$\mathcal{N}$=6 Chern Simons Matter Theories, M2 branes and Supergravity

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Strings 2008
Cern

Based on: Aharony, Bergman, Jafferis & J.M. 0806.1218

Motivation

- Understand the M2 brane field theory

- Study conformal field theories in 3d with large amount of supersymmetry

Schwarz, Bagger, Lambert, Gustavsson,...

- Find simple examples of AdS_4/CFT_3

WHY?

- Landscape
- Condensed matter

The theory

U(N) x U(N) gauge theory with bifundamental fields in 2+1 dimensions.

Field content

$$A_\mu \ , \qquad \hat{A}_\mu$$ gauge fields

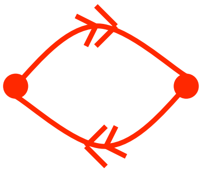
$$C_I \ , \quad \psi^I \quad \text{in} \quad (N, \bar{N})$$

$$(C_I)^* \ , \quad (\psi^I)^* \quad \text{in} (\bar{N}, N)$$

matter fields

+ complex conjugates

I=1,…,4 is an SU(4) (or SO(6)) index



$$L = L_{CS} + L_{kin} + L_{\psi^2 C^2} + L_{C^6}$$

$$L_{CS} = k \int Tr[AdA + \frac{2}{3}A^3] - k \int Tr[\tilde{A}d\tilde{A} + \frac{2}{3}\tilde{A}^3]$$

No kinetic term for the gauge fields, only Chern-Simons terms

Scale invariant (Chern Simons naturally scale invariant)

SO(6) = SU(4) symmetric

6 supercharges $Q^a_\alpha \ , \quad a = 1, \cdots, 6$

$U(1)_b$ global symmetry. Related to: $C^I \rightarrow e^{i\alpha} C^I$

$$j_b = k *_3 Tr[F + \tilde{F}]; \qquad j_C + j_b = 0$$

Why does it have $\mathcal{N}$=6 SUSY ?

- Start with the $\mathcal{N}$=3 theory with this matter content and susy

Zupnik Khetselius Kao Gaiotto Yin

$\mathcal{N}$=4 : Gaiotto Witten (Lin & JM)

- In $\mathcal{N}$=2 notation the superpotential is the same as in the Klebanov Witten theory

$$\mathrm{W} = \epsilon_{ab}\epsilon_{\dot{a}\dot{b}} Tr[A_a B_{\dot{a}} A_b B_{\dot{b}}]$$

- Has SU(2) x SU(2) symmetry that does not commute with SO(3) of $\mathcal{N}$=3
- Thus we have SU(4) R-symmetry → $\mathcal{N}$=6

$$\mathrm{C}^I = (A_a, B^*_{\dot{a}}) = (A_1, A_2, B^*_1, B^*_2)$$

Benna Klebanov Klose Smedback. Bandres, Epstein, Schwarz. Schnabl, Tachikawa.

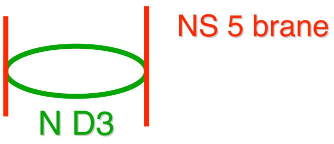
Relation to M2 branes

- Start with a brane construction of an $\mathcal{N}$=3 U(N)xU(N) Yang-Mills-Chern-Simons theory with the same matter fields.
- Relate it to M2 branes probing a particular 8-dimensional manifold.
- Take the IR limit

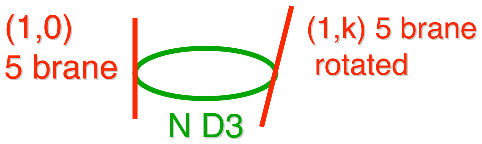
$$z^I \to e^{i\frac{2\pi}{k}} z^I \qquad \mathrm{R}^8/Z_k$$

- Get that $\mathrm{X}_8 \to R^8/Z_k$

In more detail



$\mathcal{N}=4$ Yang Mills
+ bifundamental hyper



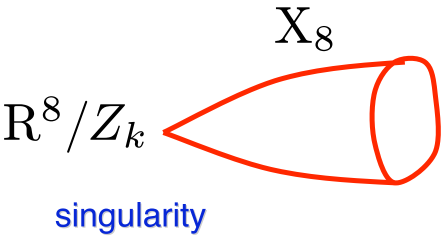
$\mathcal{N}=3$ Yang Mills CS
+ bifundamental hyper

CS: mass to fields in the vector multiplet

T-duality along the circle and lift to M-theory
The 5 branes become KK monopoles in M-theory.
We get a space X_8 which contains two intersecting KK monopoles.
These are called “Toric Hyperkahler Manifolds”.

Gauntlett, Gibbons, Papadopulos, Townsend

IR limit



$\mathcal{N}$=6 conformal CS matter theory

M2 branes at the singularity.
Locally the space preserves more susy.

- Coupling constant ~ 1/k
- k→ ∞ is the weak coupling limit
- There is a 't Hooft limit, N →∞, λ=N/k fixed.

- Field theory Moduli space: $\mathrm{Sym}\ (\ \mathrm{R}^8/Z_k)^N$
- Same as N M2 branes probing R^8/Z_k

- For k=1,2 it is SO(8) invariant and the theory should have $\mathcal{N}$=8 susy.
- For N=2 and gauge group SU(2)xSU(2) the theory has $\mathcal{N}$=8 and it coincides with the Bagger-Lambert-Gustavsson theory

Gravity dual

$$\mathrm{AdS}_4 \times S^7/Z_k$$

N units of flux of F_4

↓ k large

$$\mathrm{AdS}_4 \times CP^3$$

N units of flux of F_4
k units of flux of F_2 on CP^1 in CP^3

$$\frac{R^2}{l_s^2} \sim \lambda^{1/2} \sim \sqrt{\frac{N}{k}}$$

Nilsson Pope

Weakly coupled string theory.

We can vary the 't Hooft coupling between the perturbative gauge theory regime ($\lambda \ll 1$) and the gravity regime ($\lambda \gg 1$).

Thermal free energy

$$\beta F \sim N^{3/2} k^{1/2} V_2 T^2 \sim N^2 \frac{1}{\lambda^{1/2}} V_2 T^2$$

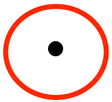
Like the ¾ of $\mathcal{N}$=4 SYM

Spectrum of operators

$$Tr[CC^*CC^* \cdots]$$

$$Tr[A_a B_b A_c B_d \cdots]$$

Operators with ‘t Hooft operators.

 S^2 Magnetic flux on this two-sphere

1 unit of magnetic flux → k units of baryon charge. Insert also k C^I fields

Borokhov
Kapustin
Wu

$$\mathrm{O} \sim T_1 C^k$$

Carry $U(1)_b$ charge

$$Tr[CC^*CC^* \cdots]$$

$$Tr[A_aB_bA_cB_d \cdots]$$

Kaluza Klein modes on S^7 with no momentum along the 11th direction.

Ordinary string states in the IIA description.

$$\text{O} \sim T_1 C^k$$

Modes with momentum in the 11th direction.

D0 branes in the IIA description

k=1,2

Extra symmetries.
Look at the scalars in the current supermultiplet.

$Tr[C^I (C^J)^*]$ Ordinary SU(4)

$T_{2,1} C^{(I} C^{J)}$ ← Dimension 1 fields → dimension 2 currents → Conserved currents

(Only for k=1,2)

Witten

For k=1 TC^I dimension ½ → free field center of mass motion

Analogy:

Compact boson in 2d at a specific radius → $SU(2)^2$ symmetry.

Integrability?

- Is this another example of an integrable gauge/string theory?

$\underline{AdS_5 \times S^5}$	$\underline{AdS_4 \times CP^3}$	
$Tr[Z^J]$	$Tr[(A_1 B_1)^J]$	vacuum
$SU(2\|2)^2$	$SU(2\|2) \times U(1)$	unbroken symmetries
$\epsilon = \sqrt{1 + \lambda \sin^2 \frac{p}{2}}$	$\epsilon = \sqrt{1 + h(\lambda) \sin^2 \frac{p}{2}}$	dispersion relation

Nishioka, Takayanagi, Minahan, Zarembo, Gaiotto, Gombi, Yin, Arutyunov Frolov Stefanski Fre Grassi D'Auria Trigiante Astolfi Giangreco Grignani Harmark Orselli Puletti Chen Wu McLoughlin Roiban Alday Bykov KrishnanAhn, Nepomechie Bak Rey

Conjectured exact phase Gromov Vieira

Generalizations

Aharony
Bergman
Jafferis

- $U(N)_k \times U(M)_{-k}$ $N > M$

Discrete Wilson lines of C_3 field

$$C_3 = \frac{N-M}{k} d\varphi J_2$$

(Torsion F_4 flux)

Theory does not exist if $N-M > k$

- $O(2N)_{2k} \times USp(2N)_{-k}$

$\mathcal{N}=5$ susy .
Extra orbifold in M-theory D_k
Orientifold of IIA theory

- Massive deformations

Gomis, Rodriguez-Gomez, Van Raamsdonk, Verlinde.

Interesting set of vacua M2 → M5 on S^3
M2 ending on M5

some puzzles

Basu, Harvey

Hanaki, Lin. Terashima.

- More general quivers

Hosomichi, $(\text{Lee})^3$, Park
Jefferis, Tomasiello. Martelli, Sparks.
Hanany, Zaffaroni.

- Squashed S^7

Ooguri, Park

- Flows to deformed S^7 with SU(3) symmetry (add 't Hooft operator)

Ahn, Benna Klebanov Klose Smedback

- Non susy

Armoni Navqi Giveon Kutasov

Conclusions

- Presented an $\mathcal{N}$=6 conformal Chern Simons Matter theory
- Has a discrete parameter, k, that allows us to go to weak coupling.
- At strong coupling, k=1,2, the theory has enhanced symmetry
- There is an interesting (and possibly integrable) ‘t Hooft limit

Future

- Understand better the ‘t Hooft operators.
  Develop techniques for seeing and exploiting the appearance of symmetries at strong coupling.

- Integrability?

- More general AdS_4 vacua.

- Condensed matter applications ?