Loop Quantum Gravity

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i. Loop Quantum Gravity

- The problem addressed
- Why loops ?

ii. The theory

- Kinematics: spin networks and quanta of space
- Dynamics: spinfoams

iii. Physics

- Cosmology, black holes, singularities, low-energy limit

iv. Conclusion

- What has been achieved? What is missing?
- Future

i. Loop Quantum Gravity

- Develops since the late 80s.
- About 200 people, 30 research groups.
- Several books (CR: "Quantum Gravity"):

The problem addressed:

How to describe
the fundamental degrees of freedom
when there is no fixed background space

Hypotheses

- A radical conceptual change in our concept of space and time is required.

- The problem can be addressed already in the context of *current* physical theory: general relativity, coupled with the standard model.

- In gravity, (unrenormalizable) UV divergences are consequences of a perturbation expansion around a wrong vacuum. → Confirmed *a posteriori* in LQG.

- Guiding principle: a symmetry: *Diffeomorphism invariance*.

Consistency with quantum mechanics and (in the low-energy limit) General Relativity, and full *diff-invariance* are extremely strong constraints on the theory.

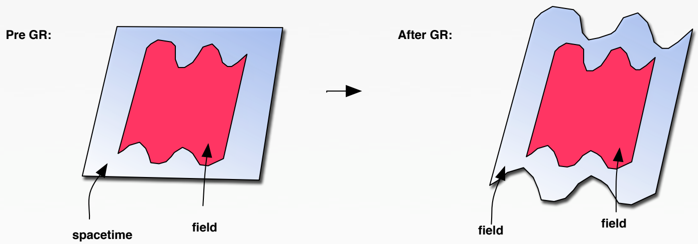
Main result

→ Definition of *Diffeomorphisms invariant* quantum field theory (for gauge fields plus fermions), in canonical and in covariant form.

A comment on general relativity

- GR is: i) A *specific* field theory for the gravitational field $g_{\mu\nu}(x)$: $S[g] = \int d^4x\,\sqrt{-g}\,(R + \lambda)$
- ii) A *general* modification of our understanding of spacetime :

spacetime ⟷ gravitational field



This modification is expressed by the invariance of the theory under the active action of the group of the *diffeomorphisms* on all the fields of the theory.

Why Loops ?

“Old” nonperturbative quantum gravity

Canonical (*Wheeler 64, DeWitt 63 ...*)

- *States*: $\Psi(\mathbf{q})$. $\mathbf{q}$: 3d metric of a t=0 surface
- *3d diff*: $\Psi(\mathbf{q}) = \Psi(\mathbf{q'})$ if there is a diff: $\mathbf{q} \longrightarrow \mathbf{q'}$
- *Dynamics*: Wheler-DeWitt eq $\mathbf{H}\Psi(\mathbf{q}) = 0$ **.**

difficulties**:**

- Which states $\Psi(\mathbf{q})$**?** Which scalar product $\mathbf{<} \Psi, \Phi \mathbf{>}$ **?**
- Operator **H** badly defined and UV divergent.
- No calculation possible.

Covariant (*Misner 57, Hawking 79 ...*)

- $$Z = \int Dg \; e^{iS[g]}$$

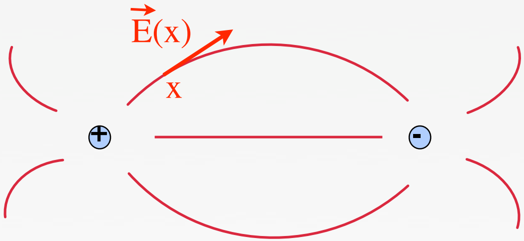
difficulties**:**

- Integral very badly defined
- Perturbative calculations bring UV divergences back.

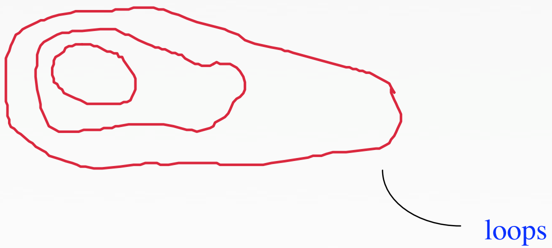
An old idea : Gauge fields are naturally "made up of lines"

(Polyakov, Mandelstan, Wilson, Migdal, ... Faraday)

Faraday lines:



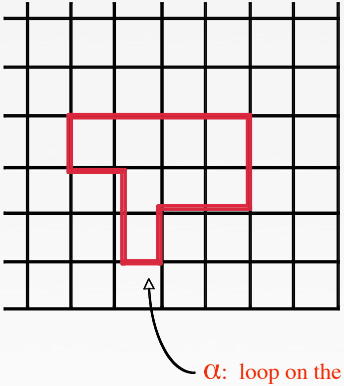
In vacuum:



The gravitational field too, can be described as a gauge field, with a connection as main variable. (Cartan, Weyl, Swinger, Utyama, ..., Ashtekar)

Can we describe a quantum gauge-field-theory in terms of these lines?

Yes, on the lattice. *(Wilson, Kogut, Susskind, ...)*



Variable: U_e in the gauge group **G**

State: $\Psi(U_e)$ in $\mathcal{H} = L_2[G^{(\text{number of edges})}, d\mu_{Haar}]$

Operators: Magnetic field $B_{Plaquette}$: $(U_1 U_2 U_3 U_4)$

Electric field: $\mathbf{E_e} = -i\hbar\, \partial/\partial U_e$ (Left invariant vector field)

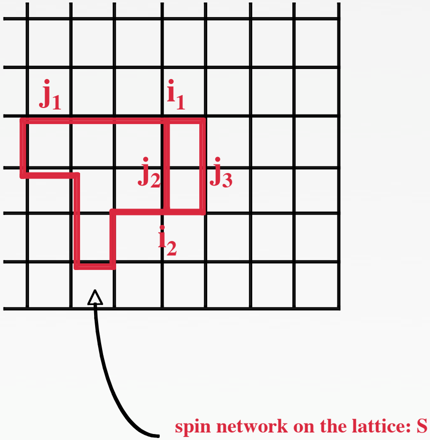
Dynamics: $H = B^2 + E^2$

Loop state: $\Psi_\alpha(U_e) = \text{Tr}\,(U_1 \ldots U_e \ldots U_N) = \langle U_e | \alpha \rangle$

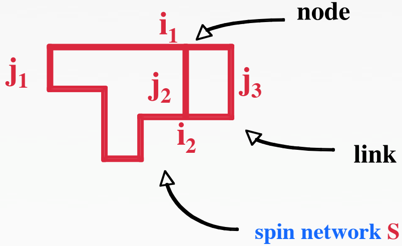
→ $|\alpha\rangle$ is an eigenstate of E_l with eigenvalue having support on α .

$|\alpha\rangle$ is a “quantum excitation of a single Faraday line”.

Generalization: spin-networks and spin-network states



Spin network $S = (\Gamma, j_l, i_n)$
- graph Γ
- spins j_l on links
- intertwiners i_n on nodes

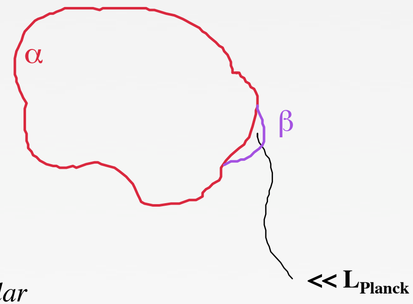


Spin network state: $\Psi_S(U_e) = R^{(j_1)}(U_l) \ldots R^{(j_L)}(U_L) \cdot i_1 \ldots i_N = \langle U_e | S \rangle$

→ The spin network states $| S \rangle$ form an othonormal basis in $\mathcal{H}$ (Peter-Weyl).

Can we use these loop states as a basis of states in the continuum ?

No !



- < α | α > = ∞

  The loop states | α > in the continuum are *too singular*

- < α | β > = 0 for β infinitely close to α

  States are *"too many", as a basis of the Hilbert space*.

- An *infinitesimal* displacement in space yields a *different* loop state

So far, just difficulties:

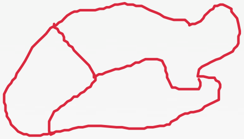
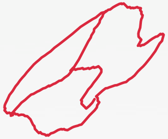
- "Old" nonperturbative quantum gravity does not work.
- A loop-state formalism in the continuum for Yang Mills does not work.

But the two ideas provide the solution to each other's stumbling blocks:

A *loop* formulation of *gravity* solves both sets of difficulties

Diffeomorphism invariance :

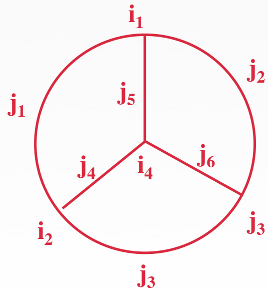
| S > and | S' > are gauge equivalent if S can be transformed into S'
by a diffeomorphism !

S =  S' =  S'' = 

$< S \mid \Psi > \; = \; < S' \mid \Psi > \; = \; < S'' \mid \Psi >$ for any Ψ.

→ States are determined only by an *abstract* graph γ with j's and i's

s-knot states | s > = $|\gamma, j, i>$, where s =



ii. The theory.

- Start from GR, or GR+standard model, or any other diff-invariant theory, in a formulation where the field is decribed by a connection $\boldsymbol{A}$ (Ashtekar).

- Do a canonical quantization of the theory, using a basis of spin network states and operators acting on these.

- Impose diffeomorphism invariance on the states.

- Study the Wheeler deWitt equation.

Result:

→ A (separable) Hilbert space $\mathcal{H}$ of states, and an operator algebra $\mathcal{A}$.

→ Basis of $\mathcal{H}$: abstract spin network states: graph labelled by spins and intertwiners.

→ A well defined UV-finite dynamics.

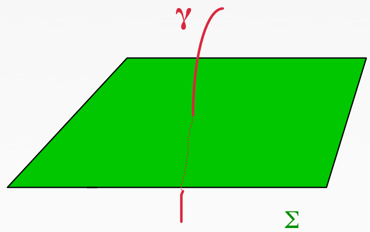
$\mathcal{H}$:

$\mathcal{H}_{\text{ext}}$: (norm-closure of the) space of the (cylindrical) functionals $\Psi_{\gamma_i,f}[A] \;=\; f(U_{\gamma_1}[A], ..., U_{\gamma_n}[A])$

where $U_\gamma[A] \;=\; e^{\int_\gamma A}$, equipped with the scalar product

$$(\Psi_{\gamma_i,f}, \Psi_{\gamma_i,g}) \;=\; \int_{SU(2)^n} dU \; \overline{f(U_1, ..., U_n)} \; g(U_1, ..., U_n)$$

→ This product is SU(2) and diff invariant. Hence $\mathcal{H}_{\text{ext}}$ carries a unitary representation of *Diff* and *local SU(2)*. The operation of factoring away the action of these groups is well-defined, and defines $\mathcal{H} = \mathcal{H}_{\text{ext}}/(Diff$ and *local SU(2)*).



$\mathcal{A}$:

→ Operators well-defined on $\mathcal{H}_{\text{ext}}$, and self-adjoint: $U_\gamma[A] \;=\; e^{\int_\gamma A}$ and $E_\Sigma = \int_\Sigma E$

where E is the variable conjugate to A, smeared on a two-surface Σ. This acts as an SU(2) Left-invariant vector field on the SU(2) of each line that intersects Σ .

The LOST uniqueness theorem

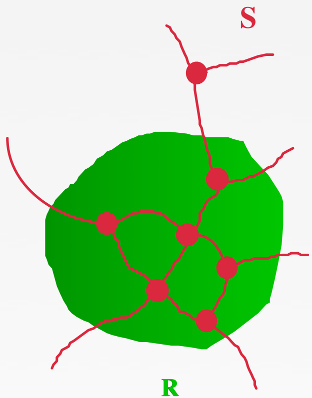
(*Fleishhack 04; Lewandowski, Okolow, Sahlmann, Thiemann 05*)

- ($\mathcal{H}_{ext}$, $\mathcal{A}$) provides a representation of the classical poisson algebra of the observables $U_\gamma[A]$ and E_Σ, carrying a unitary representation of *Diff*,

- → this representation is *unique*.

(cfr.: von Neumann theorem in nonrelativistic QM.)

Interpretation of the spin network states | S >

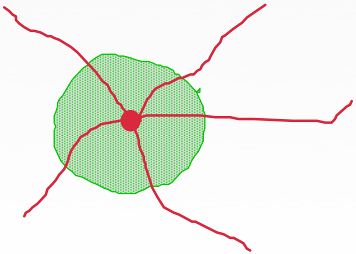


Volume: V(R): function of the gravitational field

$$V(R) = \int_R \sqrt{g} = \int_R \sqrt{EEE} \quad \rightarrow \quad V(R) \text{ operator}$$

- → V(R) is a well-defined self-adjoint operator in $\mathcal{H}_{ext}$,
- → It has discrete spectrum. Eigenstates: spin networks state | S >.

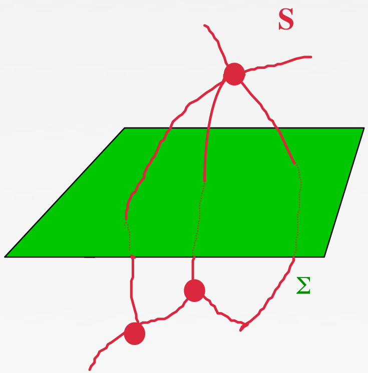
- Eigenvalues receive a contribution for each node of | S > inside R



Node = "Chunk of space"
with quantized volume

Area: A(Σ)

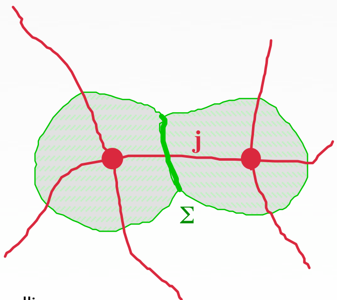
$$A(\Sigma) = \int_\Sigma \sqrt{g} = \int_\Sigma \sqrt{EE} \quad \rightarrow \quad A(\Sigma) \text{ operator}$$



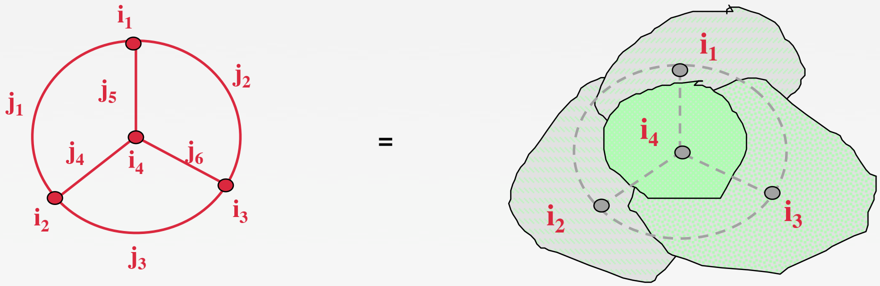
- → A(Σ) well-defined selfadjoint operator in $\mathcal{H}_{ext}$.
- → The spectrum is discrete.
- → Area gets a contribution for each link of | S > that intersects Σ.

Area eigenvalues: $$A = 8\pi\ \hbar G\ \gamma \sum_i \sqrt{j_i(j_i+1)}$$

(γ = Immirzi parameter)



Link = "Quantum of surface"
with quantized area



$$|\, s > \; = \; |\, \gamma \,,\, j_l \,,\, i_n >$$

s-knot state

connectivity between the elementary quantum chunks of space

quantum numbers of area

quantum numbers of volume

→ Spin network states represent discrete quantum excitations of spacetime

- Spin networks are not excitations in space: they are excitations of space.
  - → Background independent QFT

- → **Discrete structure of space at the Planck scale** - in quantum sense

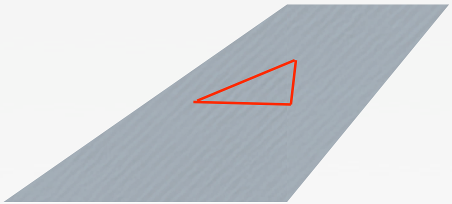
  Follows from:

  standard quantum theory (cfr granularity of oscillator's energy)

  + standard general relativity (because "space is a field").

Loops & strings: a cartoon comparison

If a string is:

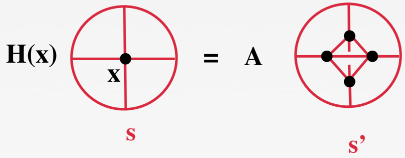


a closed lines in space, that
forms matter and forces,

a loop is:

a closed line that forms space itself
as well as matter and forces.

III. The theory - dynamics

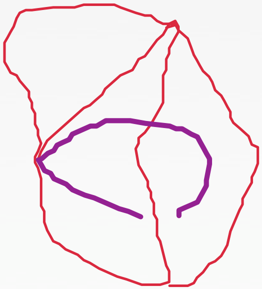


Given by a Wheeler-deWitt operator **H** in $\mathcal{H}$: **H Ψ = 0**

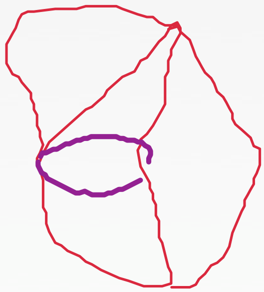
- **H** is defined by a regularization of the classical Hamiltonian constraint. In the limit in which the regularization is removed.

→ **H** is a well defined self-adjoint operator, UV finite on diff-invariant states.

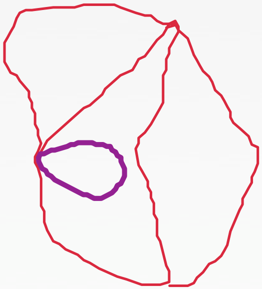
$\alpha \to 0$



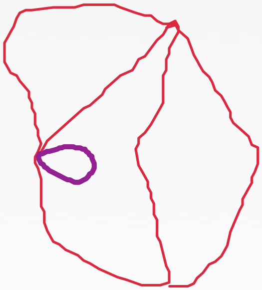
$\alpha \to 0$



$\alpha \to 0$



$\alpha \rightarrow 0$



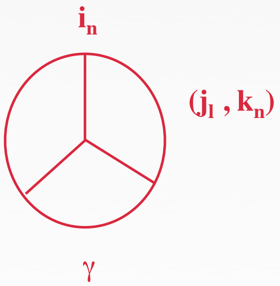
$\alpha \to 0$

The limit $\alpha \to 0$ is trivial because
there is no short distance structure at all in the theory !

- The theory is naturally ultraviolet finite

Matter

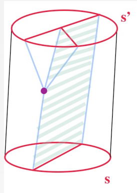
- YM, fermions
- Same techniques: The gravitational field is *not* special
- → UV finiteness remains
- YM and fermions on spin networks = on a Planck scale lattice !
  Notice: no lattice spacing to zero !
- Matter from braiding ?



quantum numbers of matter

$$| s > \; = \; | \gamma , j_l , i_n , k_l >$$

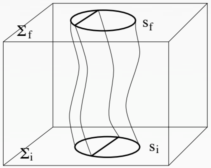
III. The theory - covariant dynamics: spinfoams



Projector $P = \delta(H)$ on the kernel of H:

$$\langle s'|P|s\rangle = \langle s'|\delta(H)|s\rangle = \langle s'|\int DN e^{iNH}|s\rangle$$
$$= c_0\langle s'|s\rangle + c_1\,\langle s'|H|s\rangle + c_2\,\langle s'|H^2|s\rangle + \ldots$$

=



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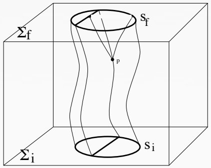

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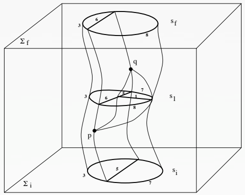
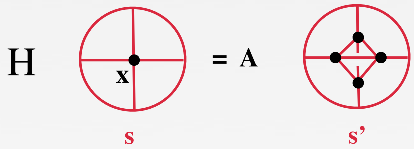
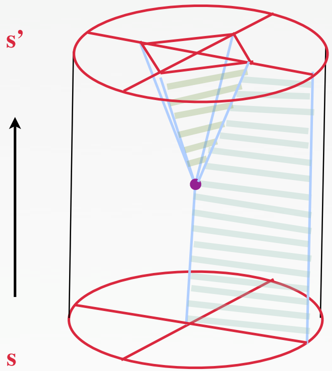


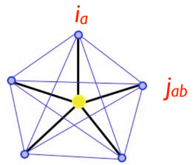
+ ...

$$\langle s'|P|s\rangle = \sum_{\partial(\sigma, j_f, i_n) = (s \cup s')} \prod_f \dim(j_f) \prod_v A_v(j_f, i_n)$$





Two-complex, colored with spins and intertwiners = spinfoam

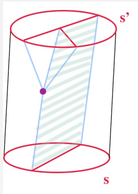


Vertex amplitude: A(j,i)

→ $$A(j_{ab}, i_a) = \sum_{i_a^+, i_a^-} 15j\left(\frac{(1+\gamma)j_{ab}}{2}, i_a^+\right) 15j\left(\frac{(|1-\gamma|)j_{ab}}{2}, i_a^-\right) \otimes_a f^{i_a}_{i_a^+, i_a^-}$$

Spinfoams

$$Z = \sum_{\sigma j_f i_v} \prod_f dim(j_f) \prod_v A_v(j_f, i_v)$$



$$A(j_{ab}, i_a) = \sum_{i_a^+, i_a^-} 15j\left(\frac{(1+\gamma)j_{ab}}{2}, i_a^+\right) 15j\left(\frac{|1-\gamma|)j_{ab}}{2}, i_a^-\right) \otimes_a f^{i_a}_{i_a^+, i_a^-}$$

- Can be directly derived from a discretization of the action of general relativity, on a *variable lattice.*

- Can be interpreted as a discrete version of Hawking's "integral over geometries"

$$Z = \int Dg\ e^{iS[g]}$$

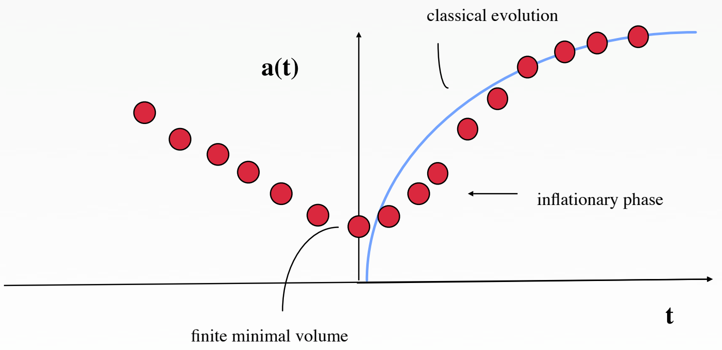
- → Z is generated as the Feynman expansion of an auxiliary field theory defined on a group manifold. This is a 4d generalization of the matrix models (à la Boulatov, Ooguri).

- → In 3d, it gives directly the old Ponzano Regge model (A=6j). (With cosmological constant: Turaev-Viro state sum model.)

III. Physics

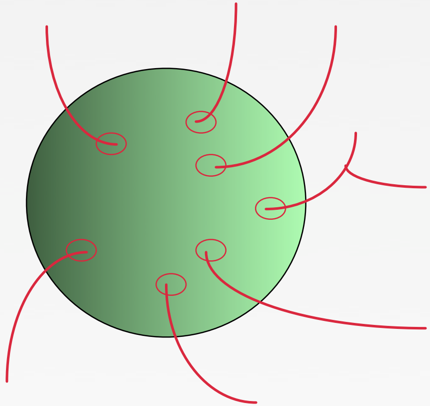
Loop cosmology

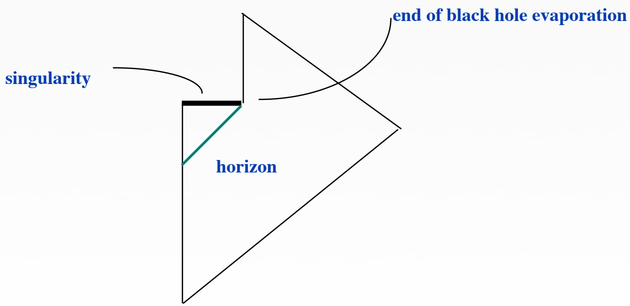
- Discrete cosmological time
- → Big Bang singularity removed (from Planck scale non-locality)
- → Evolution "across" the big bang (robust)
- → (Super-) inflationary behavior at small a(t)
- → scale invariant spectrum with the observed spectral index n_s.



Black holes

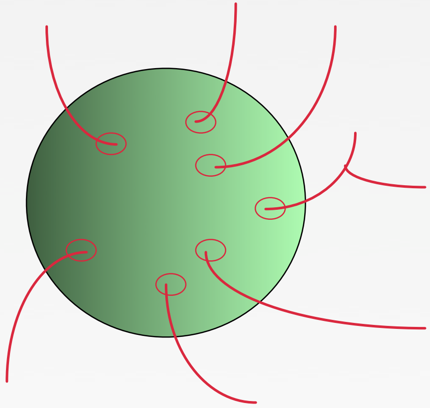
- → Entropy finite, proportional to the area
- → Physical black holes
- S = A/4   if a dimensionless free parameter (Immirzi parameter) is fixed

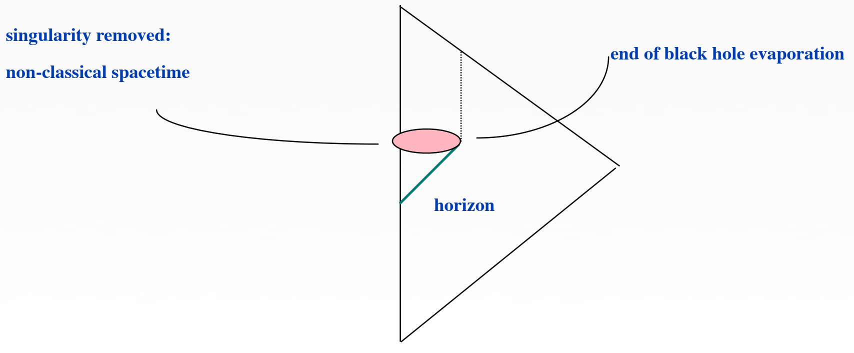




Black holes

- → Entropy finite, proportional to the area
- → Physical black holes
- S = A/4   if a dimensionless free parameter (Immirzi parameter) is fixed
- → R = 0 singularity under control:

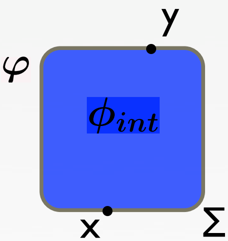




background-independent n-point functions → low-energy limit

$$W(x,y) = \int D\phi \; \phi(x)\phi(y) e^{iS[\phi]}$$

is independent from x and y if $D\varphi$ and $S[\varphi]$ are invariant under *Diff*.



Choose a closed 3-surface where x and y lie, and rewrite W as

$$W(x,y) = \int D\varphi \; \varphi(x)\varphi(y) \; W[\varphi] \; \Psi[\varphi]$$

where

$$W[\varphi] = \int_{\phi_{int}|_\Sigma = \varphi} D\phi_{int} \; e^{iS[\phi_{int}]}$$

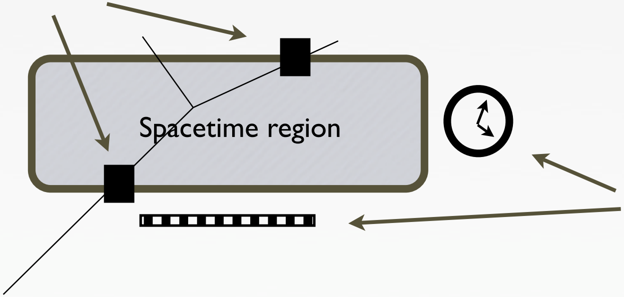
Instead of being determined by boundary conditions at infinity, let the boundary state be a state picked on a given boundary geometry q.

$$W[x,y;q] = \int D\varphi \; \varphi(x)\varphi(y) \; W[\varphi] \; \Psi_q[\varphi]$$

This expression is meanigful in a *Diff*-invariant theory, and reduces to flat space n-point function for appropriate boundary state.

Boundary values of the gravitational field = geometry of box surface
= distance and time separation of measurements

Particle detectors = field measurements



Distance and time measurements
= gravitational field measurments

In GR, distance and time measurements
are field measurements like the other ones:
they are part of the **boundary data** of the problem.

The graviton two-point function in LQC reads

$$\langle 0|g^{ab}(x)g^{cd}(y)|0\rangle = \sum_{s} W[s]\; g^{ab}(x)g^{cd}(y)\Psi_q[s]$$

And for large distances this is give at first order by

$$W[s] = \frac{\lambda}{5!}\left(\prod_{n<m} dim(j_{nm})\right) A_{vertex}(j_{nm})$$

→ The asymptotic expansion of the vertex gives the low-energy behavior of the theory.

→ Preminiary results yield: - free graviton propagator; - 3 point function; (hence Newton law); - first order corrections to the free graviton propagator. Calculations in progress.

Mathematical developments

- Diffeomorphism invariant measures (Ashtekar-Lewandowski measure)
- C* algebraic techniques
- Category theory
- "Quantum geometry"
- Uniqueness of the representation

IV. Summary

- Loop quantum gravity is a technique for defining Diff-invariant QFT. It offers a radically new description of space and time by merging in depth QFT with the *diff*-invariance introduced by GR.
- It provides a quantum theory of GR plus the standard model in 4d, which is naturally UV finite and has a discrete structure of space at Planck scale.
- Has applications in cosmology, black hole physics, astrophysics; it resolves black hole and big bang singularities.

- Unrelated to a natural unification of the forces (we are *not* at the "end of physics").
- Different versions of the dynamics exist.
- Low-energy limit still in progress.

+ Fundamental degrees of freedom explicit.
+ The theory is consistent with today's physics.
+ No *need* of higher dimensions (high-d formulation possible).
+ No *need* of supersymmetry (supersymmetric theories possible).
+ Consistent with, and based on, basic QM and GR insights.