# Four-loop perturbative Konishi from the $AdS_5 \times S^5$ string sigma model

Romuald A. Janik

Jagellonian University
Krakow

Strings 2008

Z. Bajnok, RJ: 0807.0499

# Outline

# Motivation

Basic question:

Find the spectrum of $\mathcal{N} = 4$ SYM theory (in the planar limit) for any value of 't Hooft coupling $\lambda = g_{YM}^2 N_c$

$$\begin{aligned} \text{Spectrum} \quad &\equiv \quad \text{set of eigenvalues of the dilatation operator} \\ &\equiv \quad \text{anomalous dimensions of all operators in SYM} \end{aligned}$$

Equivalently:

Find the spectrum of free (large $N_c$) type IIB superstring on $AdS_5 \times S^5$ for any value of $\lambda$

Spectrum $\equiv$ set of quantized energy levels of the worldsheet superstring QFT

# Motivation

Basic question:

Find the spectrum of $\mathcal{N} = 4$ SYM theory (in the planar limit) for any value of 't Hooft coupling $\lambda = g_{YM}^2 N_c$

Spectrum $\equiv$ set of eigenvalues of the dilatation operator
$\equiv$ anomalous dimensions of all operators in SYM

Equivalently:

Find the spectrum of free (large $N_c$) type IIB superstring on $AdS_5 \times S^5$ for any value of $\lambda$

Spectrum $\equiv$ set of quantized energy levels of the worldsheet superstring QFT

# Motivation

Basic question:

Find the spectrum of $\mathcal{N} = 4$ SYM theory (in the planar limit) for any value of 't Hooft coupling $\lambda = g_{YM}^2 N_c$

Spectrum $\equiv$ set of eigenvalues of the dilatation operator
$\equiv$ anomalous dimensions of all operators in SYM

Equivalently:

Find the spectrum of free (large $N_c$) type IIB superstring on $AdS_5 \times S^5$ for any value of $\lambda$

Spectrum $\equiv$ set of quantized energy levels of the worldsheet superstring QFT

## Motivation

Basic question:

Find the spectrum of $\mathcal{N} = 4$ SYM theory (in the planar limit) for any value of 't Hooft coupling $\lambda = g_{YM}^2 N_c$

| | | |
|---|---|---|
| Spectrum | $\equiv$ | set of eigenvalues of the dilatation operator |
| | $\equiv$ | anomalous dimensions of all operators in SYM |

Equivalently:

Find the spectrum of free (large $N_c$) type IIB superstring on $AdS_5 \times S^5$ for any value of $\lambda$

Spectrum $\equiv$ set of quantized energy levels of the worldsheet superstring QFT

# Motivation

Basic question:

Find the spectrum of $\mathcal{N} = 4$ SYM theory (in the planar limit) for any value of 't Hooft coupling $\lambda = g_{YM}^2 N_c$

Spectrum $\equiv$ set of eigenvalues of the dilatation operator
$\equiv$ anomalous dimensions of all operators in SYM

Equivalently:

Find the spectrum of free (large $N_c$) type IIB superstring on $AdS_5 \times S^5$ for any value of $\lambda$

Spectrum $\equiv$ set of quantized energy levels of the worldsheet superstring QFT

# Motivation

Basic question:

Find the spectrum of $\mathcal{N} = 4$ SYM theory (in the planar limit) for any value of 't Hooft coupling $\lambda = g_{YM}^2 N_c$

| | | |
|---|---|---|
| Spectrum | $\equiv$ | set of eigenvalues of the dilatation operator |
| | $\equiv$ | anomalous dimensions of all operators in SYM |

Equivalently:

Find the spectrum of free (large $N_c$) type IIB superstring on $AdS_5 \times S^5$ for any value of $\lambda$

Spectrum $\equiv$ set of quantized energy levels of the worldsheet superstring QFT

# Motivation

Goal:

- Continue string theory results all the way from strong to weak coupling to make contact with gauge theory perturbative computations
- Direct verification of AdS/CFT!!

Starting point:

- use worldsheet QFT of the $AdS_5 \times S^5$ superstring in the uniform light-cone gauge
- highly interacting theory
- gauge theory perturbative regime $\equiv$ deeply quantum regime of the theory
- Integrability: the exact S-matrix of the 2D worldsheet QFT of the superstring in $AdS_5 \times S^5$ is believed to be known
- use this information to perform the computation

$\longrightarrow$ compute the anomalous dimension of the Konishi operator at 4-loop order

# Motivation

Goal:

- Continue string theory results all the way from strong to weak coupling to make contact with gauge theory perturbative computations
- Direct verification of AdS/CFT!!

Starting point:

- use worldsheet QFT of the $AdS_5 \times S^5$ superstring in the uniform light-cone gauge
- highly interacting theory
- gauge theory perturbative regime $\equiv$ deeply quantum regime of the theory
- Integrability: the exact S-matrix of the 2D worldsheet QFT of the superstring in $AdS_5 \times S^5$ is believed to be known
- use this information to perform the computation

$\longrightarrow$ compute the anomalous dimension of the Konishi operator at 4-loop order

# Motivation

Goal:

- Continue string theory results all the way from strong to weak coupling to make contact with gauge theory perturbative computations
- Direct verification of AdS/CFT!!

Starting point:

- use worldsheet QFT of the $AdS_5 \times S^5$ superstring in the uniform light-cone gauge
- highly interacting theory
- gauge theory perturbative regime $\equiv$ deeply quantum regime of the theory
- Integrability: the exact S-matrix of the 2D worldsheet QFT of the superstring in $AdS_5 \times S^5$ is believed to be known
- use this information to perform the computation

$\longrightarrow$ compute the anomalous dimension of the Konishi operator at 4-loop order

## Motivation

Goal:

- Continue string theory results all the way from strong to weak coupling to make contact with gauge theory perturbative computations
- Direct verification of AdS/CFT!!

Starting point:

- use worldsheet QFT of the $AdS_5 \times S^5$ superstring in the uniform light-cone gauge
- highly interacting theory
- gauge theory perturbative regime $\equiv$ deeply quantum regime of the theory
- Integrability: the exact S-matrix of the 2D worldsheet QFT of the superstring in $AdS_5 \times S^5$ is believed to be known
- use this information to perform the computation

$\longrightarrow$ compute the anomalous dimension of the Konishi operator at 4-loop order

## Motivation

Goal:

- Continue string theory results all the way from strong to weak coupling to make contact with gauge theory perturbative computations
- Direct verification of AdS/CFT!!

Starting point:

- use worldsheet QFT of the $AdS_5 \times S^5$ superstring in the uniform light-cone gauge
- highly interacting theory
- gauge theory perturbative regime $\equiv$ deeply quantum regime of the theory
- Integrability: the exact S-matrix of the 2D worldsheet QFT of the superstring in $AdS_5 \times S^5$ is believed to be known
- use this information to perform the computation

$\longrightarrow$ compute the anomalous dimension of the Konishi operator at 4-loop order

# Motivation

Goal:

- Continue string theory results all the way from strong to weak coupling to make contact with gauge theory perturbative computations
- Direct verification of AdS/CFT!!

Starting point:

- use worldsheet QFT of the $AdS_5 \times S^5$ superstring in the uniform light-cone gauge
- highly interacting theory
- gauge theory perturbative regime $\equiv$ deeply quantum regime of the theory
- Integrability: the exact S-matrix of the 2D worldsheet QFT of the superstring in $AdS_5 \times S^5$ is believed to be known
- use this information to perform the computation

$\longrightarrow$ compute the anomalous dimension of the Konishi operator at 4-loop order

## Motivation

Goal:

- Continue string theory results all the way from strong to weak coupling to make contact with gauge theory perturbative computations
- Direct verification of AdS/CFT!!

Starting point:

- use worldsheet QFT of the $AdS_5 \times S^5$ superstring in the uniform light-cone gauge
- highly interacting theory
- gauge theory perturbative regime $\equiv$ deeply quantum regime of the theory
- Integrability: the exact S-matrix of the 2D worldsheet QFT of the superstring in $AdS_5 \times S^5$ is believed to be known
- use this information to perform the computation

$\longrightarrow$ compute the anomalous dimension of the Konishi operator at 4-loop order

# Motivation

Goal:

- Continue string theory results all the way from strong to weak coupling to make contact with gauge theory perturbative computations
- Direct verification of AdS/CFT!!

Starting point:

- use worldsheet QFT of the $AdS_5 \times S^5$ superstring in the uniform light-cone gauge
- highly interacting theory
- gauge theory perturbative regime $\equiv$ deeply quantum regime of the theory
- Integrability: the exact S-matrix of the 2D worldsheet QFT of the superstring in $AdS_5 \times S^5$ is believed to be known
- use this information to perform the computation

$\longrightarrow$ compute the anomalous dimension of the Konishi operator at 4-loop order

# Motivation

Goal:

- Continue string theory results all the way from strong to weak coupling to make contact with gauge theory perturbative computations
- Direct verification of AdS/CFT!!

Starting point:

- use worldsheet QFT of the $AdS_5 \times S^5$ superstring in the uniform light-cone gauge
- highly interacting theory
- gauge theory perturbative regime $\equiv$ deeply quantum regime of the theory
- Integrability: the exact S-matrix of the 2D worldsheet QFT of the superstring in $AdS_5 \times S^5$ is believed to be known
- use this information to perform the computation

$\longrightarrow$ compute the anomalous dimension of the Konishi operator at 4-loop order

## Motivation

Goal:

- Continue string theory results all the way from strong to weak coupling to make contact with gauge theory perturbative computations
- Direct verification of AdS/CFT!!

Starting point:

- use worldsheet QFT of the $AdS_5 \times S^5$ superstring in the uniform light-cone gauge
- highly interacting theory
- gauge theory perturbative regime $\equiv$ deeply quantum regime of the theory
- Integrability: the exact S-matrix of the 2D worldsheet QFT of the superstring in $AdS_5 \times S^5$ is believed to be known
- use this information to perform the computation

$\longrightarrow$ compute the anomalous dimension of the Konishi operator at 4-loop order

## The Konishi operator

- The Konishi operator $O_{Konishi} = \mathrm{tr}\,\Phi_i^2$
- The same supermultiplet has representatives in the $\mathfrak{su}(2)$ sector $\mathrm{tr}\, Z^2X^2 + \ldots$
- Anomalous dimension has been computed up to 4 loops ($g^2 = \frac{\lambda}{16\pi^2}$)

$$\Delta = 4 + 12g^2 - 48g^4 + 336g^6 + \underbrace{\left(-2496 + 576\zeta(3) - 1440\zeta(5)\right) g^8}_{[F.Fiamberti, A.Santambrogio, C.Sieg, D.Zanon]} + \ldots$$

(involves 139 supergraphs and 12 types of 4-loop integrals!)

- Why is it interesting to compute the 4-loop part?
  - Qualitatively new effects appear on the gauge theory side – wrapping interactions
  - First order which goes beyond the Bethe ansatz
  - A very stringent test of our understanding of the worldsheet superstring QFT
  - A computation from the string theory side will be a very nontrivial test of AdS/CFT

# The Konishi operator

- The Konishi operator $O_{Konishi} = \mathrm{tr}\, \Phi_i^2$
- The same supermultiplet has representatives in the su(2) sector $\mathrm{tr}\, Z^2 X^2 + \ldots$
- Anomalous dimension has been computed up to 4 loops ($g^2 = \frac{\lambda}{16\pi^2}$)

$$\Delta = 4 + 12g^2 - 48g^4 + 336g^6 + \underbrace{(-2496 + 576\zeta(3) - 1440\zeta(5))\, g^8}_{[F.Fiamberti, A.Santambrogio, C.Sieg, D.Zanon]} + \ldots$$

  (involves 139 supergraphs and 12 types of 4-loop integrals!)
- Why is it interesting to compute the 4-loop part?
  - Qualitatively new effects appear on the gauge theory side – wrapping interactions
  - First order which goes beyond the Bethe ansatz
  - A very stringent test of our understanding of the worldsheet superstring QFT
  - A computation from the string theory side will be a very nontrivial test of AdS/CFT

# The Konishi operator

- The Konishi operator $O_{Konishi} = \mathrm{tr}\,\Phi_i^2$
- The same supermultiplet has representatives in the su(2) sector $\mathrm{tr}\, Z^2 X^2 + \ldots$
- Anomalous dimension has been computed up to 4 loops ($g^2 = \frac{\lambda}{16\pi^2}$)

$$\Delta = 4 + 12g^2 - 48g^4 + 336g^6 + \underbrace{\left(-2496 + 576\zeta(3) - 1440\zeta(5)\right) g^8}_{[F.Fiamberti, A.Santambrogio, C.Sieg, D.Zanon]} + \ldots$$

(involves 139 supergraphs and 12 types of 4-loop integrals!)

- Why is it interesting to compute the 4-loop part?
  - Qualitatively new effects appear on the gauge theory side – wrapping interactions
  - First order which goes beyond the Bethe ansatz
  - A very stringent test of our understanding of the worldsheet superstring QFT
  - A computation from the string theory side will be a very nontrivial test of AdS/CFT

# The Konishi operator

- The Konishi operator $O_{Konishi} = \mathrm{tr}\, \Phi_i^2$
- The same supermultiplet has representatives in the $\mathtt{su(2)}$ sector $\mathrm{tr}\, Z^2 X^2 + \ldots$
- Anomalous dimension has been computed up to 4 loops ($g^2 = \frac{\lambda}{16\pi^2}$)

$$\Delta = 4 + 12g^2 - 48g^4 + 336g^6 + \underbrace{\left(-2496 + 576\zeta(3) - 1440\zeta(5)\right) g^8}_{[F.Fiamberti, A.Santambrogio, C.Sieg, D.Zanon]} + \ldots$$

(involves 139 supergraphs and 12 types of 4-loop integrals!)

- Why is it interesting to compute the 4-loop part?
  - Qualitatively new effects appear on the gauge theory side – wrapping interactions
  - First order which goes beyond the Bethe ansatz
  - A very stringent test of our understanding of the worldsheet superstring QFT
  - A computation from the string theory side will be a very nontrivial test of AdS/CFT

# The Konishi operator

- The Konishi operator $O_{Konishi} = \mathrm{tr}\, \Phi_i^2$
- The same supermultiplet has representatives in the $\mathtt{su(2)}$ sector $\mathrm{tr}\, Z^2 X^2 + \ldots$
- Anomalous dimension has been computed up to 4 loops ($g^2 = \frac{\lambda}{16\pi^2}$)

$$\Delta = 4 + 12g^2 - 48g^4 + 336g^6 + \underbrace{(-2496 + 576\zeta(3) - 1440\zeta(5))\, g^8}_{[F.Fiamberti, A.Santambrogio, C.Sieg, D.Zanon]} + \ldots$$

(involves 139 supergraphs and 12 types of 4-loop integrals!)

- Why is it interesting to compute the 4-loop part?
  - Qualitatively new effects appear on the gauge theory side – wrapping interactions
  - First order which goes beyond the Bethe ansatz
  - A very stringent test of our understanding of the worldsheet superstring QFT
  - A computation from the string theory side will be a very nontrivial test of AdS/CFT

# The Konishi operator

- The Konishi operator $O_{Konishi} = \mathrm{tr}\,\Phi_i^2$
- The same supermultiplet has representatives in the $\mathtt{su(2)}$ sector $\mathrm{tr}\, Z^2 X^2 + \ldots$
- Anomalous dimension has been computed up to 4 loops ($g^2 = \frac{\lambda}{16\pi^2}$)

$$\Delta = 4 + 12g^2 - 48g^4 + 336g^6 + \underbrace{(-2496 + 576\zeta(3) - 1440\zeta(5))\, g^8}_{[F.Fiamberti, A.Santambrogio, C.Sieg, D.Zanon]} + \ldots$$

(involves 139 supergraphs and 12 types of 4-loop integrals!)

- Why is it interesting to compute the 4-loop part?
  - Qualitatively new effects appear on the gauge theory side – wrapping interactions
  - First order which goes beyond the Bethe ansatz
  - A very stringent test of our understanding of the worldsheet superstring QFT
  - A computation from the string theory side will be a very nontrivial test of AdS/CFT

## The Konishi operator

- The Konishi operator $O_{Konishi} = \mathrm{tr}\,\Phi_i^2$
- The same supermultiplet has representatives in the $\mathtt{su(2)}$ sector $\mathrm{tr}\,Z^2X^2 + \ldots$
- Anomalous dimension has been computed up to 4 loops ($g^2 = \frac{\lambda}{16\pi^2}$)

$$\Delta = 4 + 12g^2 - 48g^4 + 336g^6 + \underbrace{(-2496 + 576\zeta(3) - 1440\zeta(5))\,g^8}_{[F.Fiamberti, A.Santambrogio, C.Sieg, D.Zanon]} + \ldots$$

(involves 139 supergraphs and 12 types of 4-loop integrals!)

- Why is it interesting to compute the 4-loop part?
  - Qualitatively new effects appear on the gauge theory side – wrapping interactions
  - First order which goes beyond the Bethe ansatz
  - A very stringent test of our understanding of the worldsheet superstring QFT
  - A computation from the string theory side will be a very nontrivial test of AdS/CFT

## The Konishi operator

- The Konishi operator $O_{Konishi} = \mathrm{tr}\,\Phi_i^2$
- The same supermultiplet has representatives in the $\mathtt{su(2)}$ sector $\mathrm{tr}\, Z^2 X^2 + \ldots$
- Anomalous dimension has been computed up to 4 loops ($g^2 = \frac{\lambda}{16\pi^2}$)

$$\Delta = 4 + 12g^2 - 48g^4 + 336g^6 + \underbrace{\left(-2496 + 576\zeta(3) - 1440\zeta(5)\right) g^8}_{[F.Fiamberti, A.Santambrogio, C.Sieg, D.Zanon]} + \ldots$$

(involves 139 supergraphs and 12 types of 4-loop integrals!)

- Why is it interesting to compute the 4-loop part?
  - Qualitatively new effects appear on the gauge theory side – wrapping interactions
  - First order which goes beyond the Bethe ansatz
  - A very stringent test of our understanding of the worldsheet superstring QFT
  - A computation from the string theory side will be a very nontrivial test of AdS/CFT

# The Konishi operator

- The Konishi operator $O_{Konishi} = \mathrm{tr}\, \Phi_i^2$
- The same supermultiplet has representatives in the $\mathtt{su(2)}$ sector $\mathrm{tr}\, Z^2 X^2 + \ldots$
- Anomalous dimension has been computed up to 4 loops ($g^2 = \frac{\lambda}{16\pi^2}$)

$$\Delta = 4 + 12g^2 - 48g^4 + 336g^6 + \underbrace{\left(-2496 + 576\zeta(3) - 1440\zeta(5)\right) g^8}_{[F.Fiamberti, A.Santambrogio, C.Sieg, D.Zanon]} + \ldots$$

(involves 139 supergraphs and 12 types of 4-loop integrals!)

- Why is it interesting to compute the 4-loop part?
  - Qualitatively new effects appear on the gauge theory side – wrapping interactions
  - First order which goes beyond the Bethe ansatz
  - A very stringent test of our understanding of the worldsheet superstring QFT
  - A computation from the string theory side will be a very nontrivial test of AdS/CFT

# Asymptotic Bethe Ansatz

- Description through the Asymptotic Bethe Ansatz [Beisert,Staudacher]
- Has $J = 2$ and two excitations with opposite momenta. Length $L = 4$.
- Bethe equations
- where $e^{2i\theta(p,-p)}$ is the dressing factor and

$$u(p) = \frac{1}{2} \cot \frac{p}{2} \sqrt{1 + 16g^2 \sin^2 \frac{p}{2}}$$

- This gives the solution for the momentum:

$$p = \frac{2\pi}{3} - \sqrt{3}g^2 + \frac{9\sqrt{3}}{2}g^4 - \frac{72\sqrt{3} + 8 \cdot 9\sqrt{3}\zeta(3)}{3}g^6 + \ldots$$

- Plug the momenta $p, -p$ into the dispersion relation:

$$\Delta_{Bethe} = 2 + 2\sqrt{1 + 16g^2 \sin^2 \frac{p}{2}}$$

## Asymptotic Bethe Ansatz

- Description through the Asymptotic Bethe Ansatz [Beisert,Staudacher]
- Has $J = 2$ and two excitations with opposite momenta. Length $L = 4$.
- Bethe equations
- where $e^{2i\theta(p,-p)}$ is the dressing factor and

$$u(p) = \frac{1}{2} \cot \frac{p}{2} \sqrt{1 + 16g^2 \sin^2 \frac{p}{2}}$$

- This gives the solution for the momentum:

$$p = \frac{2\pi}{3} - \sqrt{3}g^2 + \frac{9\sqrt{3}}{2}g^4 - \frac{72\sqrt{3} + 8 \cdot 9\sqrt{3}\zeta(3)}{3}g^6 + \dots$$

- Plug the momenta $p, -p$ into the dispersion relation:

$$\Delta_{Bethe} = 2 + 2\sqrt{1 + 16g^2 \sin^2 \frac{p}{2}}$$

## Asymptotic Bethe Ansatz

- Description through the Asymptotic Bethe Ansatz [Beisert,Staudacher]
- Has $J = 2$ and two excitations with opposite momenta. Length $L = 4$.
- Bethe equations

$$e^{ipL} = S_{\mathtt{su(2)}}(p, -p)$$

- where $e^{2i\theta(p,-p)}$ is the dressing factor and

$$u(p) = \frac{1}{2} \cot \frac{p}{2} \sqrt{1 + 16g^2 \sin^2 \frac{p}{2}}$$

- This gives the solution for the momentum:

$$p = \frac{2\pi}{3} - \sqrt{3}g^2 + \frac{9\sqrt{3}}{2}g^4 - \frac{72\sqrt{3} + 8 \cdot 9\sqrt{3}\zeta(3)}{3}g^6 + \dots$$

- Plug the momenta $p, -p$ into the dispersion relation:

$$\Delta_{Bethe} = 2 + 2\sqrt{1 + 16g^2 \sin^2 \frac{p}{2}}$$

## Asymptotic Bethe Ansatz

- Description through the Asymptotic Bethe Ansatz [Beisert,Staudacher]
- Has $J = 2$ and two excitations with opposite momenta. Length $L = 4$.
- Bethe equations

$$e^{i4p} = \frac{2u(p) + i}{2u(p) - i} e^{2i\theta(p,-p)}$$

- where $e^{2i\theta(p,-p)}$ is the dressing factor and

$$u(p) = \frac{1}{2} \cot \frac{p}{2} \sqrt{1 + 16g^2 \sin^2 \frac{p}{2}}$$

- This gives the solution for the momentum:

$$p = \frac{2\pi}{3} - \sqrt{3}g^2 + \frac{9\sqrt{3}}{2}g^4 - \frac{72\sqrt{3} + 8 \cdot 9\sqrt{3}\zeta(3)}{3}g^6 + \ldots$$

- Plug the momenta $p, -p$ into the dispersion relation:

$$\Delta_{Bethe} = 2 + 2\sqrt{1 + 16g^2 \sin^2 \frac{p}{2}}$$

## Asymptotic Bethe Ansatz

- Description through the Asymptotic Bethe Ansatz [Beisert,Staudacher]
- Has $J = 2$ and two excitations with opposite momenta. Length $L = 4$.
- Bethe equations

$$e^{i4p} = \frac{2u(p) + i}{2u(p) - i} e^{2i\theta(p,-p)}$$

- where $e^{2i\theta(p,-p)}$ is the dressing factor and

$$u(p) = \frac{1}{2} \cot \frac{p}{2} \sqrt{1 + 16g^2 \sin^2 \frac{p}{2}}$$

- This gives the solution for the momentum:

$$p = \frac{2\pi}{3} - \sqrt{3}g^2 + \frac{9\sqrt{3}}{2} g^4 - \frac{72\sqrt{3} + 8 \cdot 9\sqrt{3}\zeta(3)}{3} g^6 + \ldots$$

- Plug the momenta $p, -p$ into the dispersion relation:

$$\Delta_{Bethe} = 2 + 2\sqrt{1 + 16g^2 \sin^2 \frac{p}{2}}$$

# Asymptotic Bethe Ansatz

- Description through the Asymptotic Bethe Ansatz [Beisert,Staudacher]
- Has $J = 2$ and two excitations with opposite momenta. Length $L = 4$.
- Bethe equations

$$e^{i4p} = \frac{2u(p) + i}{2u(p) - i} e^{2i\theta(p,-p)}$$

- where $e^{2i\theta(p,-p)}$ is the dressing factor and

$$u(p) = \frac{1}{2} \cot \frac{p}{2} \sqrt{1 + 16g^2 \sin^2 \frac{p}{2}}$$

- This gives the solution for the momentum:

$$p = \frac{2\pi}{3} - \sqrt{3}g^2 + \frac{9\sqrt{3}}{2} g^4 - \frac{72\sqrt{3} + 8 \cdot 9\sqrt{3}\zeta(3)}{3} g^6 + \dots$$

- Plug the momenta $p$,$-p$ into the dispersion relation:

$$\Delta_{Bethe} = 2 + 2\sqrt{1 + 16g^2 \sin^2 \frac{p}{2}}$$

# Asymptotic Bethe Ansatz

- Description through the Asymptotic Bethe Ansatz [Beisert,Staudacher]
- Has $J = 2$ and two excitations with opposite momenta. Length $L = 4$.
- Bethe equations

$$e^{i4p} = \frac{2u(p) + i}{2u(p) - i} e^{2i\theta(p,-p)}$$

- where $e^{2i\theta(p,-p)}$ is the dressing factor and

$$u(p) = \frac{1}{2} \cot \frac{p}{2} \sqrt{1 + 16g^2 \sin^2 \frac{p}{2}}$$

- This gives the solution for the momentum:

$$p = \frac{2\pi}{3} - \sqrt{3}g^2 + \frac{9\sqrt{3}}{2}g^4 - \frac{72\sqrt{3} + 8 \cdot 9\sqrt{3}\zeta(3)}{3}g^6 + \ldots$$

- Plug the momenta $p, -p$ into the dispersion relation:

$$\Delta_{Bethe} = 2 + 2\sqrt{1 + 16g^2 \sin^2 \frac{p}{2}}$$

## Asymptotic Bethe Ansatz

- Description through the Asymptotic Bethe Ansatz [Beisert,Staudacher]
- Has $J = 2$ and two excitations with opposite momenta. Length $L = 4$.
- Bethe equations

$$e^{i4p} = \frac{2u(p) + i}{2u(p) - i} e^{2i\theta(p,-p)}$$

- where $e^{2i\theta(p,-p)}$ is the dressing factor and

$$u(p) = \frac{1}{2} \cot \frac{p}{2} \sqrt{1 + 16g^2 \sin^2 \frac{p}{2}}$$

- This gives the solution for the momentum:

$$p = \frac{2\pi}{3} - \sqrt{3}g^2 + \frac{9\sqrt{3}}{2} g^4 - \frac{72\sqrt{3} + 8 \cdot 9\sqrt{3}\zeta(3)}{3} g^6 + \dots$$

- Plug the momenta $p$,$-p$ into the dispersion relation:

$$\Delta_{Bethe} = 2 + 2\sqrt{1 + 16g^2 \sin^2 \frac{p}{2}}$$

- The resulting anomalous dimension is

$$\Delta_{Bethe} = 4 + 12g^2 - 48g^4 + 336g^6 - (2820 + 288\zeta(3))g^8 + \dots$$

- The result can be trusted only up to $g^{2L-2}$ where $L$ is the length of the operator. For higher orders wrapping interactions contribute!
- In this case the result is valid up to 3-loop order (terms $\propto g^6$) which has been verified perturbatively
- Wrapping:
  Asymptotic Bethe Ansatz incorporates all graphs of the type

  but not

- At 4-loops new contribution will arise:

$$\Delta = \Delta_{Bethe} + \Delta_{wrapping}$$

- The perturbative computation of [F.Fiamberti, A.Santambrogio, C.Sieg, D.Zanon] gives

$$\Delta_{wrapping} = (324 + 864\zeta(3) - 1440\zeta(5))g^8$$

- The resulting anomalous dimension is

$$\Delta_{Bethe} = 4 + 12g^2 - 48g^4 + 336g^6 - (2820 + 288\zeta(3))g^8 + \ldots$$

- The result can be trusted only up to $g^{2L-2}$ where $L$ is the length of the operator. For higher orders wrapping interactions contribute!
- In this case the result is valid up to 3-loop order (terms $\propto g^6$) which has been verified perturbatively
- Wrapping:
  Asymptotic Bethe Ansatz incorporates all graphs of the type

  but not

- At 4-loops new contribution will arise:

$$\Delta = \Delta_{Bethe} + \Delta_{wrapping}$$

- The perturbative computation of [F.Fiamberti, A.Santambrogio, C.Sieg, D.Zanon] gives

$$\Delta_{wrapping} = (324 + 864\zeta(3) - 1440\zeta(5))g^8$$

- The resulting anomalous dimension is

$$\Delta_{Bethe} = 4 + 12g^2 - 48g^4 + 336g^6 - (2820 + 288\zeta(3))g^8 + \ldots$$

- The result can be trusted only up to $g^{2L-2}$ where $L$ is the length of the operator. For higher orders wrapping interactions contribute!
- In this case the result is valid up to 3-loop order (terms $\propto g^6$) which has been verified perturbatively
- Wrapping:
  Asymptotic Bethe Ansatz incorporates all graphs of the type

  but not

- At 4-loops new contribution will arise:

$$\Delta = \Delta_{Bethe} + \Delta_{wrapping}$$

- The perturbative computation of [F.Fiamberti, A.Santambrogio, C.Sieg, D.Zanon] gives

$$\Delta_{wrapping} = (324 + 864\zeta(3) - 1440\zeta(5))g^8$$

- The resulting anomalous dimension is

$$\Delta_{Bethe} = 4 + 12g^2 - 48g^4 + 336g^6 - (2820 + 288\zeta(3))g^8 + \dots$$

- The result can be trusted only up to $g^{2L-2}$ where $L$ is the length of the operator. For higher orders wrapping interactions contribute!
- In this case the result is valid up to 3-loop order (terms $\propto g^6$) which has been verified perturbatively
- Wrapping:
  Asymptotic Bethe Ansatz incorporates all graphs of the type

  but not

- At 4-loops new contribution will arise:

$$\Delta = \Delta_{Bethe} + \Delta_{wrapping}$$

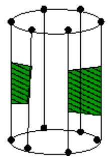
- The perturbative computation of [F.Fiamberti, A.Santambrogio, C.Sieg, D.Zanon] gives

$$\Delta_{wrapping} = (324 + 864\zeta(3) - 1440\zeta(5))g^8$$

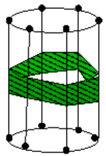
- The resulting anomalous dimension is

$$\Delta_{Bethe} = 4 + 12g^2 - 48g^4 + 336g^6 - (2820 + 288\zeta(3))g^8 + \ldots$$

- The result can be trusted only up to $g^{2L-2}$ where $L$ is the length of the operator. For higher orders wrapping interactions contribute!
- In this case the result is valid up to 3-loop order (terms $\propto g^6$) which has been verified perturbatively
- Wrapping:
  Asymptotic Bethe Ansatz incorporates all graphs of the type



but not



- At 4-loops new contribution will arise:

$$\Delta = \Delta_{Bethe} + \Delta_{wrapping}$$

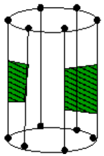
- The perturbative computation of [F.Fiamberti, A.Santambrogio, C.Sieg, D.Zanon] gives

$$\Delta_{wrapping} = (324 + 864\zeta(3) - 1440\zeta(5))g^8$$

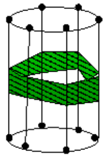
- The resulting anomalous dimension is

$$\Delta_{Bethe} = 4 + 12g^2 - 48g^4 + 336g^6 - (2820 + 288\zeta(3))g^8 + \ldots$$

- The result can be trusted only up to $g^{2L-2}$ where $L$ is the length of the operator. For higher orders wrapping interactions contribute!
- In this case the result is valid up to 3-loop order (terms $\propto g^6$) which has been verified perturbatively
- Wrapping:
  Asymptotic Bethe Ansatz incorporates all graphs of the type



but not



- At 4-loops new contribution will arise:

$$\Delta = \Delta_{Bethe} + \Delta_{wrapping}$$

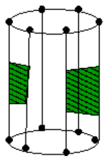
- The perturbative computation of [F.Fiamberti, A.Santambrogio, C.Sieg, D.Zanon] gives

$$\Delta_{wrapping} = (324 + 864\zeta(3) - 1440\zeta(5))g^8$$

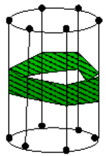
- The resulting anomalous dimension is

$$\Delta_{Bethe} = 4 + 12g^2 - 48g^4 + 336g^6 - (2820 + 288\zeta(3))g^8 + \dots$$

- The result can be trusted only up to $g^{2L-2}$ where $L$ is the length of the operator. For higher orders wrapping interactions contribute!
- In this case the result is valid up to 3-loop order (terms $\propto g^6$) which has been verified perturbatively
- Wrapping:
  Asymptotic Bethe Ansatz incorporates all graphs of the type



but not



- At 4-loops new contribution will arise:

$$\Delta = \Delta_{Bethe} + \Delta_{wrapping}$$

- The perturbative computation of [F.Fiamberti, A.Santambrogio, C.Sieg, D.Zanon] gives

$$\Delta_{wrapping} = (324 + 864\zeta(3) - 1440\zeta(5))g^8$$

## How to compute wrapping interactions?

- Within the spin chain point of view it is not clear how to incorporate wrapping effects – e.g. for the Heisenberg XXX spin chain, Bethe ansatz is exact
- Worldsheet QFT in principle provides a unique prescription how to describe these effects
- The spectrum at finite $J \equiv$ spectrum of the theory defined on a cylinder of fixed size $J$ (for the string in appropriate light cone gauge)
- Suppose we know the theory exactly on the infinite plane:
  1. complete set of asymptotic states
  2. S-matrices for these states
- How the spectrum changes when the theory is put on a cylinder?
- For single particle states, universal formulas exist – first derived by Lüscher
- For (some) integrable relativistic QFT's the complete exact finite size spectrum is encoded in nonlinear integral equations

# How to compute wrapping interactions?

- Within the spin chain point of view it is not clear how to incorporate wrapping effects – e.g. for the Heisenberg XXX spin chain, Bethe ansatz is exact
- Worldsheet QFT in principle provides a unique prescription how to describe these effects
- The spectrum at finite $J \equiv$ spectrum of the theory defined on a cylinder of fixed size $J$ (for the string in appropriate light cone gauge)
- Suppose we know the theory exactly on the infinite plane:
  1. complete set of asymptotic states
  2. S-matrices for these states
- How the spectrum changes when the theory is put on a cylinder?
- For single particle states, universal formulas exist – first derived by Lüscher
- For (some) integrable relativistic QFT's the complete exact finite size spectrum is encoded in nonlinear integral equations

## How to compute wrapping interactions?

- Within the spin chain point of view it is not clear how to incorporate wrapping effects – e.g. for the Heisenberg XXX spin chain, Bethe ansatz is exact
- Worldsheet QFT in principle provides a unique prescription how to describe these effects
- The spectrum at finite $J \equiv$ spectrum of the theory defined on a cylinder of fixed size $J$ (for the string in appropriate light cone gauge)
- Suppose we know the theory exactly on the infinite plane:
    1. complete set of asymptotic states
    2. S-matrices for these states
- How the spectrum changes when the theory is put on a cylinder?
- For single particle states, universal formulas exist – first derived by Lüscher
- For (some) integrable relativistic QFT's the complete exact finite size spectrum is encoded in nonlinear integral equations

## How to compute wrapping interactions?

- Within the spin chain point of view it is not clear how to incorporate wrapping effects – e.g. for the Heisenberg XXX spin chain, Bethe ansatz is exact
- Worldsheet QFT in principle provides a unique prescription how to describe these effects
- The spectrum at finite $J \equiv$ spectrum of the theory defined on a cylinder of fixed size $J$ (for the string in appropriate light cone gauge)
- Suppose we know the theory exactly on the infinite plane:
  1. complete set of asymptotic states
  2. S-matrices for these states
- How the spectrum changes when the theory is put on a cylinder?
- For single particle states, universal formulas exist – first derived by Lüscher
- For (some) integrable relativistic QFT's the complete exact finite size spectrum is encoded in nonlinear integral equations

# How to compute wrapping interactions?

- Within the spin chain point of view it is not clear how to incorporate wrapping effects – e.g. for the Heisenberg XXX spin chain, Bethe ansatz is exact
- Worldsheet QFT in principle provides a unique prescription how to describe these effects
- The spectrum at finite $J \equiv$ spectrum of the theory defined on a cylinder of fixed size $J$ (for the string in appropriate light cone gauge)
- Suppose we know the theory exactly on the infinite plane:
  1. complete set of asymptotic states
  2. S-matrices for these states
- How the spectrum changes when the theory is put on a cylinder?
- For single particle states, universal formulas exist – first derived by Lüscher
- For (some) integrable relativistic QFT's the complete exact finite size spectrum is encoded in nonlinear integral equations

## How to compute wrapping interactions?

- Within the spin chain point of view it is not clear how to incorporate wrapping effects – e.g. for the Heisenberg XXX spin chain, Bethe ansatz is exact
- Worldsheet QFT in principle provides a unique prescription how to describe these effects
- The spectrum at finite $J \equiv$ spectrum of the theory defined on a cylinder of fixed size $J$ (for the string in appropriate light cone gauge)
- Suppose we know the theory exactly on the infinite plane:
    1. complete set of asymptotic states
    2. S-matrices for these states
- How the spectrum changes when the theory is put on a cylinder?
- For single particle states, universal formulas exist – first derived by Lüscher
- For (some) integrable relativistic QFT's the complete exact finite size spectrum is encoded in nonlinear integral equations

## How to compute wrapping interactions?

- Within the spin chain point of view it is not clear how to incorporate wrapping effects – e.g. for the Heisenberg XXX spin chain, Bethe ansatz is exact
- Worldsheet QFT in principle provides a unique prescription how to describe these effects
- The spectrum at finite $J \equiv$ spectrum of the theory defined on a cylinder of fixed size $J$ (for the string in appropriate light cone gauge)
- Suppose we know the theory exactly on the infinite plane:
  1. complete set of asymptotic states
  2. S-matrices for these states
- How the spectrum changes when the theory is put on a cylinder?
- For single particle states, universal formulas exist – first derived by Lüscher
- For (some) integrable relativistic QFT's the complete exact finite size spectrum is encoded in nonlinear integral equations

## How to compute wrapping interactions?

- Within the spin chain point of view it is not clear how to incorporate wrapping effects – e.g. for the Heisenberg XXX spin chain, Bethe ansatz is exact
- Worldsheet QFT in principle provides a unique prescription how to describe these effects
- The spectrum at finite $J \equiv$ spectrum of the theory defined on a cylinder of fixed size $J$ (for the string in appropriate light cone gauge)
- Suppose we know the theory exactly on the infinite plane:
  1. complete set of asymptotic states
  2. S-matrices for these states
- How the spectrum changes when the theory is put on a cylinder?
- For single particle states, universal formulas exist – first derived by Lüscher
- For (some) integrable relativistic QFT's the complete exact finite size spectrum is encoded in nonlinear integral equations

## How to compute wrapping interactions?

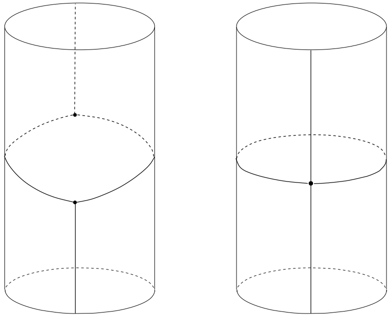
- Within the spin chain point of view it is not clear how to incorporate wrapping effects – e.g. for the Heisenberg XXX spin chain, Bethe ansatz is exact
- Worldsheet QFT in principle provides a unique prescription how to describe these effects
- The spectrum at finite $J \equiv$ spectrum of the theory defined on a cylinder of fixed size $J$ (for the string in appropriate light cone gauge)
- Suppose we know the theory exactly on the infinite plane:
  1. complete set of asymptotic states
  2. S-matrices for these states
- How the spectrum changes when the theory is put on a cylinder?
- For single particle states, universal formulas exist – first derived by Lüscher
- For (some) integrable relativistic QFT's the complete exact finite size spectrum is encoded in nonlinear integral equations

# Lüscher corrections

- For relativistic theories, leading finite size effects arise due to virtual corrections 'encircling' the cylinder
- Universal expression in terms of the (infinite volume) S-matrix
- Can be generalized to the AdS worldsheet theory (lack of relativistic invariance!) [RJ,Łukowski]
- Apply this method to the case of the Konishi operator
- Why is $L = 4$ large enough?
- Can we derive similar formulas for multi-(two-)particle states?

## Lüscher corrections

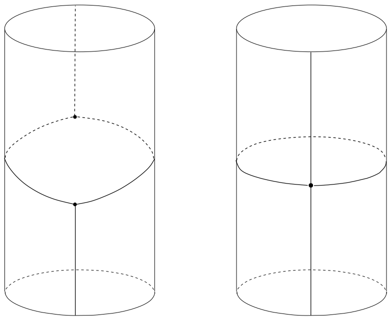
- For relativistic theories, leading finite size effects arise due to virtual corrections 'encircling' the cylinder



- Universal expression in terms of the (infinite volume) S-matrix
- Can be generalized to the AdS worldsheet theory (lack of relativistic invariance!) [RJ,Łukowski]
- Apply this method to the case of the Konishi operator
- Why is $L = 4$ large enough?
- Can we derive similar formulas for multi-(two-)particle states?

# Lüscher corrections

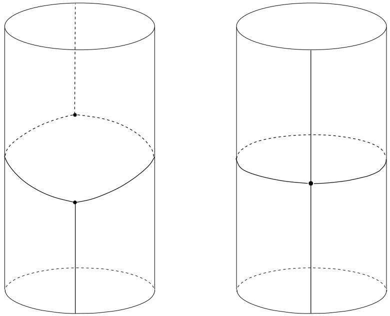
- For relativistic theories, leading finite size effects arise due to virtual corrections 'encircling' the cylinder



- Universal expression in terms of the (infinite volume) S-matrix
- Can be generalized to the AdS worldsheet theory (lack of relativistic invariance!) [RJ,Łukowski]
- Apply this method to the case of the Konishi operator
- Why is $L = 4$ large enough?
- Can we derive similar formulas for multi-(two-)particle states?

# Lüscher corrections

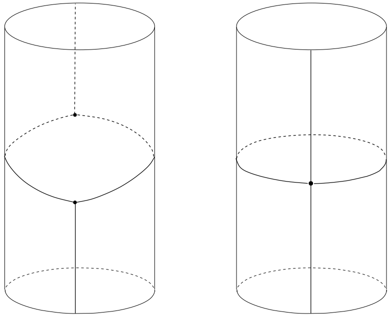
- For relativistic theories, leading finite size effects arise due to virtual corrections ‘encircling’ the cylinder



- Universal expression in terms of the (infinite volume) S-matrix
- Can be generalized to the AdS worldsheet theory (lack of relativistic invariance!) [RJ,Łukowski]
- Apply this method to the case of the Konishi operator
- Why is $L = 4$ large enough?
- Can we derive similar formulas for multi-(two-)particle states?

# Lüscher corrections

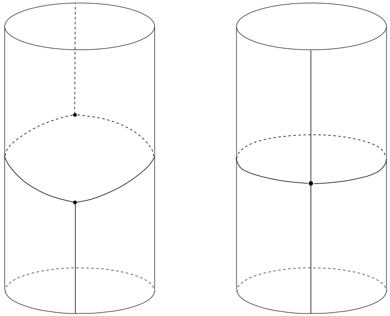
- For relativistic theories, leading finite size effects arise due to virtual corrections 'encircling' the cylinder



- Universal expression in terms of the (infinite volume) S-matrix
- Can be generalized to the AdS worldsheet theory (lack of relativistic invariance!) [RJ,Łukowski]
- Apply this method to the case of the Konishi operator
- Why is $L = 4$ large enough?
- Can we derive similar formulas for multi-(two-)particle states?

# Lüscher corrections

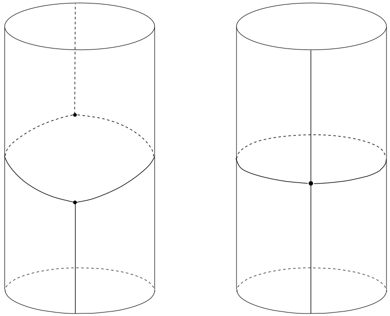
- For relativistic theories, leading finite size effects arise due to virtual corrections 'encircling' the cylinder



- Universal expression in terms of the (infinite volume) S-matrix
- Can be generalized to the AdS worldsheet theory (lack of relativistic invariance!) [RJ,Łukowski]
- Apply this method to the case of the Konishi operator
- Why is $L = 4$ large enough?
- Can we derive similar formulas for multi-(two-)particle states?

# Lüscher corrections

- For relativistic theories, leading finite size effects arise due to virtual corrections 'encircling' the cylinder



- Universal expression in terms of the (infinite volume) S-matrix
- Can be generalized to the AdS worldsheet theory (lack of relativistic invariance!) [RJ,Łukowski]
- Apply this method to the case of the Konishi operator
- Why is $L = 4$ large enough?
- Can we derive similar formulas for multi-(two-)particle states?

# Why is $L = 4$ large enough?

- One can estimate the magnitude of the finite size corrections

$$magnitude \sim e^{-L E_{TBA}(p_{TBA})}$$

where $E_{TBA}$ is obtained by space-time interchange $E_{TBA} = ip$ and $p_{TBA} = iE$ and using the mass-shell condition $E = E(p)$

- We get the estimate: [Ambjorn,RJ,Kristjansen]

$$e^{-2L \,\mathrm{arcsinh}\, \frac{\sqrt{1+q^2}}{4g}}$$

- At strong coupling:

$$e^{-2L \,\mathrm{arcsinh}\, \frac{\sqrt{1+q^2}}{4g}} \longrightarrow e^{-\frac{L}{2g}} = e^{-\frac{2\pi L}{\sqrt{\lambda}}}$$

- At weak coupling:

$$e^{-2L \,\mathrm{arcsinh}\, \frac{\sqrt{1+q^2}}{4g}} \longrightarrow \frac{4^L g^{2L}}{(1+q^2)^L}$$

- For Konishi operator we expect to get contributions at 4 loop order ($L \sim 4$)

# Why is $L = 4$ large enough?

- One can estimate the magnitude of the finite size corrections

$$magnitude \sim e^{-L E_{TBA}(p_{TBA})}$$

where $E_{TBA}$ is obtained by space-time interchange $E_{TBA} = ip$ and $p_{TBA} = iE$ and using the mass-shell condition $E = E(p)$

- We get the estimate: [Ambjorn,RJ,Kristjansen]

$$e^{-2L \operatorname{arcsinh} \frac{\sqrt{1+q^2}}{4g}}$$

- At strong coupling:

$$e^{-2L \operatorname{arcsinh} \frac{\sqrt{1+q^2}}{4g}} \longrightarrow e^{-\frac{L}{2g}} = e^{-\frac{2\pi L}{\sqrt{\lambda}}}$$

- At weak coupling:

$$e^{-2L \operatorname{arcsinh} \frac{\sqrt{1+q^2}}{4g}} \longrightarrow \frac{4^L g^{2L}}{(1+q^2)^L}$$

- For Konishi operator we expect to get contributions at 4 loop order ($L \sim 4$)

## Why is $L = 4$ large enough?

- One can estimate the magnitude of the finite size corrections

$$magnitude \sim e^{-L E_{TBA}(p_{TBA})}$$

where $E_{TBA}$ is obtained by space-time interchange $E_{TBA} = ip$ and $p_{TBA} = iE$ and using the mass-shell condition $E = E(p)$

- We get the estimate: [Ambjorn,RJ,Kristjansen]

$$e^{-2L \operatorname{arcsinh} \frac{\sqrt{1+q^2}}{4g}}$$

- At strong coupling:

$$e^{-2L \operatorname{arcsinh} \frac{\sqrt{1+q^2}}{4g}} \longrightarrow e^{-\frac{L}{2g}} = e^{-\frac{2\pi L}{\sqrt{\lambda}}}$$

- At weak coupling:

$$e^{-2L \operatorname{arcsinh} \frac{\sqrt{1+q^2}}{4g}} \longrightarrow \frac{4^L g^{2L}}{(1+q^2)^L}$$

- For Konishi operator we expect to get contributions at 4 loop order ($L \sim 4$)

# Why is $L = 4$ large enough?

- One can estimate the magnitude of the finite size corrections

$$magnitude \sim e^{-L E_{TBA}(p_{TBA})}$$

where $E_{TBA}$ is obtained by space-time interchange $E_{TBA} = ip$ and $p_{TBA} = iE$ and using the mass-shell condition $E = E(p)$

- We get the estimate: [Ambjorn,RJ,Kristjansen]

$$e^{-2L \operatorname{arcsinh} \frac{\sqrt{1+q^2}}{4g}}$$

- At strong coupling:

$$e^{-2L \operatorname{arcsinh} \frac{\sqrt{1+q^2}}{4g}} \longrightarrow e^{-\frac{L}{2g}} = e^{-\frac{2\pi L}{\sqrt{\lambda}}}$$

- At weak coupling:

$$e^{-2L \operatorname{arcsinh} \frac{\sqrt{1+q^2}}{4g}} \longrightarrow \frac{4^L g^{2L}}{(1+q^2)^L}$$

- For Konishi operator we expect to get contributions at 4 loop order ($L \sim 4$)

## Why is $L = 4$ large enough?

- One can estimate the magnitude of the finite size corrections

$$magnitude \sim e^{-L E_{TBA}(p_{TBA})}$$

where $E_{TBA}$ is obtained by space-time interchange $E_{TBA} = ip$ and $p_{TBA} = iE$ and using the mass-shell condition $E = E(p)$

- We get the estimate: [Ambjorn,RJ,Kristjansen]

$$e^{-2L \,\mathrm{arcsinh}\, \frac{\sqrt{1+q^2}}{4g}}$$

- At strong coupling:

$$e^{-2L \,\mathrm{arcsinh}\, \frac{\sqrt{1+q^2}}{4g}} \longrightarrow e^{-\frac{L}{2g}} = e^{-\frac{2\pi L}{\sqrt{\lambda}}}$$

- At weak coupling:

$$e^{-2L \,\mathrm{arcsinh}\, \frac{\sqrt{1+q^2}}{4g}} \longrightarrow \frac{4^L g^{2L}}{(1+q^2)^L}$$

- For Konishi operator we expect to get contributions at 4 loop order ($L \sim 4$)

## Why is $L = 4$ large enough?

- One can estimate the magnitude of the finite size corrections

$$magnitude \sim e^{-L E_{TBA}(p_{TBA})}$$

where $E_{TBA}$ is obtained by space-time interchange $E_{TBA} = ip$ and $p_{TBA} = iE$ and using the mass-shell condition $E = E(p)$

- We get the estimate: [Ambjorn,RJ,Kristjansen]

$$e^{-2L \operatorname{arcsinh} \frac{\sqrt{1+q^2}}{4g}}$$

- At strong coupling:

$$e^{-2L \operatorname{arcsinh} \frac{\sqrt{1+q^2}}{4g}} \longrightarrow e^{-\frac{L}{2g}} = e^{-\frac{2\pi L}{\sqrt{\lambda}}}$$

- At weak coupling:

$$e^{-2L \operatorname{arcsinh} \frac{\sqrt{1+q^2}}{4g}} \longrightarrow \frac{4^L g^{2L}}{(1+q^2)^L}$$

- For Konishi operator we expect to get contributions at 4 loop order ($L \sim 4$)

## Lüscher corrections for multiparticle states

- Look at relativistic integrable theories solvable by the excited state Thermodynamic Bethe Ansatz (TBA) e.g. sinh-Gordon, SLYM
- Answer given in terms of a nonlinear integral equation
- Solve by iteration:
- $0^{th}$ order $\longrightarrow$ Bethe equations
- $1^{st}$ order $\longrightarrow$ two main effects:
  1. shift of the momenta $\{p_i\}$
  2. a direct F-term like virtual contribution to the energy
  3. in addition, residues of the above at possible poles...
- Highly universal structure of the leading correction $\longrightarrow$ generalize to AdS case

# Lüscher corrections for multiparticle states

- Look at relativistic integrable theories solvable by the excited state Thermodynamic Bethe Ansatz (TBA) e.g. sinh-Gordon, SLYM
- Answer given in terms of a nonlinear integral equation
- Solve by iteration:
- $0^{th}$ order $\longrightarrow$ Bethe equations
- $1^{st}$ order $\longrightarrow$ two main effects:
  1. shift of the momenta $\{p_i\}$
  2. a direct F-term like virtual contribution to the energy
  3. in addition, residues of the above at possible poles...
- Highly universal structure of the leading correction $\longrightarrow$ generalize to AdS case

## Lüscher corrections for multiparticle states

- Look at relativistic integrable theories solvable by the excited state Thermodynamic Bethe Ansatz (TBA) e.g. sinh-Gordon, SLYM
- Answer given in terms of a nonlinear integral equation
- Solve by iteration:
- $0^{th}$ order $\longrightarrow$ Bethe equations
- $1^{st}$ order $\longrightarrow$ two main effects:
  1. shift of the momenta $\{p_i\}$
  2. a direct F-term like virtual contribution to the energy
  3. in addition, residues of the above at possible poles...
- Highly universal structure of the leading correction $\longrightarrow$ generalize to AdS case

## Lüscher corrections for multiparticle states

- Look at relativistic integrable theories solvable by the excited state Thermodynamic Bethe Ansatz (TBA) e.g. sinh-Gordon, SLYM
- Answer given in terms of a nonlinear integral equation
- Solve by iteration:
- $0^{th}$ order $\longrightarrow$ Bethe equations
- $1^{st}$ order $\longrightarrow$ two main effects:
  1. shift of the momenta $\{p_i\}$
  2. a direct F-term like virtual contribution to the energy
  3. in addition, residues of the above at possible poles...
- Highly universal structure of the leading correction $\longrightarrow$ generalize to AdS case

## Lüscher corrections for multiparticle states

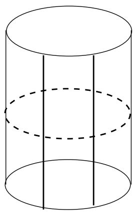
- Look at relativistic integrable theories solvable by the excited state Thermodynamic Bethe Ansatz (TBA) e.g. sinh-Gordon, SLYM
- Answer given in terms of a nonlinear integral equation
- Solve by iteration:
- $0^{th}$ order $\longrightarrow$ Bethe equations
- $1^{st}$ order $\longrightarrow$ two main effects:
  1. shift of the momenta $\{p_i\}$
  2. a direct F-term like virtual contribution to the energy
  3. in addition, residues of the above at possible poles...
- Highly universal structure of the leading correction $\longrightarrow$ generalize to AdS case

## Lüscher corrections for multiparticle states

- Look at relativistic integrable theories solvable by the excited state Thermodynamic Bethe Ansatz (TBA) e.g. sinh-Gordon, SLYM
- Answer given in terms of a nonlinear integral equation
- Solve by iteration:
- $0^{th}$ order $\longrightarrow$ Bethe equations
- $1^{st}$ order $\longrightarrow$ two main effects:
  1. shift of the momenta $\{p_i\}$
  2. a direct F-term like virtual contribution to the energy
  3. in addition, residues of the above at possible poles...
- Highly universal structure of the leading correction $\longrightarrow$ generalize to AdS case

## Lüscher corrections for multiparticle states

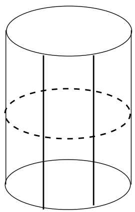
- Look at relativistic integrable theories solvable by the excited state Thermodynamic Bethe Ansatz (TBA) e.g. sinh-Gordon, SLYM
- Answer given in terms of a nonlinear integral equation
- Solve by iteration:
- $0^{th}$ order $\longrightarrow$ Bethe equations
- $1^{st}$ order $\longrightarrow$ two main effects:
  1. shift of the momenta $\{p_i\}$
  2. a direct F-term like virtual contribution to the energy



  3. in addition, residues of the above at possible poles...
- Highly universal structure of the leading correction $\longrightarrow$ generalize to AdS case

# Lüscher corrections for multiparticle states

- Look at relativistic integrable theories solvable by the excited state Thermodynamic Bethe Ansatz (TBA) e.g. sinh-Gordon, SLYM
- Answer given in terms of a nonlinear integral equation
- Solve by iteration:
- $0^{th}$ order $\longrightarrow$ Bethe equations
- $1^{st}$ order $\longrightarrow$ two main effects:
  1. shift of the momenta $\{p_i\}$
  2. a direct F-term like virtual contribution to the energy

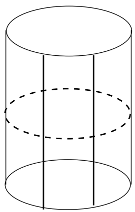


  3. in addition, residues of the above at possible poles...

- Highly universal structure of the leading correction $\longrightarrow$ generalize to AdS case

# Lüscher corrections for multiparticle states

- Look at relativistic integrable theories solvable by the excited state Thermodynamic Bethe Ansatz (TBA) e.g. sinh-Gordon, SLYM
- Answer given in terms of a nonlinear integral equation
- Solve by iteration:
- $0^{th}$ order $\longrightarrow$ Bethe equations
- $1^{st}$ order $\longrightarrow$ two main effects:
  1. shift of the momenta $\{p_i\}$
  2. a direct F-term like virtual contribution to the energy



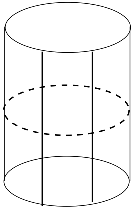
  3. in addition, residues of the above at possible poles...
- Highly universal structure of the leading correction $\longrightarrow$ generalize to AdS case

- For the Konishi at 4 loops only the F-term like expression contributes

$$\Delta E = \frac{-1}{2\pi} \sum_{Q=1}^{\infty} \int_{-\infty}^{\infty} dq \left( \frac{z^-}{z^+} \right)^2 \sum_b (-1)^{F_b} \left[ S_{Q-1}(z^\pm, x_i^\pm) S_{Q-1}(z^\pm, x_{ii}^\pm) \right]_{b(11)}^{b(11)}$$

- What particles should circulate in the loop?
  - fundamental magnons
  - $Q$-magnon bound states
    Two choices:
    1. su(2) bound states – symmetric representation – physical in the original theory
    2. sl(2) bound states – antisymmetric representation – physical in the mirror theory
- Our conclusion is that sl(2) bound states should be used
- Find $S_{Q-1}$ S-matrix for $Q > 2$...

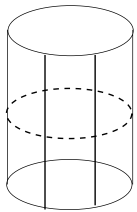
- For the Konishi at 4 loops only the F-term like expression contributes



$$\Delta E = \frac{-1}{2\pi} \sum_{Q=1}^{\infty} \int_{-\infty}^{\infty} dq \left( \frac{z^-}{z^+} \right)^2 \sum_b (-1)^{F_b} \left[ S_{Q-1}(z^\pm, x_i^\pm) S_{Q-1}(z^\pm, x_{ii}^\pm) \right]_{b(11)}^{b(11)}$$

- What particles should circulate in the loop?
  - fundamental magnons
  - $Q$-magnon bound states
    Two choices:
    1. su(2) bound states – symmetric representation – physical in the original theory
    2. sl(2) bound states – antisymmetric representation – physical in the mirror theory
- Our conclusion is that sl(2) bound states should be used
- Find $S_{Q-1}$ S-matrix for $Q > 2$...

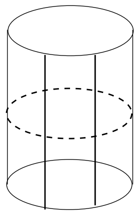
- For the Konishi at 4 loops only the F-term like expression contributes



$$\Delta E = \frac{-1}{2\pi} \sum_{Q=1}^{\infty} \int_{-\infty}^{\infty} dq \left( \frac{z^-}{z^+} \right)^2 \sum_b (-1)^{F_b} \left[ S_{Q-1}(z^\pm, x_i^\pm) S_{Q-1}(z^\pm, x_{ii}^\pm) \right]_{b(11)}^{b(11)}$$

- What particles should circulate in the loop?
  - fundamental magnons
  - $Q$-magnon bound states
    Two choices:
    1. `su(2)` bound states – symmetric representation – physical in the original theory
    2. `sl(2)` bound states – antisymmetric representation – physical in the mirror theory
- Our conclusion is that `sl(2)` bound states should be used
- Find $S_{Q-1}$ S-matrix for $Q > 2\ldots$

- For the Konishi at 4 loops only the F-term like expression contributes

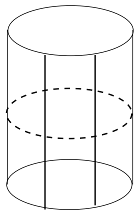


$$\Delta E = \frac{-1}{2\pi} \sum_{Q=1}^{\infty} \int_{-\infty}^{\infty} dq \left( \frac{z^-}{z^+} \right)^2 \sum_b (-1)^{F_b} \left[ S_{Q-1}(z^\pm, x_i^\pm) S_{Q-1}(z^\pm, x_{ii}^\pm) \right]_{b(11)}^{b(11)}$$

- What particles should circulate in the loop?
  - fundamental magnons
  - $Q$-magnon bound states

    Two choices:
    1. `su(2)` bound states – symmetric representation – physical in the original theory
    2. `sl(2)` bound states – antisymmetric representation – physical in the mirror theory
- Our conclusion is that `sl(2)` bound states should be used
- Find $S_{Q-1}$ S-matrix for $Q > 2 \ldots$

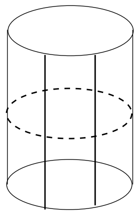
- For the Konishi at 4 loops only the F-term like expression contributes



$$\Delta E = \frac{-1}{2\pi} \sum_{Q=1}^{\infty} \int_{-\infty}^{\infty} dq \left( \frac{z^-}{z^+} \right)^2 \sum_b (-1)^{F_b} \left[ S_{Q-1}(z^\pm, x_i^\pm) S_{Q-1}(z^\pm, x_{ii}^\pm) \right]_{b(11)}^{b(11)}$$

- What particles should circulate in the loop?
  - fundamental magnons
  - $Q$-magnon bound states
    Two choices:
    1. su(2) bound states – symmetric representation – physical in the original theory
    2. sl(2) bound states – antisymmetric representation – physical in the mirror theory
- Our conclusion is that sl(2) bound states should be used
- Find $S_{Q-1}$ S-matrix for $Q > 2$...

- For the Konishi at 4 loops only the F-term like expression contributes

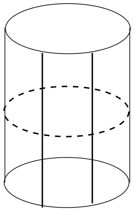


$$\Delta E = \frac{-1}{2\pi} \sum_{Q=1}^{\infty} \int_{-\infty}^{\infty} dq \left( \frac{z^-}{z^+} \right)^2 \sum_b (-1)^{F_b} \left[ S_{Q-1}(z^\pm, x_i^\pm) S_{Q-1}(z^\pm, x_{ii}^\pm) \right]_{b(11)}^{b(11)}$$

- What particles should circulate in the loop?
  - fundamental magnons
  - $Q$-magnon bound states

    Two choices:
    1. `su(2)` bound states – symmetric representation – physical in the original theory
    2. `sl(2)` bound states – antisymmetric representation – physical in the mirror theory
- Our conclusion is that `sl(2)` bound states should be used
- Find $S_{Q-1}$ S-matrix for $Q > 2 \ldots$

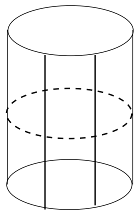
- For the Konishi at 4 loops only the F-term like expression contributes



$$\Delta E = \frac{-1}{2\pi} \sum_{Q=1}^{\infty} \int_{-\infty}^{\infty} dq \left( \frac{z^-}{z^+} \right)^2 \sum_b (-1)^{F_b} \left[ S_{Q-1}(z^\pm, x_i^\pm) S_{Q-1}(z^\pm, x_{ii}^\pm) \right]_{b(11)}^{b(11)}$$

- What particles should circulate in the loop?
  - fundamental magnons
  - $Q$-magnon bound states
    Two choices:
    1. su(2) bound states – symmetric representation – physical in the original theory
    2. sl(2) bound states – antisymmetric representation – physical in the mirror theory
- Our conclusion is that sl(2) bound states should be used
- Find $S_{Q-1}$ S-matrix for $Q > 2 \ldots$

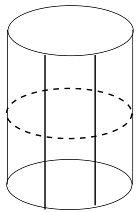
- For the Konishi at 4 loops only the F-term like expression contributes



$$\Delta E = \frac{-1}{2\pi} \sum_{Q=1}^{\infty} \int_{-\infty}^{\infty} dq \left( \frac{z^-}{z^+} \right)^2 \sum_b (-1)^{F_b} \left[ S_{Q-1}(z^\pm, x_i^\pm) S_{Q-1}(z^\pm, x_{ii}^\pm) \right]_{b(11)}^{b(11)}$$

- What particles should circulate in the loop?
  - fundamental magnons
  - $Q$-magnon bound states
    Two choices:
    1. su(2) bound states – symmetric representation – physical in the original theory
    2. sl(2) bound states – antisymmetric representation – physical in the mirror theory
- Our conclusion is that sl(2) bound states should be used
- Find $S_{Q-1}$ S-matrix for $Q > 2$...

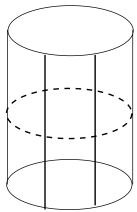
- For the Konishi at 4 loops only the F-term like expression contributes



$$\Delta E = \frac{-1}{2\pi} \sum_{Q=1}^{\infty} \int_{-\infty}^{\infty} dq \left( \frac{z^-}{z^+} \right)^2 \sum_b (-1)^{F_b} \left[ S_{Q-1}(z^\pm, x_i^\pm) S_{Q-1}(z^\pm, x_{ii}^\pm) \right]_{b(11)}^{b(11)}$$

- What particles should circulate in the loop?
  - fundamental magnons
  - $Q$-magnon bound states
    Two choices:
    1. `su(2)` bound states – symmetric representation – physical in the original theory
    2. `sl(2)` bound states – antisymmetric representation – physical in the mirror theory
- Our conclusion is that `sl(2)` bound states should be used
- Find $S_{Q-1}$ S-matrix for $Q > 2\ldots$

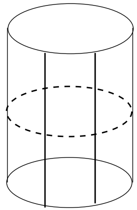
- For the Konishi at 4 loops only the F-term like expression contributes



$$\Delta E = \frac{-1}{2\pi} \sum_{Q=1}^{\infty} \int_{-\infty}^{\infty} dq \left( \frac{z^-}{z^+} \right)^2 \sum_b (-1)^{F_b} \left[ S_{Q-1}(z^\pm, x_i^\pm) S_{Q-1}(z^\pm, x_{ii}^\pm) \right]_{b(11)}^{b(11)}$$

- What particles should circulate in the loop?
  - fundamental magnons
  - $Q$-magnon bound states
    Two choices:
    1. `su(2)` bound states – symmetric representation – physical in the original theory
    2. `sl(2)` bound states – antisymmetric representation – physical in the mirror theory
- Our conclusion is that `sl(2)` bound states should be used
- Find $S_{Q-1}$ S-matrix for $Q > 2 \ldots$

- For the Konishi at 4 loops only the F-term like expression contributes



$$\Delta E = \frac{-1}{2\pi} \sum_{Q=1}^{\infty} \int_{-\infty}^{\infty} dq \left( \frac{z^-}{z^+} \right)^2 \sum_b (-1)^{F_b} \left[ S_{Q-1}(z^\pm, x_i^\pm) S_{Q-1}(z^\pm, x_{ii}^\pm) \right]_{b(11)}^{b(11)}$$

- What particles should circulate in the loop?
  - fundamental magnons
  - $Q$-magnon bound states
    Two choices:
    1. `su(2)` bound states – symmetric representation – physical in the original theory
    2. `sl(2)` bound states – antisymmetric representation – physical in the mirror theory
- Our conclusion is that `sl(2)` bound states should be used
- Find $S_{Q-1}$ S-matrix for $Q > 2 \ldots$

- The above expression is of order $g^8$ as expected from gauge theory
- The integral over $q$ can be carried out analytically by residues:

$$g^8 \cdot \sum_{Q=1}^{\infty} \left\{ -\frac{num(Q)}{(9Q^4-3Q^2+1)^4(27Q^6-27Q^4+36Q^2+16)} + \frac{864}{Q^3} - \frac{1440}{Q^5} \right\}$$

where

$$num(Q) = 7776Q(19683Q^{18} - 78732Q^{16} + 150903Q^{14} - 134865Q^{12} + \\ + 1458Q^{10} + 48357Q^8 - 13311Q^6 - 1053Q^4 + 369Q^2 - 10)$$

- Two last terms give at once $864\,\zeta(3) - 1440\,\zeta(5)$
- The remaining rational function remarkably sums up to an integer giving finally

$$\Delta_{wrapping} = (324 + 864\zeta(3) - 1440\zeta(5))g^8$$

- Exactly agrees with the 4-loop perturbative computation of [F.Fiamberti, A.Santambrogio, C.Sieg and D.Zanon]

- The above expression is of order $g^8$ as expected from gauge theory
- The integral over $q$ can be carried out analytically by residues:

$$g^8 \cdot \sum_{Q=1}^{\infty} \left\{ -\frac{num(Q)}{(9Q^4 - 3Q^2 + 1)^4 (27Q^6 - 27Q^4 + 36Q^2 + 16)} + \frac{864}{Q^3} - \frac{1440}{Q^5} \right\}$$

where

$$\begin{aligned} num(Q) =& 7776Q(19683Q^{18} - 78732Q^{16} + 150903Q^{14} - 134865Q^{12} + \\ & + 1458Q^{10} + 48357Q^8 - 13311Q^6 - 1053Q^4 + 369Q^2 - 10) \end{aligned}$$

- Two last terms give at once $864\,\zeta(3) - 1440\,\zeta(5)$
- The remaining rational function remarkably sums up to an integer giving finally

$$\Delta_{wrapping} = (324 + 864\zeta(3) - 1440\zeta(5))g^8$$

- Exactly agrees with the 4-loop perturbative computation of [F.Fiamberti, A.Santambrogio, C.Sieg and D.Zanon]

- The above expression is of order $g^8$ as expected from gauge theory
- The integral over $q$ can be carried out analytically by residues:

$$g^8 \cdot \sum_{Q=1}^{\infty} \left\{ -\frac{num(Q)}{(9Q^4 - 3Q^2 + 1)^4 (27Q^6 - 27Q^4 + 36Q^2 + 16)} + \frac{864}{Q^3} - \frac{1440}{Q^5} \right\}$$

where

$$num(Q) = 7776Q(19683Q^{18} - 78732Q^{16} + 150903Q^{14} - 134865Q^{12} + \\ + 1458Q^{10} + 48357Q^8 - 13311Q^6 - 1053Q^4 + 369Q^2 - 10)$$

- Two last terms give at once $864\,\zeta(3) - 1440\,\zeta(5)$
- The remaining rational function remarkably sums up to an integer giving finally

$$\Delta_{wrapping} = (324 + 864\zeta(3) - 1440\zeta(5))g^8$$

- Exactly agrees with the 4-loop perturbative computation of [F.Fiamberti, A.Santambrogio, C.Sieg and D.Zanon]

- The above expression is of order $g^8$ as expected from gauge theory
- The integral over $q$ can be carried out analytically by residues:

$$g^8 \cdot \sum_{Q=1}^{\infty} \left\{ -\frac{num(Q)}{(9Q^4 - 3Q^2 + 1)^4 (27Q^6 - 27Q^4 + 36Q^2 + 16)} + \frac{864}{Q^3} - \frac{1440}{Q^5} \right\}$$

where

$$num(Q) = 7776Q(19683Q^{18} - 78732Q^{16} + 150903Q^{14} - 134865Q^{12} + \\ + 1458Q^{10} + 48357Q^8 - 13311Q^6 - 1053Q^4 + 369Q^2 - 10)$$

- Two last terms give at once $864\,\zeta(3) - 1440\,\zeta(5)$
- The remaining rational function remarkably sums up to an integer giving finally

$$\Delta_{wrapping} = (324 + 864\zeta(3) - 1440\zeta(5))g^8$$

- Exactly agrees with the 4-loop perturbative computation of [F.Fiamberti, A.Santambrogio, C.Sieg and D.Zanon]

- The above expression is of order $g^8$ as expected from gauge theory
- The integral over $q$ can be carried out analytically by residues:

$$g^8 \cdot \sum_{Q=1}^{\infty} \left\{ -\frac{num(Q)}{(9Q^4-3Q^2+1)^4(27Q^6-27Q^4+36Q^2+16)} + \frac{864}{Q^3} - \frac{1440}{Q^5} \right\}$$

where

$$num(Q) = 7776Q(19683Q^{18} - 78732Q^{16} + 150903Q^{14} - 134865Q^{12} + \\ + 1458Q^{10} + 48357Q^8 - 13311Q^6 - 1053Q^4 + 369Q^2 - 10)$$

- Two last terms give at once $864\,\zeta(3) - 1440\,\zeta(5)$
- The remaining rational function remarkably sums up to an integer giving finally

$$\Delta_{wrapping} = (324 + 864\zeta(3) - 1440\zeta(5))g^8$$

- Exactly agrees with the 4-loop perturbative computation of [F.Fiamberti, A.Santambrogio, C.Sieg and D.Zanon]

- The above expression is of order $g^8$ as expected from gauge theory
- The integral over $q$ can be carried out analytically by residues:

$$g^8 \cdot \sum_{Q=1}^{\infty} \left\{ -\frac{num(Q)}{(9Q^4-3Q^2+1)^4(27Q^6-27Q^4+36Q^2+16)} + \frac{864}{Q^3} - \frac{1440}{Q^5} \right\}$$

where

$$num(Q) = 7776Q(19683Q^{18} - 78732Q^{16} + 150903Q^{14} - 134865Q^{12} + \\ + 1458Q^{10} + 48357Q^8 - 13311Q^6 - 1053Q^4 + 369Q^2 - 10)$$

- Two last terms give at once $864\,\zeta(3) - 1440\,\zeta(5)$
- The remaining rational function remarkably sums up to an integer giving finally

$$\Delta_{wrapping} = (324 + 864\zeta(3) - 1440\zeta(5))g^8$$

- Exactly agrees with the 4-loop perturbative computation of [F.Fiamberti, A.Santambrogio, C.Sieg and D.Zanon]

## Conclusions

- The appearance of the transcendentality structure of

$$\Delta_{wrapping} = (324 + 864\zeta(3) - 1440\zeta(5))g^8$$

  is far from obvious. A similar computation using $\mathfrak{su}(2)$ bound states in the symmetric representation leads to extremely complicated expressions
- Finite size effects involve a loop integral over all states in the theory – thus they form a nontrivial test of the completeness of the worldsheet theory
- This is especially important at weak coupling, where e.g. all higher bound states contribute equally
- In particular, magnons and $Q$ bound states seem to form a complete basis of asymptotic states of the superstring worldsheet QFT (e.g. there does not seem to be a place for a composite structure of the magnons)
- The computation of the finite size effects through Lüscher corrections is of a distinctly (2D) quantum field theoretical nature
- The agreement with the weak coupling perturbative gauge theory result is an extremely nontrivial test of AdS/CFT!

## Conclusions

- The appearance of the transcendentality structure of

$$\Delta_{wrapping} = (324 + 864\zeta(3) - 1440\zeta(5))g^8$$

  is far from obvious. A similar computation using $\mathtt{su(2)}$ bound states in the symmetric representation leads to extremely complicated expressions
- Finite size effects involve a loop integral over all states in the theory – thus they form a nontrivial test of the completeness of the worldsheet theory
- This is especially important at weak coupling, where e.g. all higher bound states contribute equally
- In particular, magnons and $Q$ bound states seem to form a complete basis of asymptotic states of the superstring worldsheet QFT (e.g. there does not seem to be a place for a composite structure of the magnons)
- The computation of the finite size effects through Lüscher corrections is of a distinctly (2D) quantum field theoretical nature
- The agreement with the weak coupling perturbative gauge theory result is an extremely nontrivial test of AdS/CFT!

## Conclusions

- The appearance of the transcendentality structure of

$$\Delta_{wrapping} = (324 + 864\zeta(3) - 1440\zeta(5))g^8$$

  is far from obvious. A similar computation using su(2) bound states in the symmetric representation leads to extremely complicated expressions
- Finite size effects involve a loop integral over all states in the theory – thus they form a nontrivial test of the completeness of the worldsheet theory
- This is especially important at weak coupling, where e.g. all higher bound states contribute equally
- In particular, magnons and $Q$ bound states seem to form a complete basis of asymptotic states of the superstring worldsheet QFT (e.g. there does not seem to be a place for a composite structure of the magnons)
- The computation of the finite size effects through Lüscher corrections is of a distinctly (2D) quantum field theoretical nature
- The agreement with the weak coupling perturbative gauge theory result is an extremely nontrivial test of AdS/CFT!

# Conclusions

- The appearance of the transcendentality structure of

$$\Delta_{wrapping} = (324 + 864\zeta(3) - 1440\zeta(5))g^8$$

  is far from obvious. A similar computation using $\mathtt{su}(2)$ bound states in the symmetric representation leads to extremely complicated expressions
- Finite size effects involve a loop integral over all states in the theory – thus they form a nontrivial test of the completeness of the worldsheet theory
- This is especially important at weak coupling, where e.g. all higher bound states contribute equally
- In particular, magnons and $Q$ bound states seem to form a complete basis of asymptotic states of the superstring worldsheet QFT (e.g. there does not seem to be a place for a composite structure of the magnons)
- The computation of the finite size effects through Lüscher corrections is of a distinctly (2D) quantum field theoretical nature
- The agreement with the weak coupling perturbative gauge theory result is an extremely nontrivial test of AdS/CFT!

## Conclusions

- The appearance of the transcendentality structure of

$$\Delta_{wrapping} = (324 + 864\zeta(3) - 1440\zeta(5))g^8$$

  is far from obvious. A similar computation using $\mathtt{su}(2)$ bound states in the symmetric representation leads to extremely complicated expressions
- Finite size effects involve a loop integral over all states in the theory – thus they form a nontrivial test of the completeness of the worldsheet theory
- This is especially important at weak coupling, where e.g. all higher bound states contribute equally
- In particular, magnons and $Q$ bound states seem to form a complete basis of asymptotic states of the superstring worldsheet QFT (e.g. there does not seem to be a place for a composite structure of the magnons)
- The computation of the finite size effects through Lüscher corrections is of a distinctly (2D) quantum field theoretical nature
- The agreement with the weak coupling perturbative gauge theory result is an extremely nontrivial test of AdS/CFT!

## Conclusions

- The appearance of the transcendentality structure of

$$\Delta_{wrapping} = (324 + 864\zeta(3) - 1440\zeta(5))g^8$$

  is far from obvious. A similar computation using $\mathtt{su}(2)$ bound states in the symmetric representation leads to extremely complicated expressions
- Finite size effects involve a loop integral over all states in the theory – thus they form a nontrivial test of the completeness of the worldsheet theory
- This is especially important at weak coupling, where e.g. all higher bound states contribute equally
- In particular, magnons and $Q$ bound states seem to form a complete basis of asymptotic states of the superstring worldsheet QFT (e.g. there does not seem to be a place for a composite structure of the magnons)
- The computation of the finite size effects through Lüscher corrections is of a distinctly (2D) quantum field theoretical nature
- The agreement with the weak coupling perturbative gauge theory result is an extremely nontrivial test of AdS/CFT!

## Conclusions

- The appearance of the transcendentality structure of

$$\Delta_{wrapping} = (324 + 864\zeta(3) - 1440\zeta(5))g^8$$

  is far from obvious. A similar computation using $\mathtt{su}(2)$ bound states in the symmetric representation leads to extremely complicated expressions
- Finite size effects involve a loop integral over all states in the theory – thus they form a nontrivial test of the completeness of the worldsheet theory
- This is especially important at weak coupling, where e.g. all higher bound states contribute equally
- In particular, magnons and $Q$ bound states seem to form a complete basis of asymptotic states of the superstring worldsheet QFT (e.g. there does not seem to be a place for a composite structure of the magnons)
- The computation of the finite size effects through Lüscher corrections is of a distinctly (2D) quantum field theoretical nature
- The agreement with the weak coupling perturbative gauge theory result is an extremely nontrivial test of AdS/CFT!