
# 40 years of high-energy string collisions

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# 40 years of string theory*

* Nuovo Cim. 57A (1st Sept. 1968); submitted 29.07.'68

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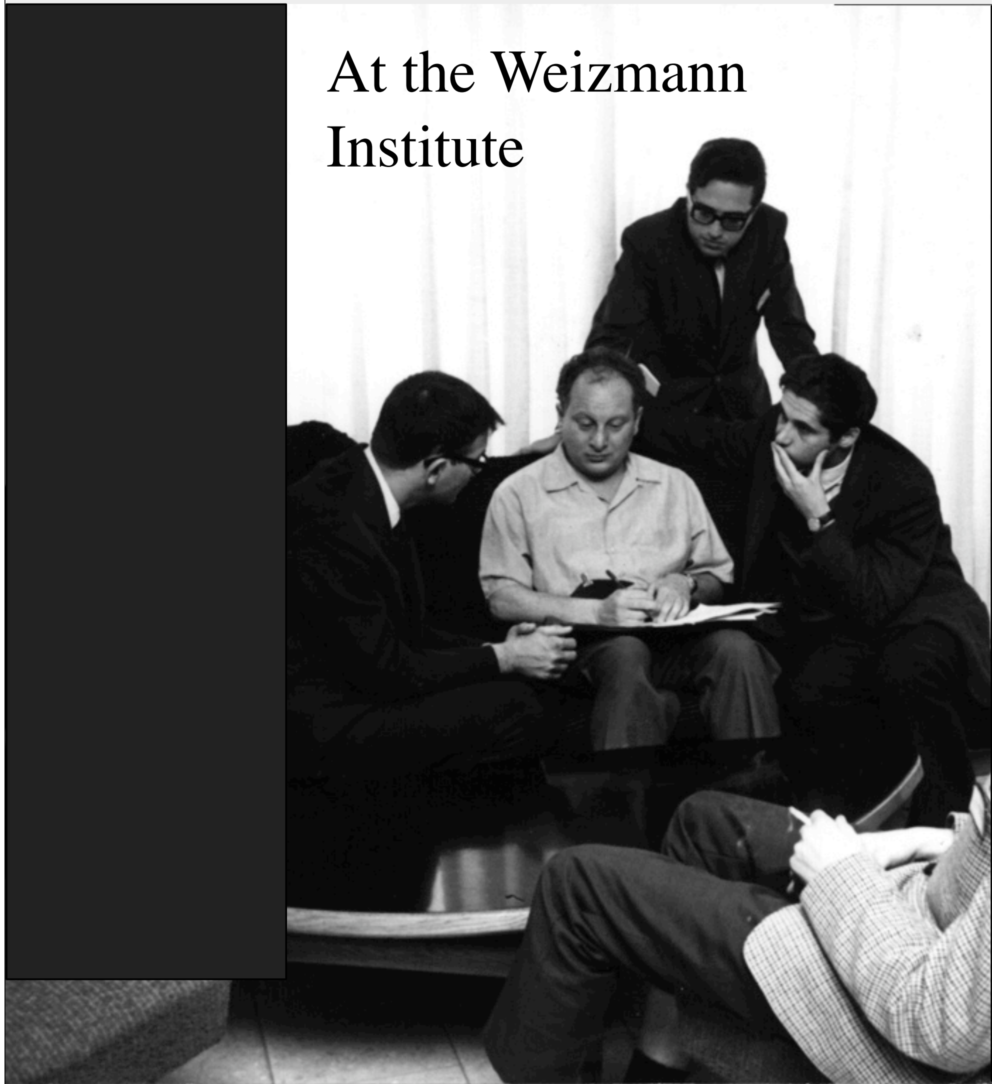
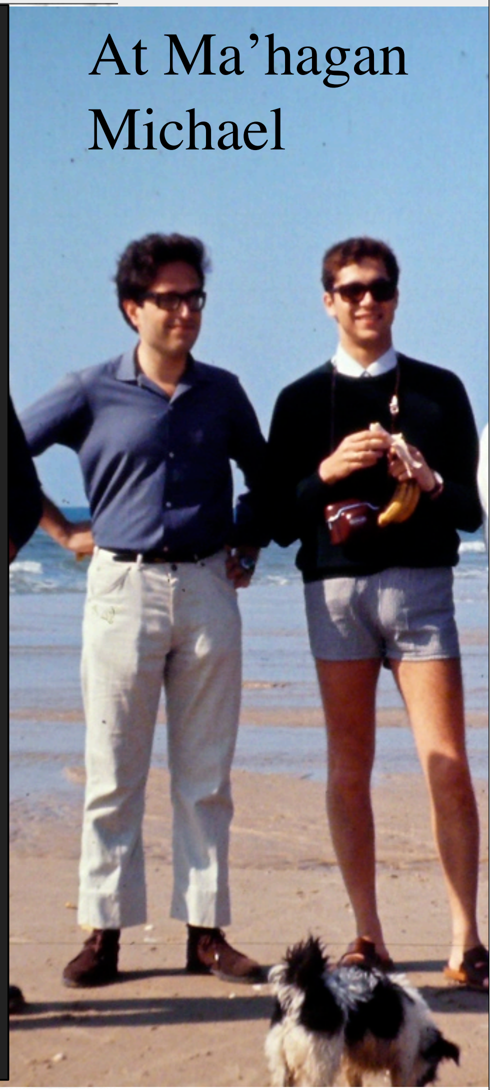
\# On leave from WIS, on the way to MIT

# Outline

- HE string collisions 40 years ago (very briefly)
- HE string collisions 20 years ago (briefly)
- HE string collisions 2008 (??)

# 40 years ago: no strings attached yet! (ARVV, 1967-'68)

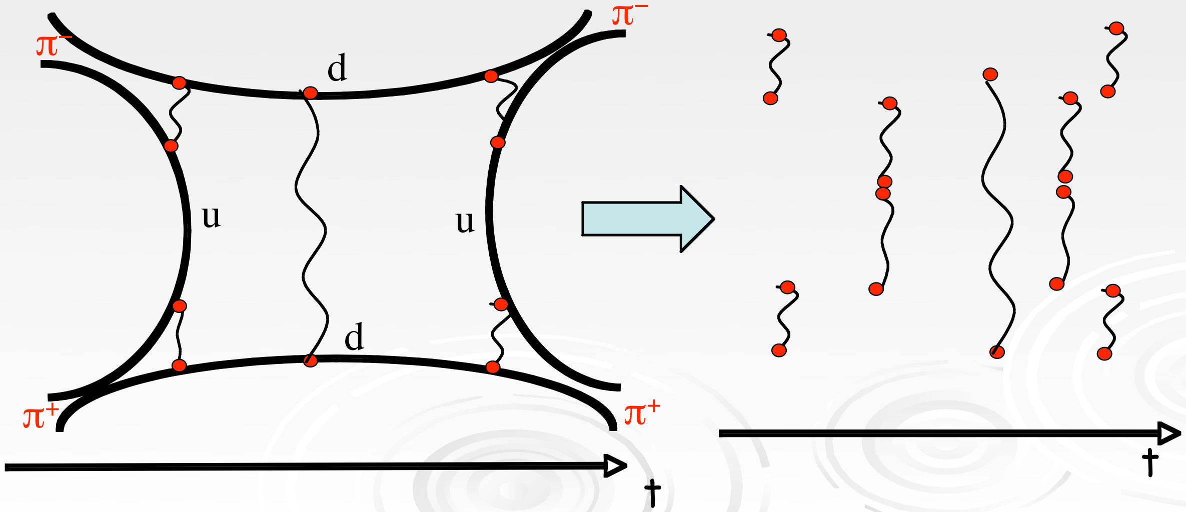


- Strong interactions @ "high" energy: resonances in s-channel vs. Regge poles in t-channel
- To sum or not to sum?
- DHS duality (1967) gave the correct (-ve) answer
- DRM and linear Regge trajectories: a first (missed) hint of an underlying string?

$$\alpha' = dJ/dM^2 = dl/dE = 1/T \quad \text{(N.B. no } \hbar\text{!)}$$

Duality diagrams A second missed hint?



# 20 years later/ago

- QCD well established
- DRM (= String Theory) reinterpreted as a TOE, in particular of quantum gravity. Huge rescaling needed:

$$\alpha' \to 10^{-38}\alpha' \quad , \quad l_s \to 10^{-19} l_s$$

- HE => transplackian-energy (TPE); semiclassical analysis should be reliable, reproduce GR, but
  - Naive tree-level turns out to be bad:
    - Partial-wave unitarity violated at TPE
    - Fixed-angle too much suppressed (wrt GR)

Qs: 1. Can loops come to the rescue?
2. Is a semiclassical approximation still meaningful?

Examples of GR expectations

based on the construction of

Closed Trapped Surfaces (CTS)

0805.3880 [gr-qc] 26 May 2008, 594 pages

# The Formation of Black Holes in General Relativity

Demetrios Christodoulou

May 18, 2008

# CTS (sufficiency) criteria => bounds

- Point-particle collisions:

1. Penrose ('74), b=0: $M_{BH} > E/\sqrt{2} \sim 0.71E$
2. D'Eath & Payne ('92), Pretorius ('08): $M_{BH} \sim 0.86\ E$
3. Eardley and Giddings, b≠0, gr-qc/0201034:

$$\left(\frac{R}{b}\right)_{cr} \leq 1.25\ ,\ \ D = 4 \qquad\qquad (R = 2G\sqrt{s} = 4GE_1 = 4GE_2)$$

- Extended sources:

1. Yurtsever ('88) gave arguments for critical size O(R)
2. Kohlprath and GV, gr-qc/0203093, using EG method: e.g.

$$\left(\frac{R}{L}\right)_{cr} \leq 1\ ,\ \ D = 4$$

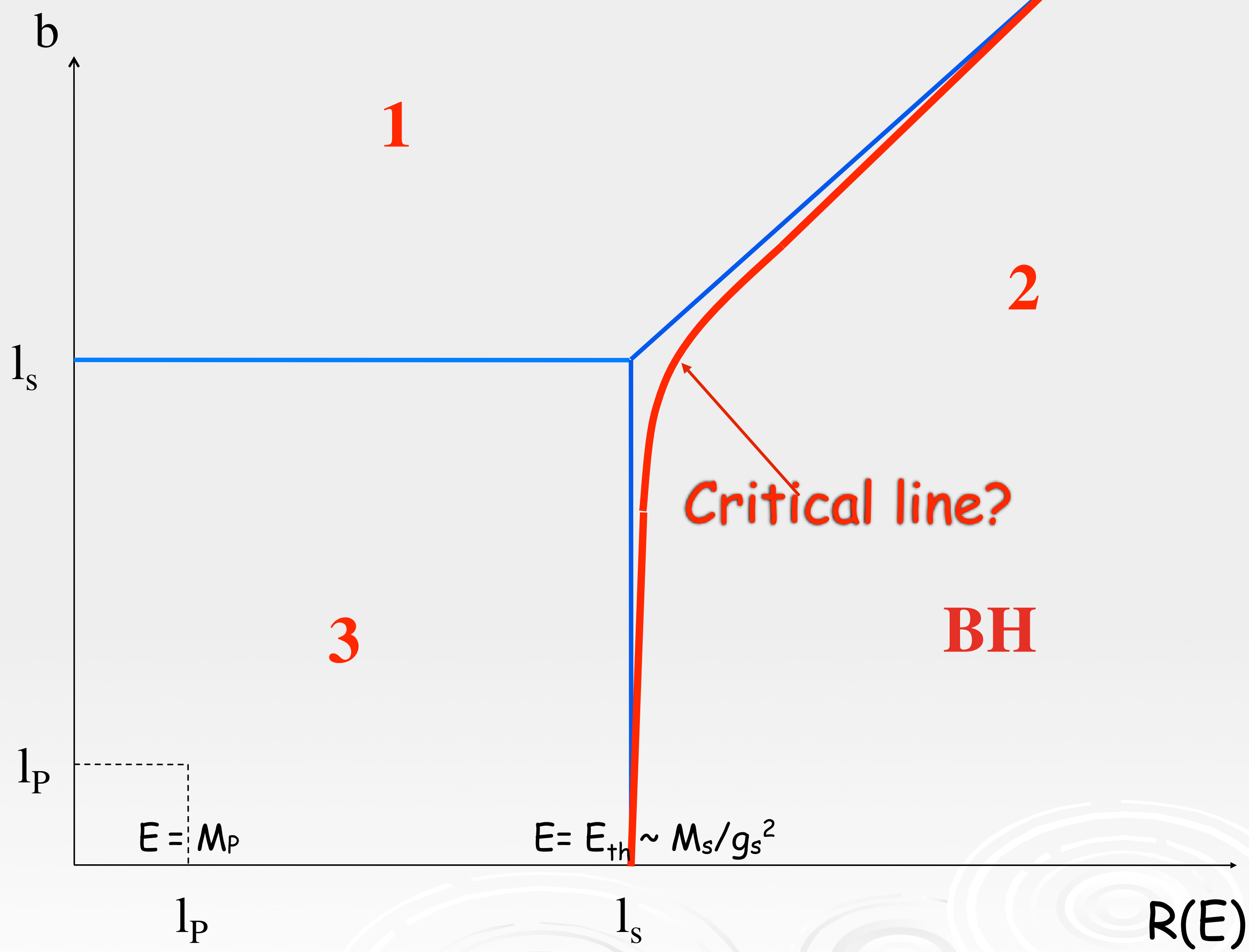
for central collision of 2 homogeneous null beams of radius L

The string length parameter $l_s$ plays the role of the beam size!

3 length scales: b, R and $l_s$ =>

## 3 broad-band regimes in trans-Planckian superstring scattering

1) Small angle scattering ($b \gg R, l_s$)
2) Large angle scattering ($b \sim R > l_s$), collapse ($b, l_s < R$)
3) Stringy ($l_s > R, b$)



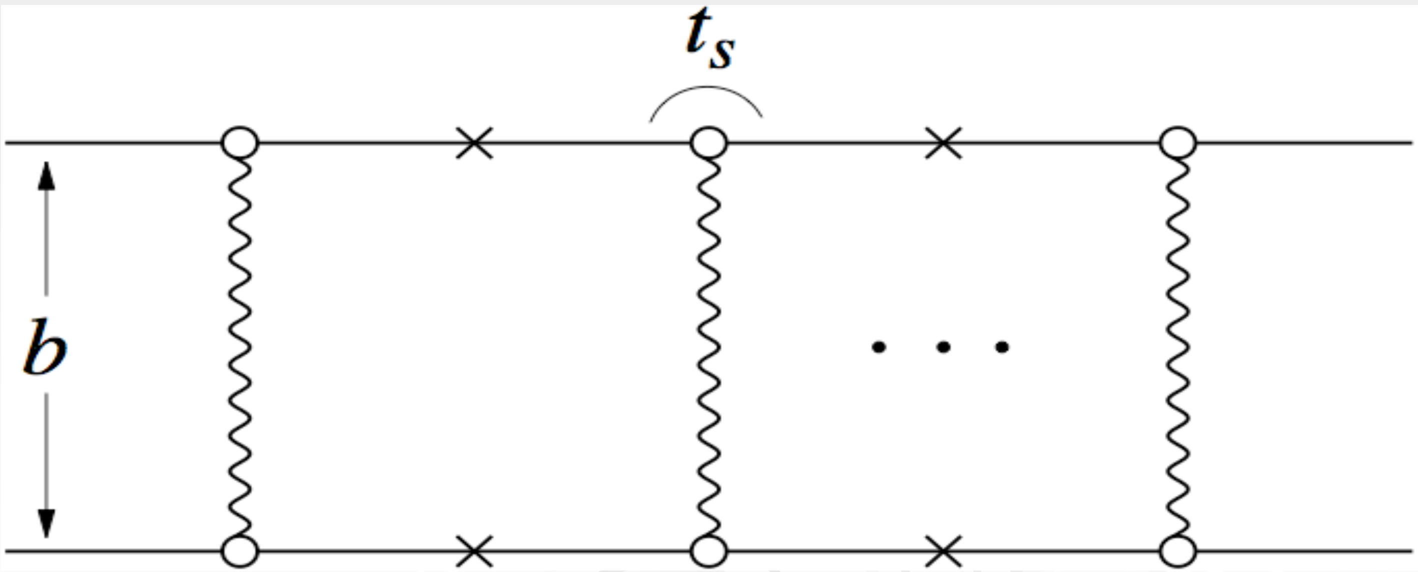

NB: we take the string coupling $g_s$ very small so that $l_s >> l_P$

# The semiclassical S-matrix

General arguments as well as string-loop calculations suggest the following form for the TPE S-matrix:

$$S(E,b) \sim exp\left(i\frac{A_{cl}}{\hbar}\right) \quad ; \quad \frac{A_{cl}}{\hbar} \sim \frac{Gs}{\hbar} c_D b^{4-D} \left(1 + O((R/b)^{2(D-3)}) + O(l_s^2/b^2) + O((l_P/b)^{D-2}) + \dots\right)$$

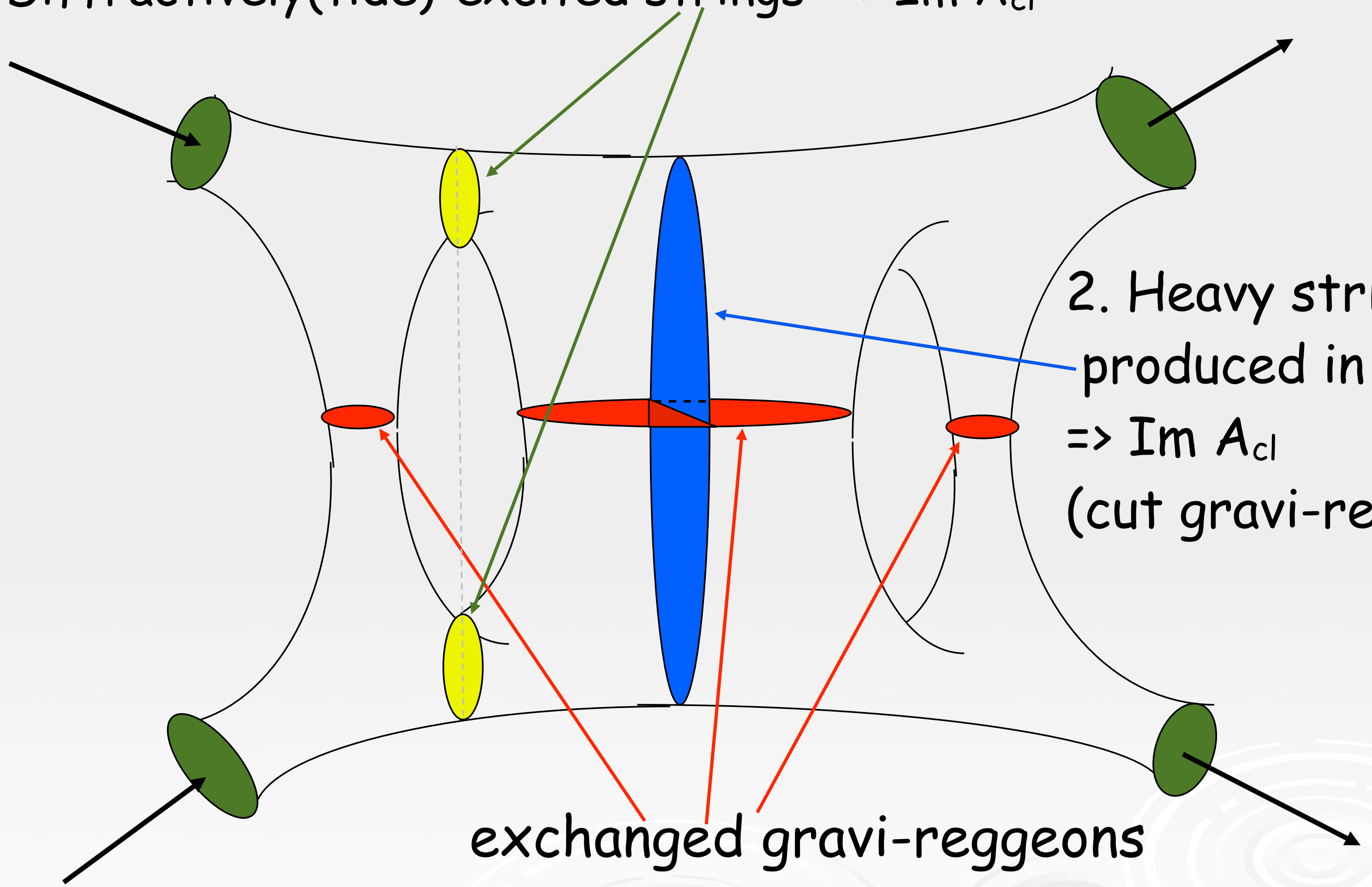
Leading eikonal diagrams (crossed ladders included)




NB: For Im $A_{cl}$ some terms may be more than just corrections...

# Two examples of string corrections (controlled by $l_s$)

1. Diffractively(tide)-excited strings => Im $A_{cl}$



2. Heavy strings produced in s-ch. => Im $A_{cl}$ (cut gravi-reggeons)

exchanged gravi-reggeons

Classical corrections (controlled by R/b) to be discussed later

The existence of these corrections complicates the previous diagram with new regions appearing in our parameter space. We may roughly distinguish 6 (increasingly difficult) regimes:

I) Small-angle elastic scattering (leading eikonal)

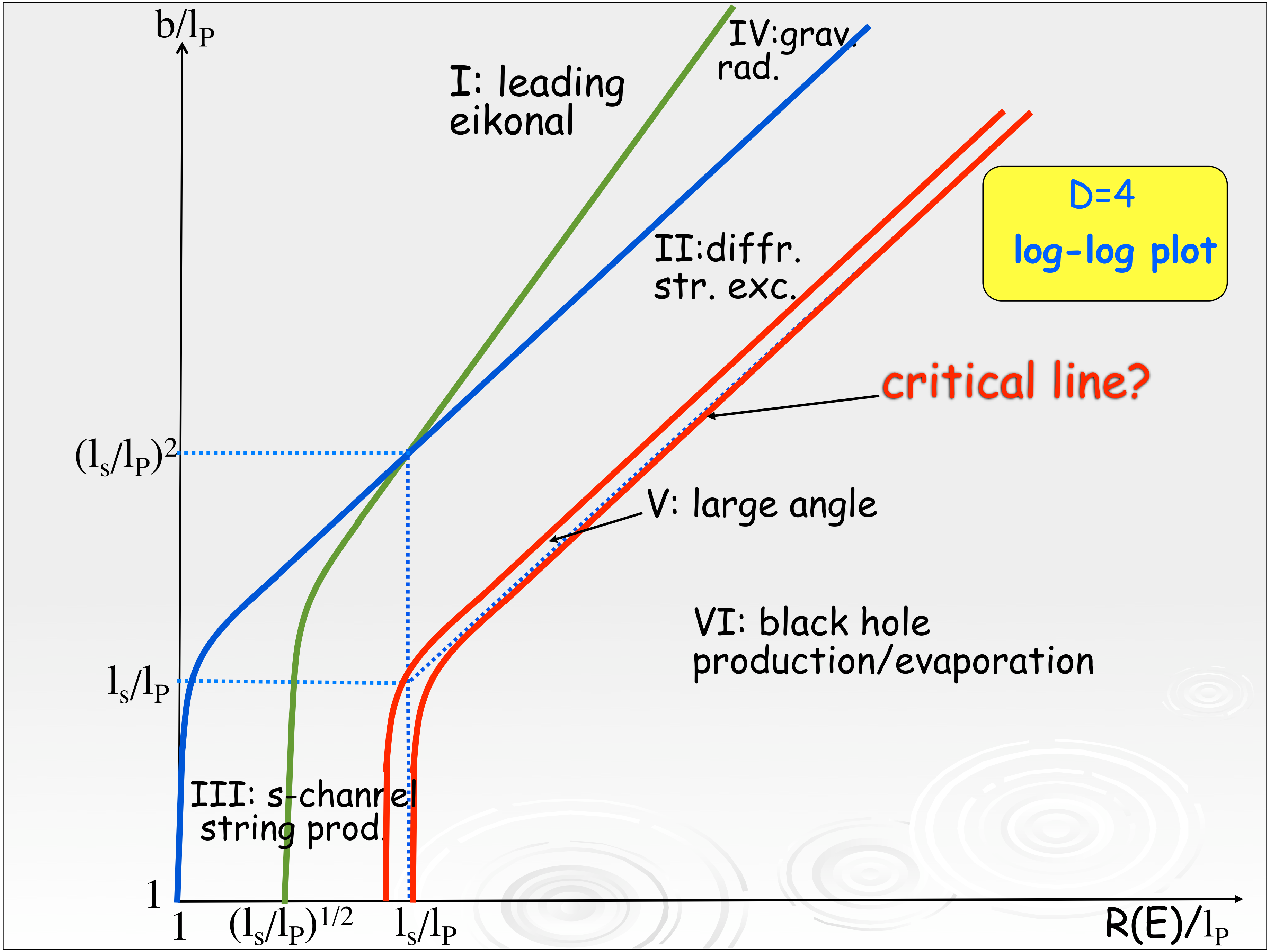
II) Small-angle inelastic scattering (a.string excitation)

III) Small-angle inelastic scattering (b.string formation)

IV) Small-angle inelastic scattering (c.graviton emission)

V) Large-angle inelastic scattering

VI) Classical Collapse

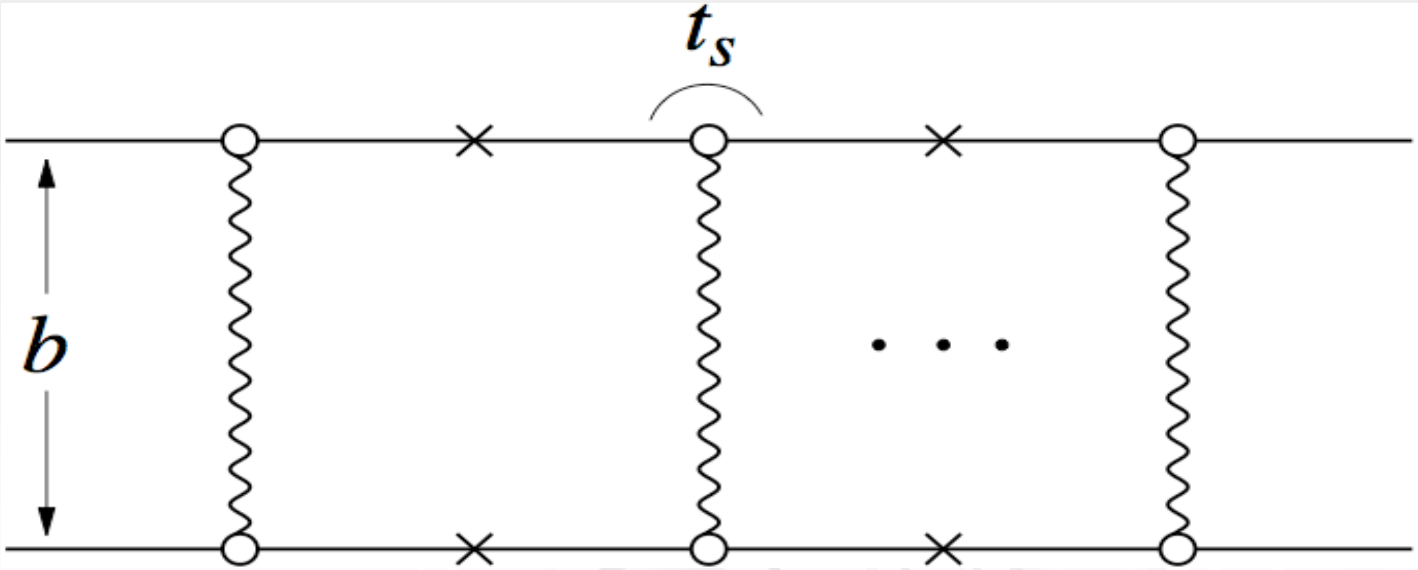


b/$l_P$
I: leading eikonal
IV:grav. rad.
II:diffr. str. exc.
D=4
log-log plot
critical line?
$(l_s/l_P)^2$
V: large angle
VI: black hole production/evaporation
$l_s/l_P$
III: s-channel string prod.
1
1
$(l_s/l_P)^{1/2}$
$l_s/l_P$
R(E)/$l_P$

# I: Small-angle elastic scattering (leading eikonal)

$$S(E,b) \sim exp\left(i\frac{A_{cl}}{\hbar}\right) \quad ; \quad \frac{A_{cl}}{\hbar} \sim \frac{Gs}{\hbar} c_D b^{4-D}\left(1 + O((R/b)^{2(D-3)}) + O(l_s^2/b^2) + O((l_P/b)^{D-2}) + \dots\right)$$

Leading eikonal diagrams (crossed ladders included)



# Recovering CGR expectations @ large distance

$$S = \mathrm{e}^{2i\delta} \qquad\qquad Re\delta \sim Gsb^{4-D}$$

$$\delta(E, b) = \int d^{D-2}q \frac{A_{tree}(s,t)}{4s} \, \mathrm{e}^{-iqb} \;, \quad s = E^2, \quad t = -q^2$$

$$\mathrm{Im}\delta \sim \frac{G_D \; s \; l_s^2}{(Y l_s)^{D-2}} \mathrm{e}^{-b^2/b_I^2} \;, \; b_I^2 \equiv l_s^2 Y^2 \;, \; Y = \sqrt{\log(\alpha' s)}$$

For b >> $l_s$Y (Region I), we can forget about Im δ

Going over to scattering angle θ, we find a saddle point at

$$b_s^{D-3} \sim \frac{G\sqrt{s}}{\theta} \quad ; \quad \theta \sim \left(\frac{R_S}{b}\right)^{D-3}$$

corresponding precisely to the relation between impact parameter and deflection angle in the (AS) metric generated by a relativistic point-particle of energy E.

# II: Small-angle inelastic scattering

## (a. diffractive/tidal string excitation)

When a string moves in an AS metric it suffers tidal forces as a result of its finite size (Giddings 0604072)
Grav. counterpart to diffractive excitation?

When does DE kick-in? Tidal-force argument (SG/GV):

$$\theta_1 \sim G_D \; E_2 \; b^{3-D} \;\; \Rightarrow \Delta\theta_1 \sim G_D \; E_2 \; l_s \; b^{2-D}$$

This angular spread provides an invariant mass:

$M_1 \sim E_1 \Delta\theta_1 \sim G_D \; s \; l_s \; b^{2-D} = M_2$ strings get excited if

$$M_{1,2} \sim M_s = \hbar l_s^{-1} \Rightarrow b = b_D \sim \left(\frac{G s l_s^2}{\hbar}\right)^{\frac{1}{D-2}}$$

... as in ACV '87

Also:

$$\sigma_{el} \sim \exp(-S(M)) \sim \exp(-M/M_s) \sim \exp(-\frac{Gs}{\hbar}\frac{l_s^2}{b^{D-2}})$$

# III: Small-angle inelastic scattering

## (b. string formation @ b, R < $l_s$)

Because of Im $\delta \neq 0$, $S_{el}$ is suppressed as exp(-2 Im $\delta$):

$$\sigma_{\rm el} \sim \exp(-4{\rm Im}\delta) = \exp\left[-\frac{G_D\ s\ l_s^2}{(Yl_s)^{D-2}}\right] \equiv \exp\left[-\frac{s}{M_*^2}\right]$$

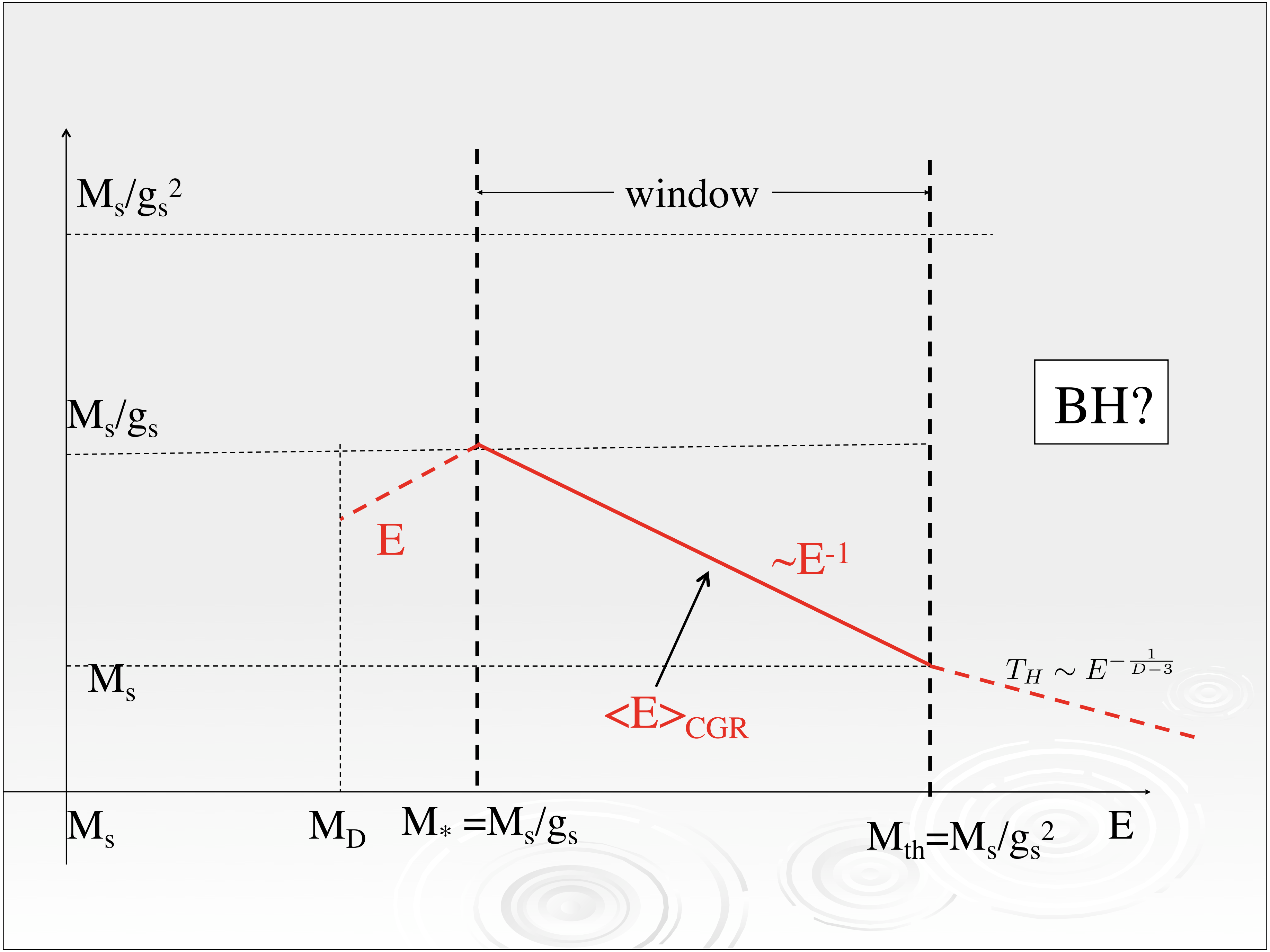
$$M_* = \sqrt{M_s M_{th}} \sim M_s g_s^{-1}$$ If we go to E= $E_{th}$ = $M_s/g_s^2$ we find:

$$\sigma_{\rm el} \sim \exp(-g_s^{-2}) \sim \exp(-S_{sh})$$

where $S_{sh}$ is the common entropy of a BH/string of mass $E_{th}$

Also: $$\langle N_{\rm CGR}\rangle = 4{\rm Im}\delta = \frac{G_D\ s\ l_s^2}{(Yl_s)^{D-2}} = O\left(\frac{s}{M_*^2}\right)$$ and thus:

$$\langle E\rangle_{\rm CGR} = \frac{\sqrt{s}}{\langle N_{\rm CGR}\rangle} \sim M_s Y^{D-2}\left(\frac{l_s}{R_S}\right)^{D-3} \sim T_{\rm eff} \equiv \frac{M_*^2}{E} = \frac{M_s^2}{g_s^2 E}$$



window
BH?
Ms/gs2
Ms/gs
Ms
E
~E-1
<E>CGR
$T_H \sim E^{-\frac{1}{D-3}}$
Ms
MD
M* =Ms/gs
Mth=Ms/gs2
E

# And today?

## An additional phenomenological motivation ?

Finding signatures of string/quantum gravity @ LHC

- ✳ In KK models with large extra dimensions;
- ✳ In brane-world scenarios; in general:
- ✳ If we can lower the true QG scale to O(TeV)

NB: In the most optimistic situation the LHC will be very marginal for producing BH, let alone semiclassical ones

➡ Find precursors of BH behaviour even below the expected production threshold

# An additional theoretical motivation: AdS/CFT

Hopes that a suitable generalization of AdS/CFT may lead to a good string model for QCD (back to the old game!). IR problems afflicting the early attempts to describe high-energy soft hadronic scattering should be absent.

Interesting recent developments include:

* Models for the soft Pomeron and connection to gravitational processes in AdS (Janik & Peschanski; Brower, Strassler, Polchinski &Tan; Cornalba, Costa, & Penedones...)
* Models for the BFKL (hard) Pomeron and connection between parton-saturation and critical collapse a la Choptuik (Alvarez-Gaume, Gomez, Vasquez-Mozo,...)

Is the old program of a string-based approach to high-energy soft hadronic scattering also to be revived?

# IV: Small-angle inelastic scattering (ACV-90's)

(graviton emission)

## => Classical corrections to leading eikonal

$$S(E,b) \sim exp\left(i\frac{A_{cl}}{\hbar}\right) \sim exp\left(-i\frac{Gs}{\hbar}(logb^2 + O(R^2/b^2) + O(l_s^2/b^2) + O(l_P^2/b^2) + \dots)\right)$$

# V: Large-angle inelastic scattering

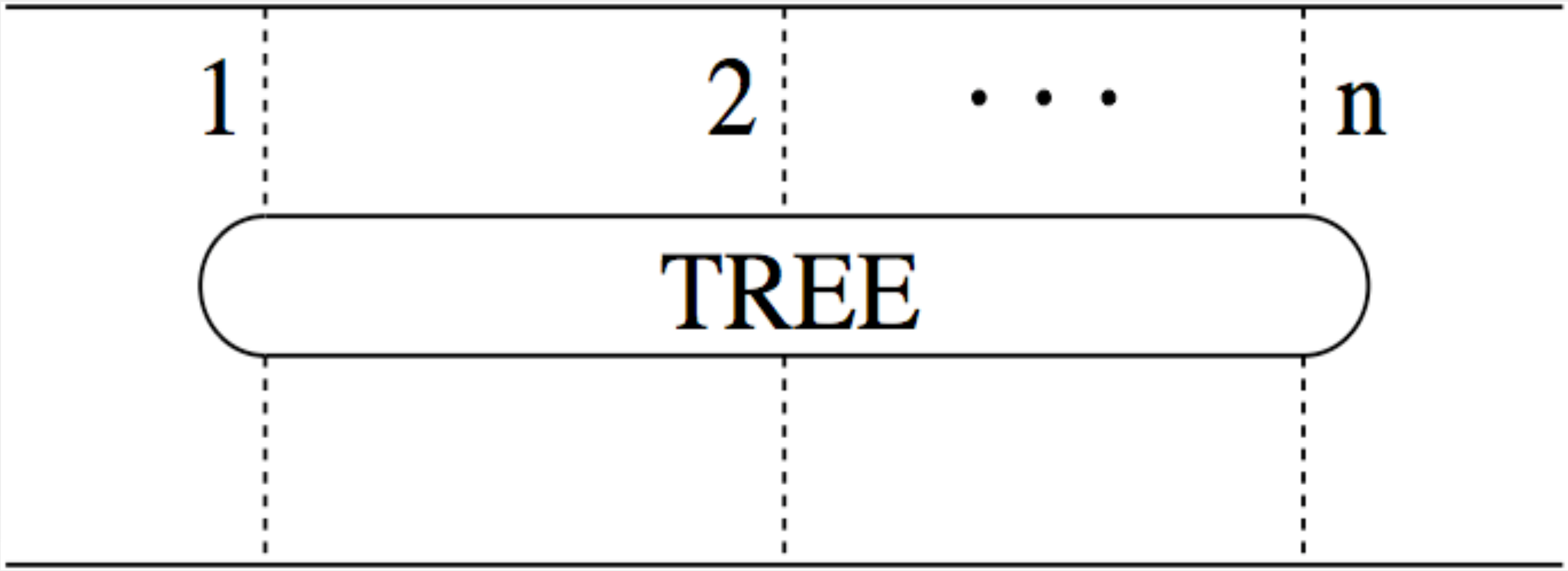
# VI: Collapse?

## => Resumming classical corrections

(ACV, hep/th-0712.1209, MO, VW, CC...'08)

D=4 hereafter

Classical corrections characterized by absence of h.
Not surprisingly, they are related to tree diagrams once the coupling to the external energetic particles is replaced by a classical source

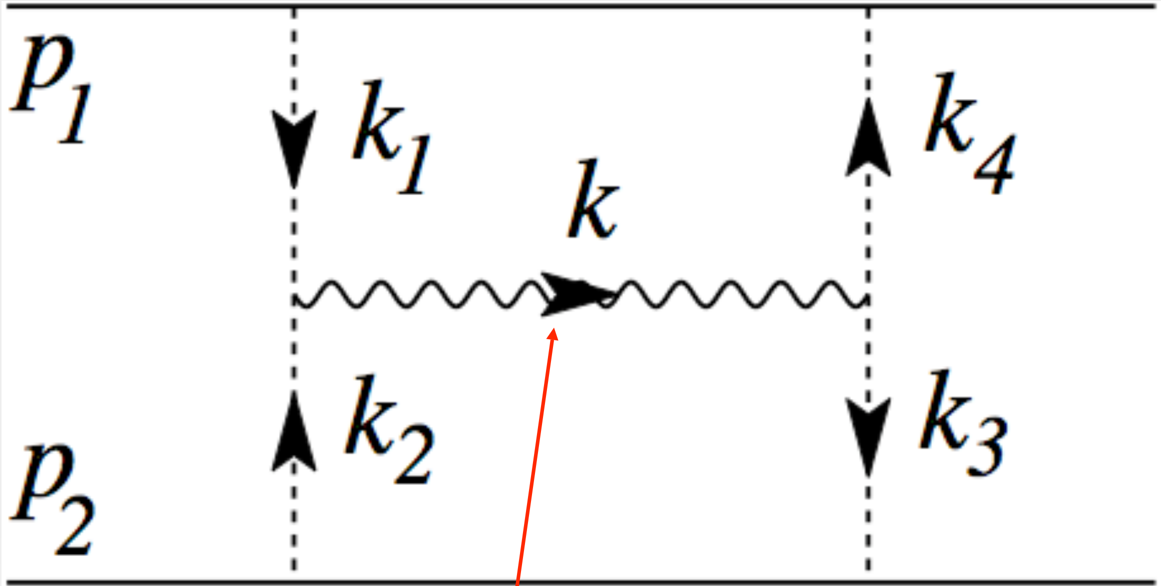



When considering the exponent (the "phase") one should restrict to connected trees

Power counting for connected trees:

$$\delta(E,b) \sim G^{2n-1} s^n \sim Gs\; R^{2(n-1)} \rightarrow Gs\; (R/b)^{2(n-1)}$$

# Next to leading order: the H diagram




$$\sim G^3 s^2 = Gs\ G^2 s = GsR^2 \rightarrow Gs\ (R/b)^2$$

One of the produced graviton's polarizations ("TT") is IR-safe
the other ("LT") is not

# Reduced effective action & field equations

There is a simple D=2 effective action generating the leading diagrams (Lipatov, ACV '93)

Neglecting the IR-unsafe (LT) polarization, it contains:
$a$ and $\bar{a}$, representing the longitudinal (++ and --) components of the gravitational field, coupled to the corresponding components of the EMT;
$\phi$, representing the TT graviton-emission field. Besides source and kinetic terms there is a trilinear derivative coupling of $a$, $\bar{a}$ and $\phi$

# The 2D action

$$\frac{\mathcal{A}}{2\pi G s} = \int d^2x \left[ a(x)\bar{s}(x) + \bar{a}(x)s(x) - \frac{1}{2}\nabla_i \bar{a} \nabla_i a \right]$$
$$+ \frac{(\pi R)^2}{2} \int d^2x \left( -(\nabla^2\phi)^2 + 2\phi\nabla^2\mathcal{H} \right) ,$$
$$-\nabla^2\mathcal{H} \equiv \nabla^2 a \, \nabla^2 \bar{a} - \nabla_i \nabla_j a \, \nabla_i \nabla_j \bar{a} \, ,$$

and the corresponding eom

$$\nabla^2 a + 2\delta(x) = 2(\pi R)^2 (\nabla^2 a \, \nabla^2 \phi - \nabla_i \nabla_j a \, \nabla_i \nabla_j \phi), \quad \bar{a}(x) = a(b-x)$$
$$\nabla^4 \phi = -(\nabla^2 a \, \nabla^2 \bar{a} - \nabla_i \nabla_j a \, \nabla_i \nabla_j \bar{a})$$

Semiclassical approximation corresponds to solving the eom and computing the classical action on the solution.

Too hard for analytic study, numerically doable (see below)

# Axisymmetric Solutions

(ACV07, J. Wosiek & G.V. 08/1 & 08/2)

## I. Particle-particle collisions @ b=0

Equations can be studied (ACV, 07121209) but are unreliable: lesson unclear

## II. Central beam-beam collisions

One example in ACV07, more systematically explored in VW (0804.3321 & 0805.2973)

# Central beam-beam collisions

A rich problem in spite of restrictive symmetry:

1. The two beams contain several parameters (total intensity, shape; same or different) & we can look for critical surfaces in their multi-dim.$^{al}$ space
2. The CTS criterion is simple (see below)
3. Numerical results should be next on line (Cf. recent talks by Choptuik & Pretorius)

Two major simplifications occur in ACV eqns:

1. PDEs become ODEs
2. The IR-singular polarization is just not produced

# Axisymmetric action and eqns (t=r$^2$)

$$\frac{\mathcal{A}}{2\pi^2 G s} = \int dt\,[a(t)\bar{s}(t) + \bar{a}(t)s(t) - 2\rho\dot{\bar{a}}\dot{a}] - \frac{2}{(2\pi R)^2}\int dt(1-\dot{\rho})^2$$

$$\rho = t\left(1 - (2\pi R)^2\dot{\phi}\right)$$

$$\pi \int^t dt' s_i(t') \;=\; R_i(t)/R$$

$$\dot{a}_i = -\frac{1}{2\pi\rho}\frac{R_i(r)}{R}$$

$$\boxed{\ddot{\rho} = \frac{1}{2}(2\pi R)^2\dot{a}_1\dot{a}_2 = \frac{1}{2}\frac{R_1(r)R_2(r)}{\rho^2}}$$

$$\boxed{\rho(0) = 0 \;\;;\;\; \dot{\rho}(\infty) = 1}$$

2$^{nd}$ order ODE w/ Sturm-Liouville-like b. conditions

# CTS criterion (KV gr-qc/0203093)

If there exists an $r_c$ such that

$$R_1(r_c)R_2(r_c) = r_c^2$$

we can construct a CTS and therefore expect a BH to form.

Theorem (VW08): whenever the KV criterion holds*) the ACV field equations do not admit regular (at r=0) real solutions. Thus:

KV criterion ==> ACV criterion

but of course not the other way around!

*) actually the r.h.s. can be replaced by $\frac{2}{3\sqrt{3}} r_c^2$

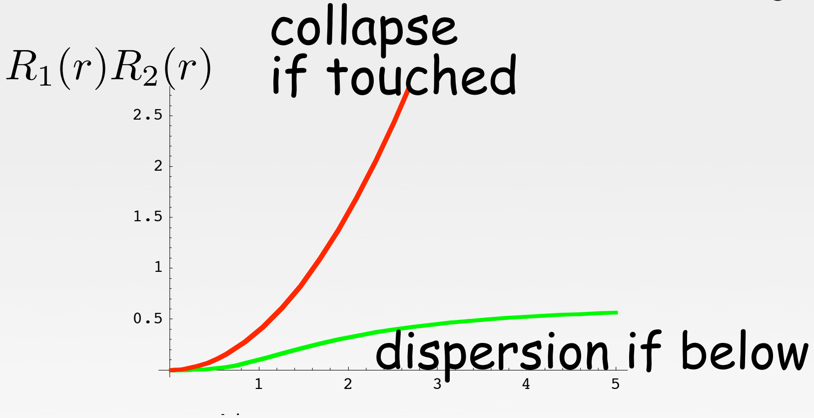
# A sufficient criterion for dispersion

## (P.-L. Lions, private comm.)

If $$R_1(r)R_2(r) \leq \frac{8}{27}\frac{r^4}{(1+r^2)^2}\left[1+\frac{1}{2}\left(1-\frac{log(1+r^2)}{r^2}\right)\right]^2$$

the ACV eqns do admit regular, real solutions.

To summarize

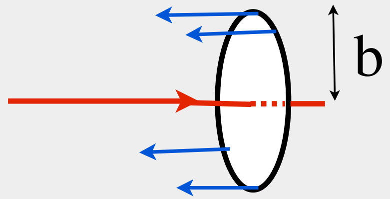



clearly, there is room for improvement...

# 3 examples

(if time/chair allows...)

# Example 1: particle-scattering off a ring




Can be dealt with analytically:

$$\ddot{\rho} = \frac{R^2}{2\rho^2}\Theta(r^2 - b^2)$$

$$\begin{aligned} \rho &= \rho(0) + r^2\dot{\rho}(0) \quad , \quad (r < b) \\ \dot{\rho} &= \sqrt{1 - R^2/\rho} \quad , \quad (r > b) \end{aligned}$$

Since ρ(0) =0:

$$\rho(b^2) = b^2\dot{\rho}(b^2) = b^2\sqrt{1 - R^2/\rho(b^2)}$$

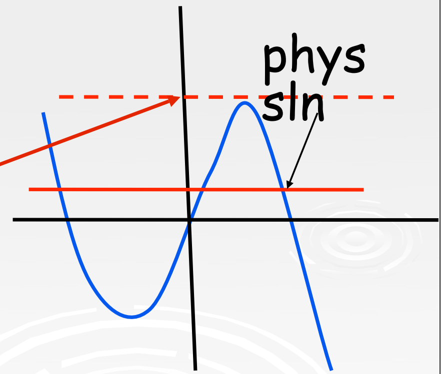
This (cubic) equation has positive real solutions iff $b^2 > \frac{3\sqrt{3}}{2}R^2 \equiv b_c^2$

$b_c \sim 1.61R$,

CTS: $b_c > R$

Amusing analogy with turning point in Schwarzschild metric

$$\frac{R}{b} = x - x^3, x \equiv \frac{r*}{b} \Rightarrow b > b_c = \frac{3\sqrt{3}}{2} R$$



# Example 2: Two hom. beams of radius L.

The equation for $\rho$ becomes

$$\ddot{\rho}(r^2) = \frac{R^2}{2\rho^2}\Theta(r-L) + \frac{R^2 r^4}{2L^4\rho^2}\Theta(L-r)$$

We can compute the critical value numerically:

$$\left(\frac{R}{L}\right)_{cr} \sim 0.47$$

It is compatible with (and close to) the CTS upper bound of KV:

$$\left(\frac{R}{L}\right)_{cr} < 1.0$$

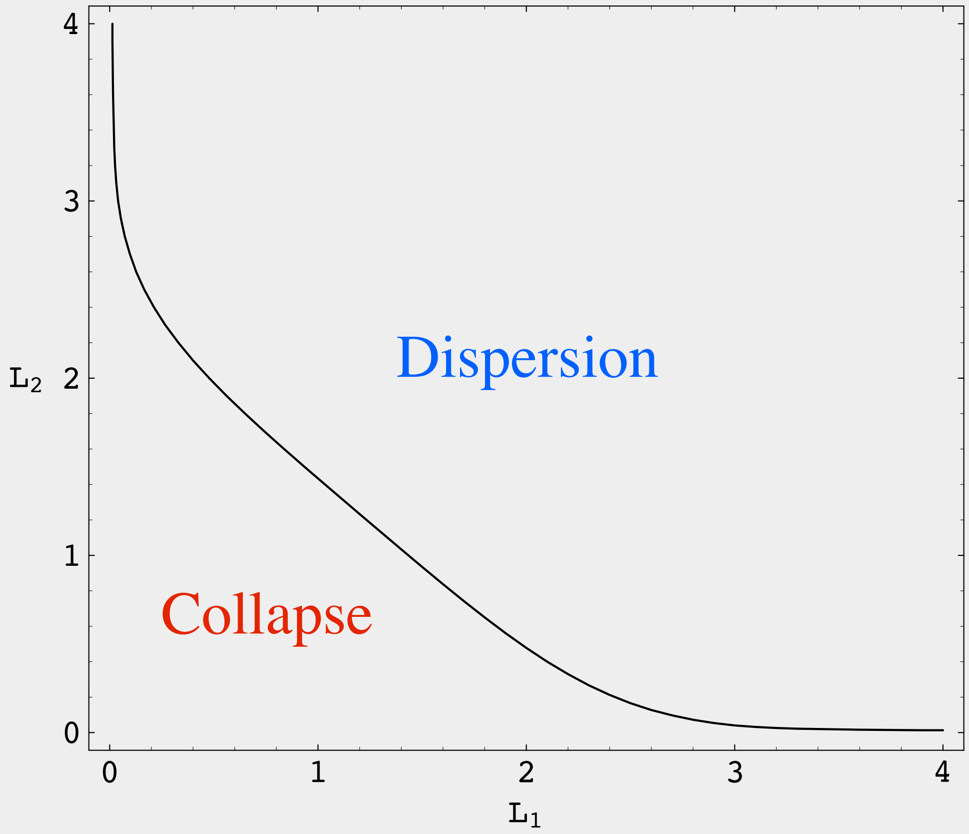
# Example 3: Two different Gaussian Beams

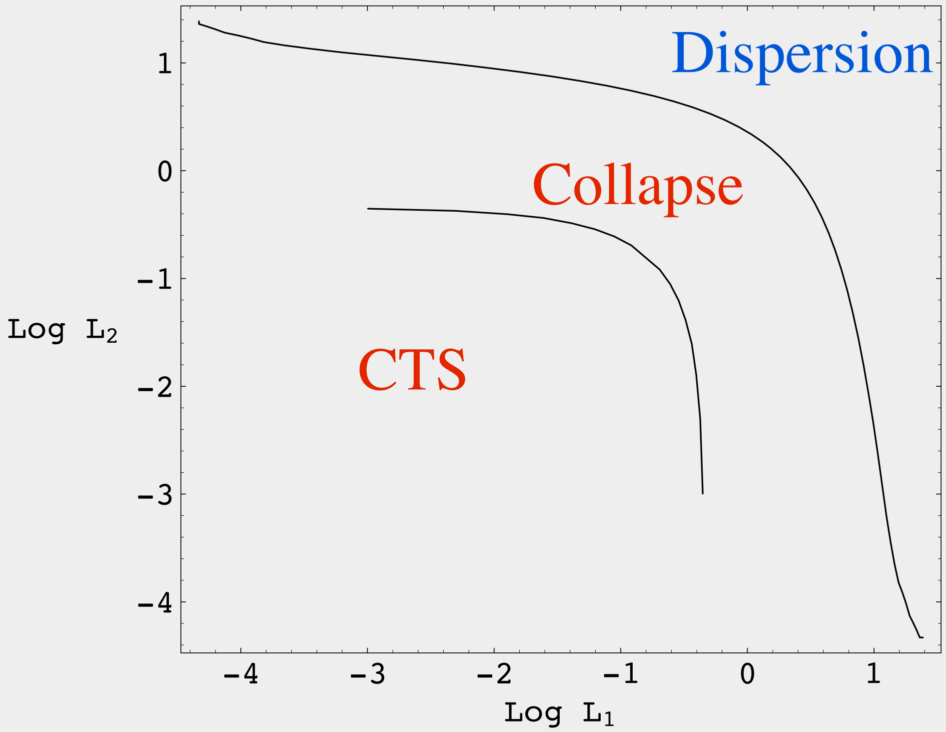
(V&Wosiek '08)

Two Gaussian profiles with widths $L_1$ and $L_2$

$$s_i(t) = \frac{1}{2\pi L_i^2} exp\left(-\frac{t}{2L_i^2}\right) \quad , \quad \frac{R_i(t)}{R} = 1 - exp\left(-\frac{t}{2L_i^2}\right)$$

We determined the critical line in the ($L_1$ , $L_2$) plane and compared it with the one coming from the CTS criterion.


4
3
$L_2$
2
1
0
0
1
2
3
4
$L_1$
Dispersion
Collapse




Many other examples: find agreement w/ CTS suff. criteria and numerical values within a factor 1.5 to 2.5

# Particle-particle collisions at finite b

## Numerical solutions

(G. Marchesini & E. Onofri, 0803.0250)

Solve directly PDEs by FFT methods in Matlab
Result: real solutions only exist only for

$$b > b_c \sim 2.28R$$

Compare with EG's CTS lower bound on $b_c$

$$b_c > 0.80R$$

# Particle Spectra

## (ACV07, VW08/2, & Ciafaloni GV in progress)

We can study the spectrum of the produced particles by looking at various contributions to the imaginary part of the elastic amplitude at fixed E & b (E-cons. important)

The final spectrum is roughly as follows (for extended sources b--> beam size):

$$\frac{1}{\sigma}\frac{d\sigma}{d^2k\,dy} = \frac{Gs}{\hbar}R^2 \exp\left(-\frac{|k||b|}{\hbar}(1+\cosh y\; R^3/b^3)\right)$$

This shows that, while for b >> R gravitons are produced at small angles, as b -> $b_c$ ~ R their distribution becomes more and more spherical w/ <n> ~ Gs and characteristic energy O(1/R ~$T_H$)

# Near & beyond $b_c$

Leaving aside imaginary part due to graviton production, for b-->$b_c^+$ the on-shell action behaves as

$$\frac{A - A_c}{Gs} = \sqrt{3}\left(1 - \frac{b^2}{b_c^2}\right) + \frac{2\sqrt{2}}{3}\left(\frac{b^2}{b_c^2} - 1\right)^{3/2}$$

The elastic amplitude picks up an extra damping below $b_c$ meaning that new channels have opened up.

Q: Do these correspond to the formation of BHs?

Ciafaloni and Colferai (08.07.2117) have formulated this as a QM tunneling problem (w/ $r^2$ playing role of time)

Just below $b_c$ the new imaginary part of the action behaves like

$$ImA \sim Gs(1 - J/Gs)^{3/2} \;, \; \sigma_{el} \sim exp(-ImA)$$

Q: Can we make the identification:

$\sigma_{el} \sim exp(-S_{BH})$ ?

A: If we can the mass of the BH should go to zero for $b \to b_c$ (Type-II critical collapse) as:

$$M_{BH} \sim \sqrt{s}(1 - b/b_c)^{3/4}$$

fixing the value of **Choptuik's exponent** to about twice his 0.37 (known to depend on $w = p/\rho$)

# Conclusions

- Gedanken **HE** collisions (e.g. ππ->πω) have played an important role in the early developments of ST.
- After the 1984 revolution **TPE collisions** may well play a similar role for understanding whether & how **QM & GR** are mutually compatible in a string theory framework
- **Superstring theory** in flat space-time (and in other consistent backgrounds) offers a **concrete framework** where the quantum scattering problem is well-posed.

- The problem simplifies by considering Gs/h >> 1 so that a suitable semiclassical approximation can be justified. Within that inematical constraint we have considered various regimes, roughly classified as follows:

- A large impact parameter regime, where an eikonal approximation w/ small corrections holds and GR expectations are recovered (AS effective metric..)
- A stringy regime, where one finds an approximate S-matrix with some characteristics of BH-physics as the expected BH threshold is approached from below
- A strong-gravity (large R) regime where an effective action approach can be (partly) justified and tested

•Critical points (lines) have emerged matching well CTS-based GR criteria

- As the critical line is approached, the final state starts resembling a Hawking-like spectrum: a fast growth (~ $E^2$) of multiplicity w/ a related softening of the final state.

•Progress was made towards constructing a unitary S-matrix and understanding the physics of the process as the critical surface is reached and possibly crossed

- Much more work remains to be done, but an understanding of the quantum analog/replacement of GR's gravitational collapse does no-longer look completely out of reach...

Thank you!