is

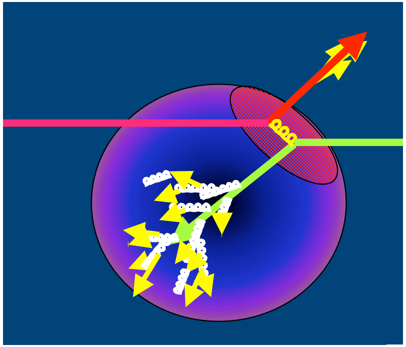
$$ds_5^2 = \frac{L^2 \pi^2 T^2}{y^2} \left[-(1-y^4)dt^2 + d\vec{x}^2 + \frac{1}{\pi^2 T^2} \frac{dy^2}{1-y^4} \right] . \qquad (29)$$

Consider observers who stay at fixed y while holding onto the trailing string:

- $d\tau^2 > 0$ if $y > y_v \equiv \sqrt[4]{1-v^2}$: "outside" the worldsheet black hole.
- $d\tau^2 < 0$ if $y < y_v$: "inside" the worldsheet black hole. The observer can't stay at fixed y, but slides down the string.

Something roughly like Hawking radiation must emanate from the worldsheet horizon, leading to stochastic $\vec{F}(t)$. Actual computations directly access $\langle F_i(t_1) F_j(t_2) \rangle$.

5. Jet-splitting?

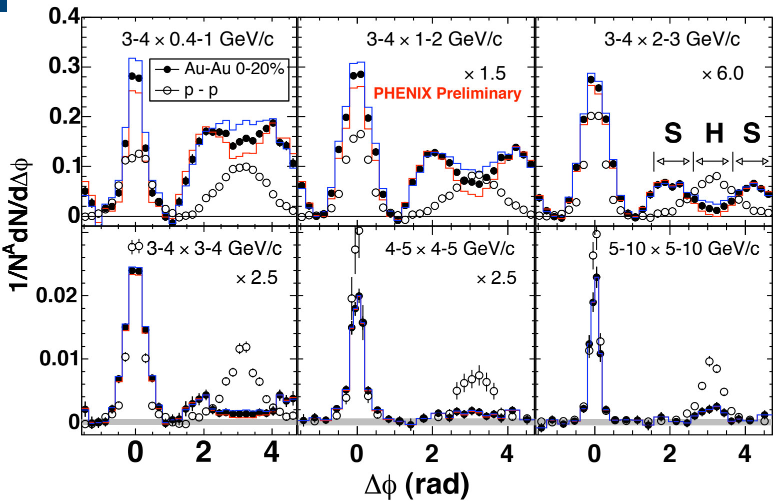


A hard process occurring near the edge of the medium produces a near-side "trigger" jet (red). The away-side parton interacts strongly with the medium. From [Jacak 2006].

Jet reconstruction is impractical, so make histograms of azimuthal separation between two energetic hadrons.

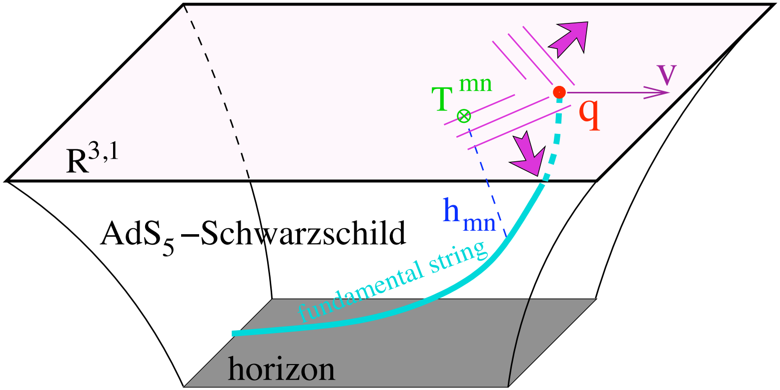
With appropriate p_T cuts, observe a double-hump structure on away-side: "jet-splitting." From [Jia 2007].

More inclusive cuts fill in the region around $\Delta\phi = \pi$: "jet-broadening" [Adams et al. 2005].



A string theory calculation has been done *for heavy quarks*: [Gubser et al. 2007; Chesler and Yaffe 2007] and refs therein.

A heavy quark trails a string behind it. The string couples to gravitons dual to $\langle T_{mn} \rangle$ in the gauge theory.

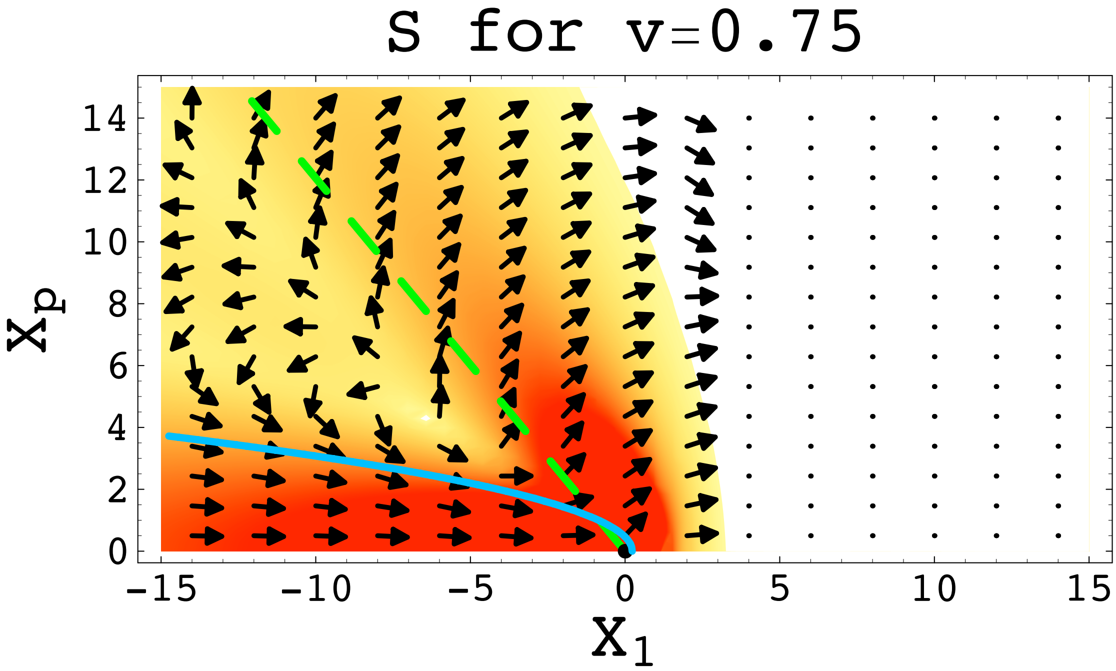


Calculate h_{mn} using linearized Einstein equations.

One big calculation gives $\langle T^{0m} \rangle$ over a broad range of scales; high k asymptotics pioneered in [Yarom 2007] turn out to be especially interesting.

Render all quantities dimensionless:

$$\vec{X} = \pi T \vec{x} \qquad S_i(\vec{X}) \equiv \frac{\sqrt{1-v^2}}{(\pi T)^4 \sqrt{\lambda}} \left\langle T^{0i}(0,\vec{x}) - T^{0i}_{\text{Coulomb}}(0,\vec{x}) \right\rangle . \tag{30}$$



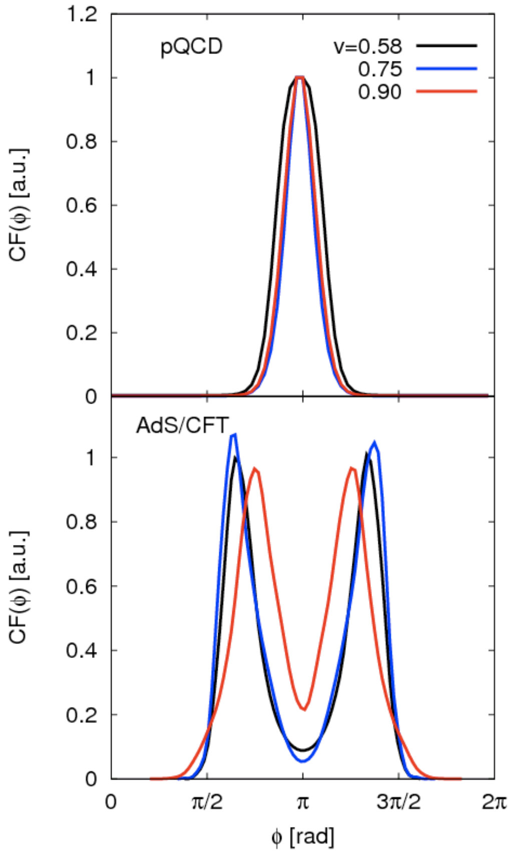
Rescaled, subtracted Poynting vector generated by a quark in an infinite, static medium. Green shows the Mach angle, and blue shows the parabolic boundary of the diffusion wake. For $T \approx 318\,\mathrm{MeV}$, $|\vec{X}| = 5$ *is a distance* $1\,\mathrm{fm}$ *from the quark. From [Gubser et al. 2007].*

A phenomenological comparison [Betz et al. 2008] including Cooper-Frye hadronization shows that AdS/CFT does lead to jet-splitting at $p_T \approx 5\,\mathrm{GeV}$.

But the reason is unexpected: it's *not* the hydro region that does it, it's the "neck" region with $|x| \lesssim 1\,\mathrm{fm}$.

Puzzles / problems remain:

- Pseudo-Mach angle is smaller than data, and gets smaller as $v \to 1$.
- This was for heavy quarks!
- Cooper-Frye isn't perfect.
- Interpretation of experimental phenomenon isn't universally agreed upon.



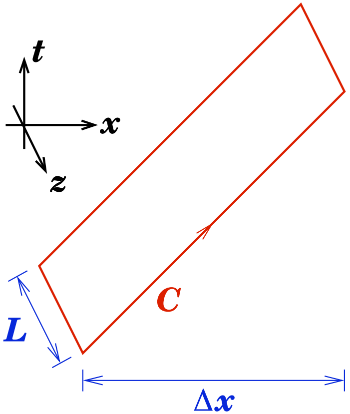
6. Jet quenching

According to pQCD (e.g. [Baier et al. 1997; Zakharov 1997; Wiedemann 2000]), radiative energy loss by light quarks and gluons is

$$\Delta E = \frac{1}{4}\alpha_s C_R \hat{q} (\Delta x)^2 , \tag{31}$$

where the jet-quenching parameter describes how fast momentum broadens as a function of path length Δx:

$$\hat{q} = \frac{\langle p_\perp^2 \rangle}{\Delta x} . \tag{32}$$



Authors including [Kovner and Wiedemann 2003; Liu et al. 2006] prefer a definition in terms of a partially light-like Wilson loop with $L \ll \Delta x$:

$$\langle W^{\rm adjoint}(\mathcal{C}) \rangle \approx \exp\left[-\frac{1}{4}\hat{q} L^2 \Delta x \right] . \tag{33}$$

A gauge-string calculation of $\langle W^{\rm fundamental} \rangle$ leads to

$$\hat{q} = \frac{\pi^{3/2}\Gamma(3/4)}{\Gamma(5/4)} \sqrt{\lambda}\, T^3 . \tag{34}$$

A correction factor $\sqrt{s_{\rm QCD}/s_{\rm SYM}}$ is advocated in [Liu et al. 2007] to correct for fewer degrees of freedom. Including this factor and using $\lambda = 6\pi$, as they prefer, I calculate

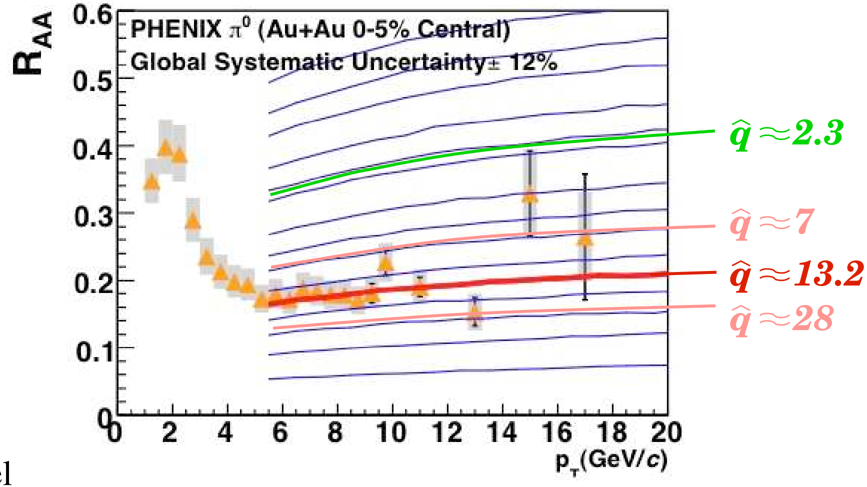
$$\hat{q} \approx 2.3 \frac{\rm GeV^2}{\rm fm} \qquad \text{at } T = 280\,{\rm MeV}, \tag{35}$$

significantly above pQCD's $\hat{q} \approx 0.77\,{\rm GeV}^2/{\rm fm}$ and almost big enough to agree with experiment (more later).

But some puzzles remain:

- Argyres and collaborators criticize the choice of saddle point [Argyres et al. 2007 2008] and find $\log\langle W^A(\mathcal{C})\rangle \sim L$ not L^2.
- $\hat{q}$ as defined through Wilson loop may not be directly related to energy loss or momentum diffusion in strongly coupled gauge theories.
- Independent calculations of $\hat{q}_T \equiv \langle p_\perp^2\rangle/\Delta x$ for heavy quarks [Herzog et al. 2006; Casalderrey-Solana and Teaney 2006 2007; Gubser 2006b] lead to larger values than (34): larger by $\sim \sqrt{\gamma}$ as $v \to 1$.

From [Adare et al. 2008], with minor additions. Predictions of PQM model [Dainese et al. 2005] versus PHENIX data. All values of $\hat{q}$ *are in* GeV^2/fm.



Best fit curve (red) has $\hat{q} = 13.2\,\text{GeV}^2/\text{fm}$.

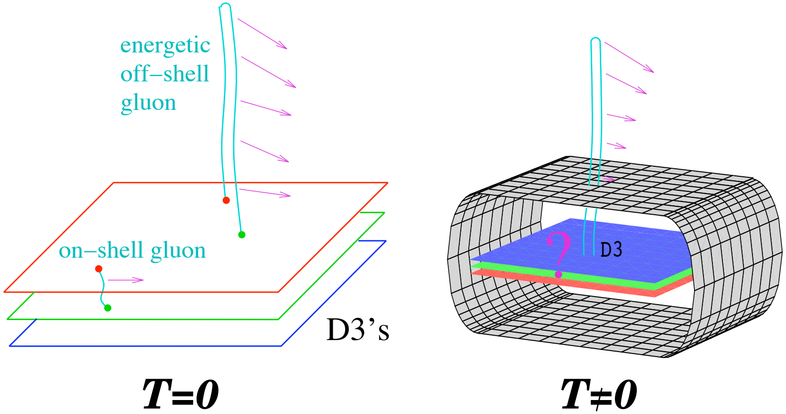
3σ range is $7 < \hat{q} < 28\,\text{GeV}^2/\text{fm}$.

Parton Quenching Model is based on many soft momentum transfers between medium and hard partons. Other formalisms exist (see e.g. [Gyulassy et al. 2001; Arnold et al. 2002; Wang 2004]) for connecting pQCD to data.

7. Falling strings

Can we calculate *ab initio* the energy loss of a gluon in strongly coupled $\mathcal{N} = 4$?

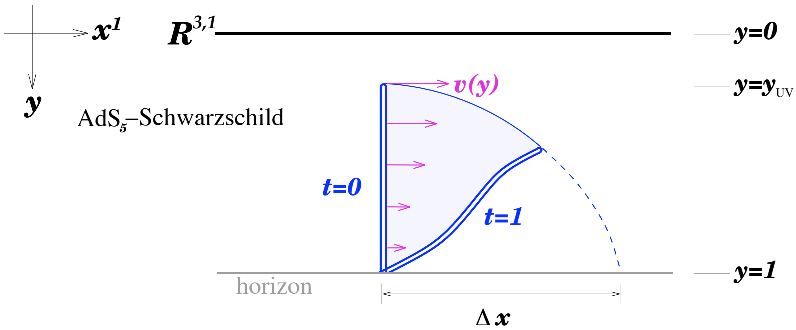
We propose [Gubser et al. 2008a] to regard an off-shell gluon as a doubled string with both ends passing through the horizon.



At zero temperature, results of [Alday and Maldacena 2007] show that gluon scattering produces approximately this type of string configuration.

At finite temperature, something funny happens: where the string crosses the horizon, it can't move! (Infinite red-shifting wrt Killing time t.)

A doubled string starts at $t = 0$ *with some total energy and virtuality, then falls into the horizon over a distance* Δx.



- Given initial E, what is Δx?
- Answer must depend on virtuality $\leftrightarrow$ $y_{\rm UV}$, so what is *maximum* Δx?
- How do we roughly convert the answer to $\hat{q}$?

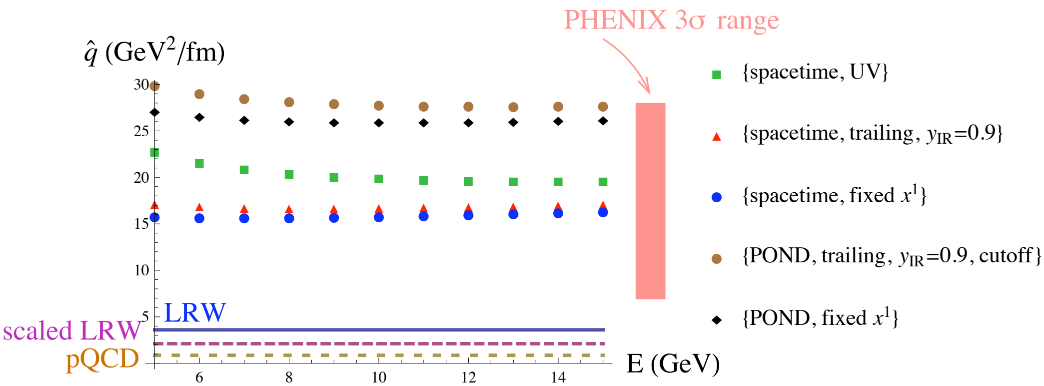
We made estimates based on assuming the shape of the falling string quickly approaches a segment of the trailing string; confirmed numerically in [Chesler et al. 2008].

For $E \gg T$, we found $\Delta\hat{x} \approx \hat{E}^{1/3}$ (see also [Hatta et al. 2008]), where

$$\hat{x} = \pi T x \qquad \hat{E} = \frac{1}{\sqrt{g_{YM}^2 N}} \frac{E}{T} \,. \tag{36}$$

This is not too different from pQCD prediction $\Delta x \propto \sqrt{E/\hat{q}}$. So let's convert to a rough prediction of $\hat{q}$:

$$\hat{q}_{\rm rough} \equiv \frac{4E}{3\alpha_s(\Delta x)^2} \,. \tag{37}$$



Estimates of the jet-quenching parameter, from (37), **comparing at fixed energy density, with** $\lambda = 5.5$*. Different symbols correspond to varying assumptions about shape of falling string. From [Gubser et al. 2008a]. LRW is from [Liu et al. 2006] at* $280\,{\rm MeV}$*, for SYM; scaled LRW is for QCD at* $280\,{\rm MeV}$*, including the* $\sqrt{s_{\rm QCD}/s_{\rm SYM}}$ *factor from [Liu et al. 2007].*

The overall picture on jet-quenching is, in my view, somewhat muddled at present:

- Good that we're within 3σ range, or close.
- Good that we can accommodate gluons that start off significantly virtual.
- Questionable to compare $\hat{q}$ from falling strings to a value in PQM model, where underlying assumptions are different.
- Bad that we don't understand relation among jet-quenching calculations, plus heavy quark drag / diffusion.
- Interesting to consider including fluctuations or graviton response, starting either from [Liu et al. 2006] or [Gubser et al. 2008a].
- Maybe good that numerical study [Chesler et al. 2008] shows larger Δx (so smaller $\hat{q}$) for falling strings; or was that due to initial conditions?

8. Total multiplicity

Central RHIC collision:

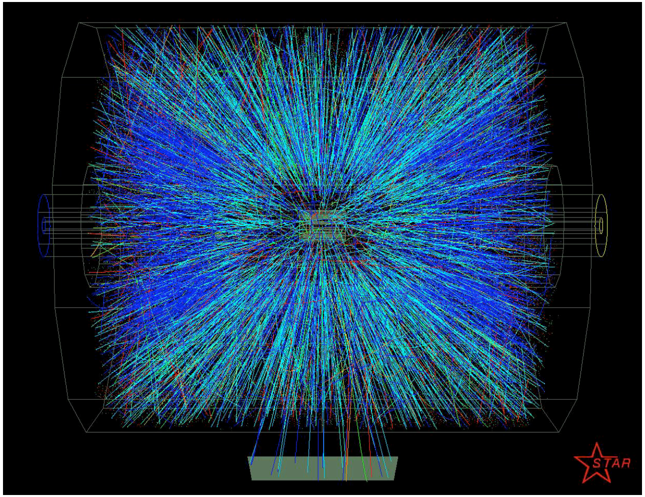
$N_{\text{part}} \approx 2 \times 197 = 394$ nucleons in

$N_{\text{ch}} \approx 5000$ charged particles out.

A reasonable estimate of the entropy produced is

$$S \approx 7.5 N_{\text{charged}} \approx 38000\,, \quad (38)$$

(E.g. consider a gas of free hadrons at T_c and compute S/N_{charged} starting from partition function.)



Charged tracks measured by STAR in a gold-gold collision [STA]. For multiplicitly estimates, see e.g. PHOBOS's [Back et al. 2005].

How well can we estimate S from the gauge-string duality?

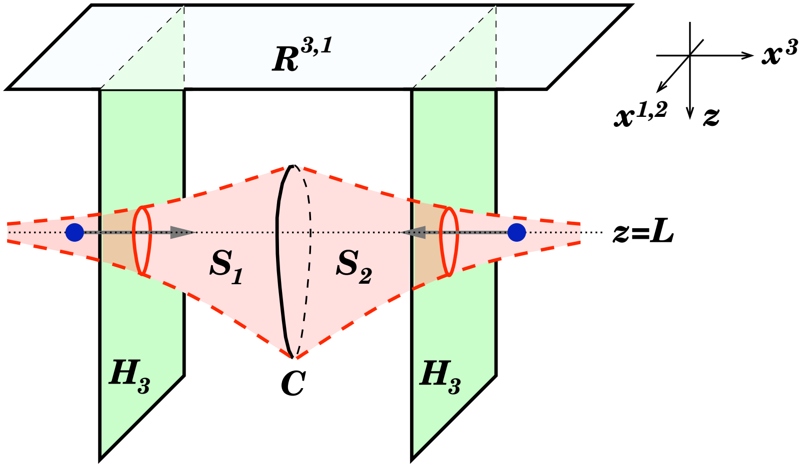
Strategy of [Gubser et al. 2008d]:

- Replace QCD by a conformal theory with $\epsilon/T^4 = 11$, as lattice predicts for QCD for $T \gtrsim 1.2T_c$. (Remarkably slow rise thereafter.)

- Replace a heavy ion with a boosted "conformal soliton," dual to a point-sourced gravitational shock wave in AdS_5: if $x^- = x^0 - x^3$, then

$$\langle T_{--} \rangle = \frac{2EL}{\pi \left[(x^1)^2 + (x^2)^2 + L^2 \right]^3} \delta(x^-) \,, \tag{39}$$

(Power law tails are not a good thing, but at least they're a big power: $1/x_\perp^6$.)



Trapped surface is typically on past light-like trajectory of shocks; shown here is projection to $t = 0$.

A standard but non-rigorous lower bound is

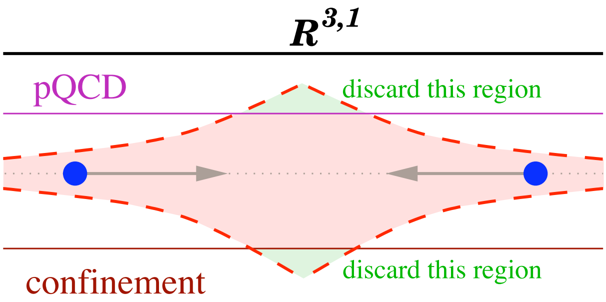
$$S \geq S_{\text{trapped}} \equiv A_{\text{trapped}}/4G_5 \,.$$

Earlier related work is reviewed in [Nastase 2008].

The final result is

$$S_{\text{trapped}} \approx \pi \left(\frac{L^3}{G_5} \right)^{1/3} (2EL)^{2/3} \approx 35000 \left(\frac{\sqrt{s_{NN}}}{200\,\text{GeV}} \right)^{2/3} . \qquad (40)$$

- I set $L = 4.3\,\text{fm}$ to match the rms transverse radius of a gold nucleus.
- $E \approx 19.7\,\text{GeV}$ is beam energy; $\sqrt{s_{NN}} = 200\,\text{GeV}$ is cm energy of a pair of nucleons (NN).



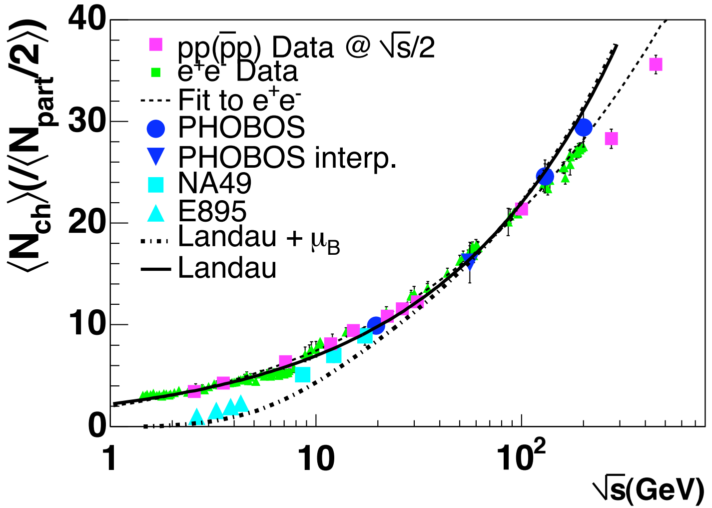
$E^{2/3}$ scaling is faster than Landau ($E^{1/2}$) [Landau 1953] and faster than data ($\approx$ Landau).

I think it's because strong-coupling conformal window covers only a range of scales. A crude solution [Gubser et al.]:

Assume that most entropy is generated within this range, above confinement and below pQCD.

UV cutoff changes scaling from $S_{\text{trapped}} \sim E^{2/3}$ to $E^{1/3}$ at large E. So anticipate $N_{\text{charged}} \sim E^{1/3}$. Maybe even for protons?

Roll-over from Landau's $E^{1/2}$ to slower growth might just be starting at top RHIC energies:



Total multiplicity per participant as a function of energy. From [Steinberg 2005].

9. Outlook

- Gauge-string / Heavy-ion connection is the closest interface we have between modern string theory and modern experiment.
- *Many* comparisons are successful at a semi-quantitative level. (Many more than I have summarized here...)
- Comparisons are invariably plagued by the difficulty of translating from AdS calculations to real-world QCD.
- We may often be measuring our successes against prevailing interpretations of data rather than data itself.
- At the least, gauge-string calculations show what happens in a truly strongly coupled thermal plasma.
- Insights from AdS/CFT complement pQCD intuitions and may sometimes be closer to capturing the true dynamics.

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STAR collision image, from http://www.bnl.gov/RHIC/full_en_images.htm.

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