Black Hole Entropy and AdS_2/CFT_1 Correspondence

A.S. arXiv:0805.0095

R. Gupta, A.S. arXiv:0806.0053

Related earlier work

Maldacena, Michelson, Strominger, hep-th/9812073

Ooguri, Strominger, Vafa, hep-th/0405146

Beasley, Gaiotto, Guica, Huang, Strominger, Yin, hep-th/0608021

One of the successes of string theory has been an explanation of the Bekenstein-Hawking entropy of a class of supersymmetric black holes in terms of microscopic quantum states.

$$S_{BH} = S_{micro}$$

Strominger, Vafa

$S_{BH} = A/4G_N$, $\qquad A =$ Area of event horizon

$$S_{micro} = \ln(\text{degeneracy})$$

Originally the comparison between black hole and statistical entropy was carried out in the limit of large charges.

Can we go beyond this limit?

In order to study this problem we need to address two separate issues.

1. We need to learn how to take into account the effect of the higher derivative terms / quantum corrections on the computation of black hole entropy.

2. We also need to know how to calculate the statistical entropy to greater accuracy.

In string theories with sufficient amount of supersymmetry we now have a good control over counting of microstates and computing statistical entropy for supersymmetric extremal black holes.

In this talk I shall address the other side of the problem, i.e. of computing higher derivative and quantum corrections to extremal black hole entropy.

A general frameork for computing higher derivative corrections to classical black hole entropy has been developed by Wald.

For extremal black holes this can be encoded in the entropy function formalism.

Our main goal will be to understand the effect of quantum corrections on extremal black hole entropy.

We shall begin with a lightening review of the entropy function formalism.

Postulate: An extremal black hole has an AdS_2 factor / ($SO(2,1)$ isometry in the near horizon geometry.

Regarding all other directions (including angular coordinates) as compact we can regard the near horizon geometry of an extremal black hole as

$AdS_2 \times$ a compact space (fibered over AdS_2)

Consider string theory in such a background containing two dimensional metric $g_{\mu\nu}$ and $U(1)$ gauge fields $A^{(i)}_\mu$ among other fields.

The most general field configuration consistent with $SO(2,1)$ isometry:

$$ds^2 \equiv g^{(2)}_{\mu\nu} dx^\mu dx^\nu = v\left(-(r^2-1)dt^2 + \frac{dr^2}{r^2-1}\right)$$

$$F^{(i)}_{rt} = e_i, \quad \cdots\cdots\cdots$$

$\mathcal{L}^{(2)}(v, \vec{e}, \cdots)$: The Lagrangian density evaluated in this background.

Define

$$\mathcal{E}(\vec{q}, v, \vec{e}, \cdots) \equiv 2\pi \left(e_i\, q_i - v\, \mathcal{L}^{(2)}\right)$$

One finds that for a black hole of charge $\vec{q}$

1. All the near horizon parameters are obtained by extremizing $\mathcal{E}$ with respect to v, e_i and the other near horizon parameters.

2. $S_{wald}(\vec{q}) = \mathcal{E}$ at this extremum.

We need to compare $S_{wald}(\vec{q})$ with $\ln d_{micro}(\vec{q})$.

We shall now try to generalize this formula taking into account quantum corrections.

1. We shall not make use of SUSY, although SUSY is undoubtedly useful in ensuring stability of the extremal BPS black holes.

2. Semiclassical part of our analysis will be close to the Euclidean approach to black hole thermodynamics.

However we shall work entirely in the near horizon geometry of the black hole instead of the full black hole solution.

3. We shall assume that there are no multi-centered black holes degenerate with the single centered extremal black hole so that $d_{micro}(\vec{q})$ contributes only to the entropy of single centered black holes.

Suggested relation between geometry and microstate counting:

$$Z_{AdS_2}(\vec{e}) = \sum_{\vec{q}} d_{micro}(\vec{q})\, e^{-2\pi\vec{e}\cdot\vec{q}}$$

(possibly as an asymptotic expansion about classical limit)

$Z_{AdS_2}(\vec{e})$: Euclidean partition function of string theory in AdS_2 background given by the attractor geometry corresponding to $\vec{e}$.

We shall try to justify this by two means.

1. In the classical limit* this corresponds to

$$S_{wald}(\vec{q}) = \ln d_{micro}(\vec{q})$$

2. This fits in with the usual rules of AdS/CFT correspondence.

* Classical limit may be defined as

$$\mathcal{L}^{(2)} \to \lambda \mathcal{L}^{(2)}, \qquad \vec{q} \to \lambda \vec{q}$$

with λ large.

$$ds^2 = v\left(-(r^2-1)dt^2 + \frac{dr^2}{r^2-1}\right)$$
$$F^{(i)}_{rt} = e_i$$

Euclidean continuation:

$t = -i\theta, \quad r = \cosh\eta, \quad \theta \equiv \theta + 2\pi, \quad 0 \le \eta < \infty$

This gives

$$\begin{aligned} ds^2 &= v\left(d\eta^2 + \sinh^2\eta\, d\theta^2\right), \\ F^{(i)}_{\theta\eta} &= ie_i \sinh\eta \end{aligned}$$

$$\rightarrow \quad A^{(i)}_\theta = i\,e_i\,(\cosh\eta - 1) = i\,e_i\,(r-1)\,.$$

Classical supergravity partition function:

$$Z_{AdS_2} \simeq e^{-A}, \qquad A = \text{Euclidean action}$$

Since AdS_2 has infinite volume, A would be infinite.

We regularize by putting a cut-off at:

$$\eta = \eta_0 \quad \to \quad r = \cosh \eta_0 = r_0$$

$$\to \quad \int \sqrt{\det g}\, dr\, d\theta = 2\pi\, v\, (r_0 - 1)$$

Result

$$A_{bulk} = -(r_0 - 1)\, 2\pi v\, \mathcal{L}^{(2)}$$

$$A_{boundary} = -K r_0 + \mathcal{O}(r_0^{-1})$$

K: some constant which depends on the details of the boundary terms.

This gives

$$\begin{aligned} Z_{AdS_2} &\simeq e^{-A_{bulk} - A_{boundary}} \\ &= e^{r_0 (2\pi v\, \mathcal{L}^{(2)} + K) - 2\pi v\, \mathcal{L}^{(2)} + \mathcal{O}(r_0^{-1})} \end{aligned}$$

in the classical limit.

$$Z_{AdS_2} = e^{r_0(2\pi v\,\mathcal{L}^{(2)} + K) - 2\pi v\,\mathcal{L}^{(2)} + \mathcal{O}(r_0^{-1})}$$

The term in the exponent proportional to r_0 can be removed by choosing appropriate boundary counterterms.

The r_0 independent piece is what we shall call $Z_{AdS_2}(\vec{e})$.

Thus our proposal reduces to

$$e^{-2\pi v\,\mathcal{L}^{(2)}(\vec{e})} = \sum_{\vec{q}} d_{micro}(\vec{q})\, e^{-2\pi \vec{e}\cdot\vec{q}}$$

$$e^{-2\pi v\,\mathcal{L}^{(2)}(\vec{e})} = \sum_{\vec{q}} d_{micro}(\vec{q})\, e^{-2\pi\vec{e}\cdot\vec{q}}$$

In classical limit the r.h.s. is sharply peaked, and we get

$$-2\pi v\mathcal{L}^{(2)}(\vec{e}) = \ln d_{micro}(\vec{q}) - 2\pi\vec{e}\cdot\vec{q}$$

at

$$\partial \ln d_{micro}(\vec{q})/\partial q_i = 2\pi e_i$$

Compare this with Wald entropy

$$S_{wald}(\vec{q}) = 2\pi(\vec{e}\cdot\vec{q} - v\mathcal{L}^{(2)})$$

$$\rightarrow S_{wald}(\vec{q}) = \ln d_{micro}(\vec{q})$$

Expected form of Z_{AdS_2} in the full quantum theory

$$Z_{AdS_2}(\vec{e}) = e^{Cr_0 - 2\pi v\, \mathcal{L}^{(2)}_{eff}(\vec{e})}$$

$\mathcal{L}^{(2)}_{eff}$: 'effective lagrangian density' evaluated in the AdS_2 background

– can be calculated using perturbation theory.

AdS_2/CFT_1 correspondence

By the usual AdS/CFT correspondence we would expect that string theory on AdS_2 should be equivalent to a CFT_1 at the boundary $r = r_0$ of AdS_2.

$$Z_{CFT_1} = Z_{AdS_2}$$

We shall now analyze Z_{CFT_1}.

Conventionally one uses units in which the size of the boundary is fixed but the UV length cut-off is of order $1/r_0$.

We shall use a convention in which the UV cut-off is fixed but the size of the boundary is of order r_0.

Parametrize the boundary by

$$w \equiv r_0\theta$$

w has period $2\pi\, r_0$.

At $r = r_0$

$$
\begin{aligned}
ds^2 &= v\,(d\eta^2 + dw^2) + \mathcal{O}(r_0^{-2}), \qquad w \equiv w + 2\pi r_0 \\
A_w^{(i)} &= i\,e_i \left(1 - r_0^{-1}\right) .
\end{aligned}
$$

Define

H: generator of w translation in CFT_1 in the $r_0 \to \infty$ limit

Q_i: Conserved charge dual to $A_\mu^{(i)}$ in CFT_1

Then

$$
Z_{CFT_1} = Tr\left[e^{-2\pi r_0 H - 2\pi e_i Q_i + \mathcal{O}(r_0^{-1})}\right]
$$

$$Z_{CFT_1} = Tr\left[e^{-2\pi r_0 H - 2\pi \vec{e}\cdot\vec{Q} + \mathcal{O}(r_0^{-1})}\right]$$

$$Z_{AdS_2}(\vec{e}) = e^{C r_0 - 2\pi v\, \mathcal{L}_{eff}^{(2)}(\vec{e}) + \mathcal{O}(r_0^{-1})}$$

Compare the two in the $r_0 \to \infty$ limit:

$\longrightarrow$ if the ground state energy of H is E_0 and there are $d(\vec{q})$ ground states of charge $\vec{q}$ then

$$e^{-2\pi E_0 r_0} \sum_{\vec{q}} d(\vec{q})\, e^{-2\pi \vec{e}\cdot\vec{q}} = e^{C r_0 - 2\pi v\, \mathcal{L}_{eff}^{(2)}(\vec{e})}$$

$$e^{-2\pi E_0 r_0} \sum_{\vec{q}} d(\vec{q})\, e^{-2\pi \vec{e}\cdot\vec{q}} = e^{C r_0 - 2\pi v\, \mathcal{L}^{(2)}_{eff}(\vec{e})}$$

$$\Downarrow$$

$$E_0 = -C/(2\pi), \quad \sum_{\vec{q}} d(\vec{q})\, e^{-2\pi \vec{e}\cdot\vec{q}} = e^{-2\pi v\, \mathcal{L}^{(2)}_{eff}(\vec{e})}$$

Compare with our proposal:

$$e^{-2\pi v\, \mathcal{L}^{(2)}_{eff}(\vec{e})} = \sum_{\vec{q}} d_{micro}(\vec{q})\, e^{-2\pi \vec{e}\cdot\vec{q}}$$

Thus our proposal amounts to equating the ground state degeneracies of CFT_1 living on the boundary of AdS_2 with the black hole microstate degeneracies.

Special case: Type IIA on CY_3

In this case Z_{AdS_2} may be computable due to SUSY.

Recall:

$$Z_{AdS_2} \simeq e^{-2\pi v \mathcal{L}^{(2)}_{eff}}$$

after removing cut-off dependent terms.

If we evaluate $v\mathcal{L}^{(2)}_{eff}$ using only the F-type terms in the effective action then

$$Z_{AdS_2} \simeq e^{-2\pi v \mathcal{L}^{(2)}_{eff}} = |Z_{top}|^2$$

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Quantum corrections should be strongly constrained due to SUSY.

Expect

$$Z_{AdS_2} = |Z_{top}|^2 \times \text{simple measure factor}$$

It may not be impossible to calculate this completely.

We can then compare this with $Z_{micro}(\vec{e})$ when the latter is known.

Summary

1. Proposal for relating the extremal black hole entropy to the microscopic degeneracy

$$Z_{AdS_2}(\vec{e}) = \sum_{\vec{q}} d_{micro}(\vec{q})\, e^{-2\pi\vec{e}\cdot\vec{q}}$$

– reduces to the relation between wald entropy and statistical entropy in the classical limit.

– in the spirit of AdS/CFT correspondence.

2. It may be possible to subject this to precision tests for supersymmetric black holes for which the microscopic degeneracies are known.