Multiple Membrane Dynamics

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Based on:

“M2 to D2”,
SM and Costis Papageorgakis,

“M2-branes on M-folds”,
Jacques Distler, SM, Costis Papageorgakis and Mark van Raamsdonk,

“D2 to D2”,
Bobby Ezhuthachan, SM and Costis Papageorgakis,

Mohsen Alishahiha and SM, to appear
Motivation and background

The Higgs mechanism

Lorentzian 3-algebras

Higher-order corrections for Lorentzian 3-algebras

Conclusions
Motivation

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- And yet, despite $\sim 200$ recent papers – and two Strings 2008 talks – on the subject, we don’t exactly know what the multiple membrane theory is.
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- The latter should hold the key to M-theory:

- While there is an obstacle (due to (anti) self-dual 2-forms) to writing the $M_5$-brane field theory, there is no obstacle for $M_2$-branes as far as we know.
- And yet, despite $\sim 200$ recent papers – and two Strings 2008 talks – on the subject, we don’t exactly know what the multiple membrane theory is.
- Even the French aristocracy doesn’t seem to know...
Of course, there is one description that is clearly right and has manifest supersymmetry. But not manifest conformal symmetry.

\( \lim_{g \to \infty} g_2 \) YM

The question is whether this conformal IR fixed point has an explicit Lagrangian description wherein all the symmetries are manifest. This includes a global \( SO(8) \) symmetry describing rotations of the space transverse to the membranes – enhanced from the \( SO(7) \) of SYM.

Let us look at the Lagrangians that have been proposed to describe this limit.
Of course, there is one description that is clearly right and has manifest $\mathcal{N} = 8$ supersymmetry (but not manifest conformal symmetry):

$$\lim_{g_{YM} \to \infty} \frac{1}{g_{YM}^2} \mathcal{L}_{SYM}$$
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Euclidean 3-algebra [Bagger-Lambert, Gustavsson]: Labelled by integer $k$. Algebra is $SU(2) \times SU(2)$.

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- **Lorentzian 3-algebra** [Gomis-Milanesi-Russo, Benvenuti-Rodriguez-Gomez-Tonni-Verlinde, Bandres-Lipstein-Schwarz, Gomis-Rodriguez-Gomez-van Raamsdonk-Verlinde]: Based on arbitrary Lie algebras, have $\mathcal{N} = 8$ superconformal invariance.
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ABJM theories [Aharony-Bergman-Jafferis-Maldacena]: Labelled by algebra $G \times G'$ and integer $k$, with $\mathcal{N}=6$ superconformal invariance. Is actually a “relaxed” 3-algebra.
⇒ Describe multiple $M2$-branes at orbifold singularities. But the $k=1$ theory is missing two manifest supersymmetries and decoupling of CM mode not visible.
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Thus the basic classification is:

(i) **Euclidean signature** 3-algebras, which are $G \times G$ Chern-Simons theories:

$$k \, \text{tr} \left( A \wedge dA + \frac{2}{3} A \wedge A \wedge A - \tilde{A} \wedge d\tilde{A} - \frac{2}{3} \tilde{A} \wedge \tilde{A} \wedge \tilde{A} \right)$$

BLG : $G = SU(2)$

ABJM : $G = SU(N)$ or $U(N)$, any $N$ (+ other choices)

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(ii) **Lorentzian signature** 3-algebras, which are $B \wedge F$ theories based on any Lie algebra.

scalars, fermions are singlet + adjoint, e.g. $X^{I}_{+}, X^{I}$
Both classes make use of the triple product $X^{IJK}$:

- **Euclidean**: $X^{IJK} \sim X^I X^J \dagger X^K$, $X^I$ bi-fundamental
- **Lorentzian**: $X^{IJK} \sim X^I [X^J, X^K] + \text{cyclic}$
  
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The potential is:

$$V(X) \sim (X^{IJK})^2$$

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However it’s also maximally superconformal, which should give us a lot of power in dealing with it.

In this talk I’ll deal with some things we have understood about the desired theory.
The Higgs mechanism

Lorentzian 3-algebras

Higher-order corrections for Lorentzian 3-algebras

Conclusions
The Higgs mechanism

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- If we give a vev $v$ to one component of the bi-fundamental fields, then at energies below this vev, the Lagrangian becomes:

$$L_{CS}^{(G \times G)} \bigg|_{vev \ v} = \frac{1}{v^2} L_{SYM}^{(G)} + O \left( \frac{1}{v^3} \right)$$

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$$L^{(G \times G)}_{CS} \bigg|_{vev \ v} = \frac{1}{v^2} L^{(G)}_{SYM} + \mathcal{O} \left( \frac{1}{v^3} \right)$$

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- This is an unusual result. In SYM with gauge group $G$, when we give a vev to one component of an adjoint scalar, at low energy the Lagrangian becomes:

$$\frac{1}{g^2_{YM}} L^{(G)}_{SYM} \bigg|_{vev \ v} = \frac{1}{g^2_{YM}} L^{(G' \subset G)}_{SYM}$$

where $G'$ is the subgroup that commutes with the vev.
Let’s give a quick derivation of this novel Higgs mechanism, first for $k = 1$:

$$L_{CS} = \text{tr} \left( A \wedge dA + \frac{2}{3} A \wedge A \wedge A - \tilde{A} \wedge d\tilde{A} - \frac{2}{3} \tilde{A} \wedge \tilde{A} \wedge \tilde{A} \right)$$

$$= \text{tr} \left( A_- \wedge F_+ + \frac{1}{6} A_- \wedge A_- \wedge A_- \right)$$

where $A_\pm = A \pm \tilde{A}, \quad F_+ = dA_+ + \frac{1}{2} A_+ \wedge A_+$. 
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If $\langle X \rangle = v 1$ then:

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Thus, $A_-$ is massive – but not dynamical. Integrating it out gives us:

$$-\frac{1}{4v^2} (F_+)^{\mu\nu} (F_+)^{\mu\nu} + O \left( \frac{1}{v^3} \right)$$

so $A_+$ becomes dynamical.
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But how should we **physically interpret** this?

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For any **finite** $v$, there are corrections to the SYM. These decouple only as $v \rightarrow \infty$. So at best we can say that:

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The RHS is by definition the theory on $M2$-branes! So this is more like a “proof” that the original Chern-Simons theory really is the theory on $M2$-branes.
However once we introduce the Chern-Simons level \( k \) then the analysis is different [Distler-SM-Papageorgakis-van Raamsdonk]:

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If we take $k \to \infty$, $v \to \infty$ with $v^2/k = g_{YM}$ fixed, then in this limit the RHS actually becomes:

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and this is definitely the Lagrangian for $D2$ branes at finite coupling.
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So this time we have compactified the theory! How can that be?
We proposed this should be understood as **deconstruction** for an orbifold $C^4/Z_k$:

![Diagram illustration](Image)

In our paper we observed that the orbifold $C^4/Z_k$ has $N=6$ supersymmetry and $SU(4)_R$ symmetry. We thought this might be enhanced to $N=8$ for some unknown reason. Instead, as $y_z J_M$ found, it's the $z L G$ field theory that needs to be modified to have $N=6$. One lesson we learn is that for large $k$ we are in the regime of weakly coupled string theory, but for understanding the basics of M-branes, that is not where we want to be.
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In stead as $yzJM$ found, it’s the LG field theory that needs to be modified to have $\mathcal{N} = 6$.

One lesson we learn is that for large $k$ we are in the regime of weakly coupled string theory. A lot can be done in that regime, but for understanding the basics of M-branes, that is not where we want to be.
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- Instead, as ABJM found, it’s the BLG field theory that needs to be modified to have $\mathcal{N} = 6$. 
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- One lesson we learn is that for large \( k \) we are in the regime of weakly coupled string theory.

- A lot can be done in that regime, but for understanding the basics of M2-branes, that is not where we want to be.
Outline

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Lorentzian 3-algebras

The Lorentzian 3-algebra theories have the following Lagrangian:

\[ L^{(G)}_{L3A} = \text{tr} \left( \frac{1}{2} \varepsilon^{\mu \nu \lambda} B_{\mu} F_{\nu \lambda} - \frac{1}{2} \hat{D}_{\mu} X^{I} \hat{D}^{\mu} X^{I} ight) \]

\[ - \frac{1}{12} \left( X^{I}[X^{J}, X^{K}] + X^{J}[X^{K}, X^{I}] + X^{K}[X^{I}, X^{J}] \right)^{2} \]

\[ + \left( C^{\mu I} - \partial^{\mu} X^{I}_{-} \right) \partial_{\mu} X^{I}_{+} + L_{\text{gauge fixing}} + L_{\text{fermions}} \]

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They have \( SO(8) \) global symmetry acting on the indices \( I, J, K \in 1, 2, \cdots, 8 \).

The equation of motion of the auxiliary gauge field \( C^I_\mu \) implies that \( X_+ = \text{constant} \).
Our Higgs mechanism works in these theories, but it works too well! [Ho-Imamura-Matsuo]
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$$\langle X^8_+ \rangle = v$$

one finds:

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In fact it can be derived [Ezhuthachan-SM-Papageorgakis] starting from $\mathcal{N} = 8$ SYM.
The procedure involves a non-Abelian (dNS) duality [deWit-Nicolai-Samtleben] on the (2+1)d gauge field.
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Start with $\mathcal{N} = 8$ SYM in (2+1)d. Introducing two new adjoint fields $B_\mu, \phi$, the dNS duality transformation is:

$$- \frac{1}{4g_{YM}^2} F^{\mu\nu} F_{\mu\nu} \rightarrow \frac{1}{2} \epsilon^{\mu\nu\lambda} B_\mu F_{\nu\lambda} - \frac{1}{2} (D_\mu \phi - g_{YM} B_\mu)^2$$

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In addition to the gauge symmetry $G$, the new action has a noncompact abelian gauge symmetry:

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where $M(x)$ is an arbitrary matrix in the adjoint of $G$.

To prove the duality, use this symmetry to set $\phi = 0$. Then integrating out $B_\mu$ gives the usual YM kinetic term for $F_{\mu\nu}$. 
The dNS-duality transformed $\mathcal{N} = 8$ SYM is:

\[
L = \text{tr} \left( \frac{1}{2} \epsilon^{\mu \nu \lambda} B_\mu F_{\nu \lambda} - \frac{1}{2} (D_\mu \phi - g_{YM} B_\mu)^2 
- \frac{1}{2} D_\mu X^i D^\mu X^i - \frac{g_{YM}^2}{4} [X^i, X^j]^2 + \text{fermions} \right)
\]
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Rename $\phi \to X^8$. Then the scalar kinetic terms are:

$$-\frac{1}{2} \hat{D}_\mu X^I \hat{D}^\mu X^I = -\frac{1}{2} \left( \partial_\mu X^I - [A_\mu, X^I] - g_{YM}^I B_\mu \right)^2$$

where $g_{YM}^I = (0, \ldots, 0, g_{YM})$. 
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where $g_{YM}^I = (0, \ldots, 0, g_{YM})$.

Next, we can allow $g_{YM}^I$ to be an arbitrary 8-vector.
The action is now $SO(8)$-invariant if we rotate both the fields $X^I$ and the coupling-constant vector $g^I_{YM}$:

$$L = \text{tr} \left( \frac{1}{2} \epsilon^{\mu \nu \lambda} B_\mu F_{\nu \lambda} - \frac{1}{2} \hat{D}_\mu X^I \hat{D}^\mu X^I \\
- \frac{1}{12} \left( g^I_{YM}[X^J, X^K] + g^J_{YM}[X^K, X^I] + g^K_{YM}[X^I, X^J] \right)^2 \right)$$
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The final step is to introduce an 8-vector of new (gauge-singlet) scalars $X_+^I$ and replace:

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The final step is to introduce an 8-vector of new (gauge-singlet) scalars $X^I_+$ and replace:

\[ g^I_{YM} \rightarrow X^I_+(x) \]

This is legitimate if and only if $X^I_+(x)$ has an equation of motion that renders it constant. Then on-shell we can recover the original theory by writing $\langle X^I_+ \rangle = g^I_{YM}$. 

\[ L = \text{tr} \left( \frac{1}{2} \epsilon^{\mu\nu\lambda} B_{\mu} F_{\nu\lambda} - \frac{1}{2} \hat{D}_\mu X^I \hat{D}^\mu X^I 
- \frac{1}{12} \left( g^I_{YM} [X^J, X^K] + g^J_{YM} [X^K, X^I] + g^K_{YM} [X^I, X^J] \right)^2 \right) \]
The action is now $SO(8)$-invariant if we rotate both the fields $X^I$ and the coupling constant vector $g^I_{YM}$. This is not yet a symmetry since it rotates the coupling constant $f$.

The final step is to introduce a vector of new "gauge ingleta" scalars $X^I$ and replace $g^I_{YM} \rightarrow X^I$.

This is legitimate if and only if $X^I$ has an equation of motion that renders it constant. Then, one shell we can recover the original theory by writing $X^I = g^I_{YM} f$ – on constancy of $X^I$ is imposed by introducing a new set of abelian gauge fields and scalars $C^I_\mu, X_I^-$ and adding the following term:

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Constancy of $X^I_+$ is imposed by introducing a new set of abelian gauge fields and scalars: $C^I_\mu, X^I_-$ and adding the following term:

$$L_C = (C^\mu_ I - \partial X^-_I)\partial_\mu X^+_I$$

This has a shift symmetry

$$\delta X^I_- = \lambda^I, \quad \delta C^I_\mu = \partial_\mu \lambda^I$$

which will remove the negative-norm states associated to $C^I_\mu$. 

We have thus ended up with the Lorentzian algebra action:

$$L = \text{tr} \left( \frac{1}{2} \epsilon_{\mu \nu \lambda} B^{\mu \nu} F^{\nu \lambda} - \frac{1}{2} \hat{D}^{\mu} X^I_+ \hat{D}_\mu X^{-I} - \frac{1}{12} \left[ X^I_+, X^J_-, X^K_+ \right] \right)$$
Constancy of $X^I_+$ is imposed by introducing a new set of abelian gauge fields and scalars: $C^I_\mu, X^I_-$ and adding the following term:

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We have thus ended up with the Lorentzian 3-algebra action [Bandres-Lipstein-Schwarz, Gomis-Rodriguez-Gomez-van Raamsdonk-Verlinde]:

$$L = \text{tr} \left( \frac{1}{2} \epsilon^{\mu\nu\lambda} B_\mu F_{\nu\lambda} - \frac{1}{2} \hat{D}_\mu X^I \hat{D}_\mu X^I ight.$$  

$$- \frac{1}{12} \left( X^I_+ [X^J, X^K] + X^J_+ [X^K, X^I] + X^K_+ [X^I, X^J] \right)^2 \bigg) + (C^I_\mu - \partial_\mu X^I_-) \partial_\mu X^I_+ + L_{\text{gauge-fixing}} + \mathcal{L}_{\text{fermions}}$$
The final action has some remarkable properties. It has manifest $SO(8)$ invariance as well as $N=8$ superconformal invariance. However, both are spontaneously broken by giving a vev $X^I + \bar{X}_I = g_I Y_M$ and the theory reduces to $N=8$ SYM with coupling $|g_{YM}|$. It will certainly describe M-branes if one can find a way to take $X^I + \bar{X}_I = \infty$. That has not yet been done.
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However, both are spontaneously broken by giving a vev $\langle X_+^I \rangle = g_{YM}^I$ and the theory reduces to $\mathcal{N} = 8$ SYM with coupling $|g_{YM}|$.

It will certainly describe M2-branes if one can find a way to take $\langle X_+^I \rangle = \infty$. That has not yet been done.
Motivation and background

The Higgs mechanism

Lorentzian 3-algebras

Higher-order corrections for Lorentzian 3-algebras

Conclusions
Higher-order corrections for Lorentzian 3-algebras

One might ask if the non-Abelian duality that we have just performed works when higher order (in $\alpha'$) corrections are included.
Higher-order corrections for Lorentzian 3-algebras

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- For the Abelian case [Duff, Townsend, Schmidhuber] we know that the analogous duality works for the entire DBI action and that fermions and supersymmetry can also be incorporated [Aganagic-Park-Popescu-Schwarz].
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One might ask if the non-Abelian duality that we have just performed works when higher order (in $\alpha'$) corrections are included.

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Recently we have shown [Alishahiha-SM] that to lowest nontrivial order ($F^4$-type corrections) one can indeed dualise the non-Abelian SYM into an $SO(8)$-invariant form.

Here of course one cannot do all orders in $\alpha'$ because a non-Abelian analogue of DBI is still not known.

However our approach may have a bearing on that unsolved problem.
Let us see how this works. In (2+1)d, the lowest correction to SYM for D2-branes is the sum of the following contributions (here $X^{ij} = [X^i, X^j]$):

\[
L_1^{(4)} = \frac{1}{12g^4_{YM}} \left[ F_{\mu\nu} F_{\rho\sigma} F^{\mu\rho} F^{\nu\sigma} + \frac{1}{2} F_{\mu\nu} F^{\nu\rho} F_{\rho\sigma} F^{\sigma\mu} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} F_{\rho\sigma} F^{\rho\sigma} - \frac{1}{8} F_{\mu\nu} F_{\rho\sigma} F^{\mu\nu} F^{\rho\sigma} \right]
\]

\[
L_2^{(4)} = \frac{1}{12g^2_{YM}} \left[ F_{\mu\nu} D^\mu X^i F^{\rho\nu} D^\rho X^i + F_{\mu\nu} D_\rho X^i F^{\mu\rho} D^\nu X^i - 2 F_{\mu\rho} F^{\rho\nu} D^\mu X^i D^\nu X^i - 2 F_{\mu\rho} F^{\rho\nu} D^\nu X^i D^\mu X^i - F_{\mu\nu} F^{\mu\nu} D^\rho X^i D_\rho X^i - \frac{1}{2} F_{\mu\nu} D_\rho X^i F^{\mu\rho} D_\rho X^i \right]
\]

\[
L_3^{(4)} = -\frac{1}{6} \left( D^\mu X^i D^\nu X^j F_{\mu\nu} + D^\nu X^j F_{\mu\nu} D^\mu X^i + F_{\mu\nu} D^\mu X^i D^\nu X^j \right) X^{ij}
\]
\[ L_4^{(4)} = \frac{1}{12} \left[ D_\mu X^i D_\nu X^j D^\nu X^i D^\mu X^j + D_\mu X^i D_\nu X^j D^\mu X^j D^\nu X^i \right. \]
\[ \left. + D_\mu X^i D_\nu X^i D^\nu X^j D^\mu X^j - D_\mu X^i D^\mu X^i D_\nu X^j D^\nu X^j \right. \]
\[ \left. - \frac{1}{2} D_\mu X^i D_\nu X^j D^\mu X^i D^\nu X^j \right] \]
\[ L_5^{(4)} = \frac{g_{YM}^2}{12} \left[ X^{kij} D_\mu X^k X^i j D^\mu X^i + X^{ij} D_\mu X^k X^i k D^\mu X^j \right. \]
\[ \left. - 2X^{kij} X^{ik} D_\mu X^j D^\mu X^i - 2X^{kij} X^{jk} D_\mu X^j D^\mu X^i \right. \]
\[ \left. - X^{ij} X^{ij} D_\mu X^k D^\mu X^k - \frac{1}{2} X^{ij} D_\mu X^k X^{ij} D^\mu X^k \right] \]
\[ L_6^{(4)} = \frac{g_{YM}^4}{12} \left[ X^{ij} X^{kl} X^{ik} X^{jl} + \frac{1}{2} X^{ij} X^{jk} X^{kl} X^{li} \right. \]
\[ \left. - \frac{1}{4} X^{ij} X^{ij} X^{kl} X^{kl} - \frac{1}{8} X^{ij} X^{kl} X^{ij} X^{kl} \right] \]
We have been able to show that this is dual, under the DNS transformation, to:

\[ L = \text{tr} \left[ \frac{1}{2} \epsilon^{\mu \nu \rho} B_{\mu} F_{\nu \rho} - \frac{1}{2} \hat{D}_{\mu} X^I \hat{D}^\mu X^I \right] \]

\[ + \frac{1}{12} \left( \hat{D}_{\mu} X^I \hat{D}_\nu X^J \hat{D}^\nu X^I \hat{D}^\mu X^J + \hat{D}_{\mu} X^I \hat{D}_\nu X^J \hat{D}^\mu X^J \hat{D}^\nu X^I \right. \]

\[ + \hat{D}_{\mu} X^I \hat{D}_\nu X^I \hat{D}^\nu X^J \hat{D}^\mu X^J - \hat{D}_{\mu} X^I \hat{D}^\mu X^I \hat{D}_\nu X^J \hat{D}^\nu X^J \]

\[ - \frac{1}{2} \hat{D}_{\mu} X^I \hat{D}_\nu X^J \hat{D}^\mu X^I \hat{D}^\nu X^J \right) \]

\[ + \frac{1}{12} \left( \frac{1}{2} \epsilon^{L K J} \hat{D}_{\mu} X^K X^{L I J} \hat{D}^\mu X^I + \frac{1}{2} \epsilon^{L I J} \hat{D}_{\mu} X^K X^{L I K} \hat{D}^\mu X^J \right. \]

\[ - X^{L K J} X^{L I K} \hat{D}_{\mu} X^J \hat{D}^\mu X^I - X^{L K I} X^{L J K} \hat{D}_{\mu} X^J \hat{D}^\mu X^J \]

\[ - \frac{1}{3} \epsilon^{L I J} \epsilon^{L I J} \hat{D}_{\mu} X^K \hat{D}^\mu X^K - \frac{1}{6} \epsilon^{L I J} \hat{D}_{\mu} X^K X^{L I J} \hat{D}^\mu X^K \right) \]

\[ - \frac{1}{6} \epsilon_{\rho \mu \nu} \hat{D}^\rho X^I \hat{D}^\mu X^J \hat{D}^\nu X^K X^{I J K} - V(X) \]
In the previous expression,

\[ \hat{D}_\mu X^I = \partial_\mu X^I - [A_\mu, X^I] - B_\mu X^I \]

\[ X^{IJK} = X_+[X^J, X^K] + X_+[X^K, X^I] + X_+[X^I, X^J] \]
In the previous expression,

\[
\hat{D}_\mu X^I = \partial_\mu X^I - [A_\mu, X^I] - B_\mu X^I + X^{IJK} = X^I + [X^J, X^K] + X^J + [X^K, X^I] + X^K + [X^I, X^J]
\]

Here \( V(X) \) is the potential:

\[
V(X) = \frac{1}{12} X^{IJK} X^{IJK} + \frac{1}{108} \left[ X^{NIJ} X^{NKL} X^{MIK} X^{MJL} + \frac{1}{2} X^{NIJ} X^{MKL} X^{NKL} X^{MLI} - \frac{1}{4} X^{NIJ} X^{NLI} X^{MKL} X^{MKL} - \frac{1}{8} X^{NII} X^{MKL} X^{NII} X^{MKL} \right]
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We see that the dual Lagrangian is \( SO(8) \) invariant.
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\]

We see that the dual Lagrangian is \( SO(8) \) invariant.

It’s worth noting that this depends crucially on the relative coefficients of various terms in the original Lagrangian.
We see from this that the 3-algebra structure remains intact when higher-derivative corrections are taken into account.
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We conjecture that $SO(8)$ enhancement holds to all orders in $\alpha'$. 
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We conjecture that $SO(8)$ enhancement holds to all orders in $\alpha'$.

Unfortunately the all-orders corrections are not known for SYM, so we don’t have a starting point from which to check this.
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Much progress has been made towards finding the multiple membrane field theory representing the IR fixed point of $\mathcal{N} = 8$ SYM.
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The existence of a large-order orbifold (deconstruction) limit provides a way (the only one so far) to relate the membrane theory to $D2$-branes. One would like to understand compactification of transverse or longitudinal directions, as we do for D-branes.
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The existence of a large-order orbifold (deconstruction) limit provides a way (the only one so far) to relate the membrane theory to $D2$-branes. One would like to understand compactification of transverse or longitudinal directions, as we do for D-branes.

An interesting mechanism has been identified to dualise the $D2$-brane action into a superconformal, $SO(8)$ invariant one. The result is a Lorentzian 3-algebra and this structure is preserved by $\alpha'$ corrections.
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