The effective action on the confining string

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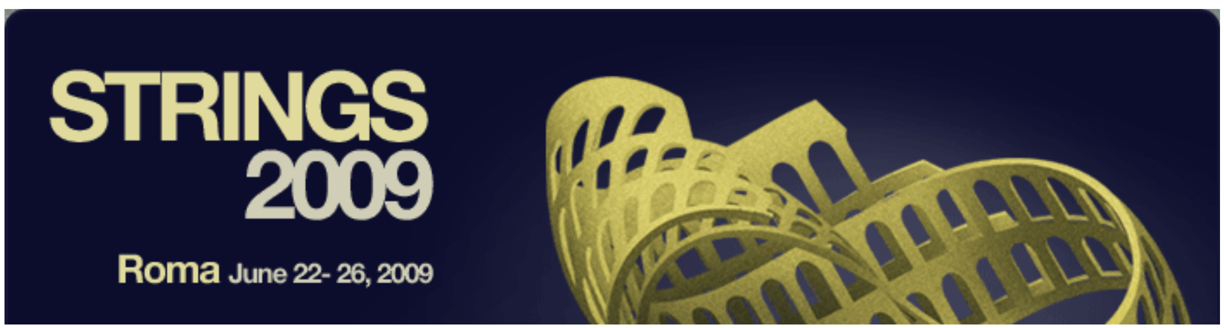


Based on: O.A. and Eyal Karzbrun, arXiv:0903.1927
O.A. and Zohar Komargodski, work in progress

Outline

1) Motivations
2) The effective action on long confining strings
3) Constraints from Lorentz invariance
4) The effective action on holographic long confining strings
5) Other constraints and future directions

How to connect string theory with experiment ?

- String theory is good for :
  - A consistent theory of quantum gravity
  - A framework for unified theories of physics beyond the standard model (from string compactifications)
  - Duals of (large N) field theories
- The best hope for making quantitative contact between string theory and experiment seems to be in the third application, where we can try to make predictions for QCD (or for other strongly coupled sectors which may be discovered at LHC).

Can we make predictions for QCD?

- We believe that SU(N) QCD has a dual description with a string coupling $g_s \sim 1/N$. This means that a classical string background controls the large N limit of QCD, and hopefully we can find this background (and solve large N QCD).
- Even if we can do this, connecting to experiment will be hard since we need to control corrections of order $1/N^2$ (closed strings) and N_f/N (open strings; recall that flavors correspond to D-branes).

- But, as a start, we could try to make predictions for large N QCD (=pure YM) which can be tested by lattice simulations. (And then compute corrections…)
- Need to find classical background of string theory which is dual to large N pure SU(N) gauge theory. What do we know about this string theory ?
- Not much. Like any other local 3+1d field theory, it should be a warped background with 3+1 infinite dimensions, one radial direction (the "scale") and maybe additional dimensions (the curvature should be of order the string scale so the number of dimensions is ill-defined).

What else do we know about the QCD string ?

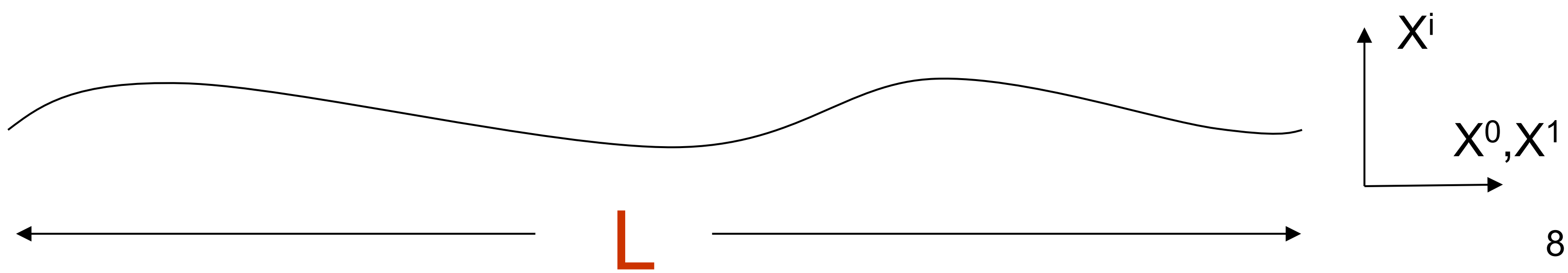
- It should be confining, and be able to screen magnetic charges ('t Hooft loops); this means that long F-strings should be dynamically localized at some radial position, and some D-branes should be able to end in the bulk.
- In known examples there are 2 ways to realize this : an internal cycle could shrink in the IR and smoothly end the space (with 't Hooft loops wrapped on it) (MN, KS, Witten), or we could have a strongly coupled region where the D-branes end ($g_s \sim \#/N \mapsto$ possible problems) (PS).

How to construct the QCD string ?

- Need to understand highly curved RR backgrounds…
- Various approaches (e.g. start from weak coupling, bottom-up action in space-time) – will say more at end if time remains – but no systematic method, and not much progress to date.
- We suggest trying a bottom-up approach on the worldsheet, analyzing light excitations on the QCD string worldsheet (for a <u>long string</u>). We can try to predict the light excitations from string theory, or use lattice results to constrain the QCD string action. Systematic but not full theory…

What is the effective action on a long QCD string ?

- Like any other solitonic object, have massless NGBs on worldvolume from broken translation symmetries, X^i (i=2,…,D-1) (for string stretching along X^0,X^1). (We'll discuss closed string on circle, but open string between quark-anti-quark is similar.) In the absence of any other symmetries, all other worldsheet excitations should be heavy, and it is natural to write the low-energy effective action on the worldsheet in these variables.



What is the effective action on a long QCD string ?

- From the point of view of a fundamental string theory, this is the effective action in a static gauge for the worldsheet diffeomorphisms. All other worldsheet fields are indeed generically heavy in this gauge.
- The effective action $L(X^i)$ is valid for stable strings (=no dynamical quarks or large N) :

a) Below the scale of massive worldsheet fields.

b) If the string theory is weakly coupled, and/or if we are below the mass of any other states (true for large L, when have mass gap in bulk).

The low-energy string action

- This action must obey all the symmetries; translation implies that it is only a function of $d_a X^i$, but it is further constrained by Lorentz invariance.
- A simple action which obeys all the symmetries, and a natural guess for the effective action, is the Nambu-Goto action :

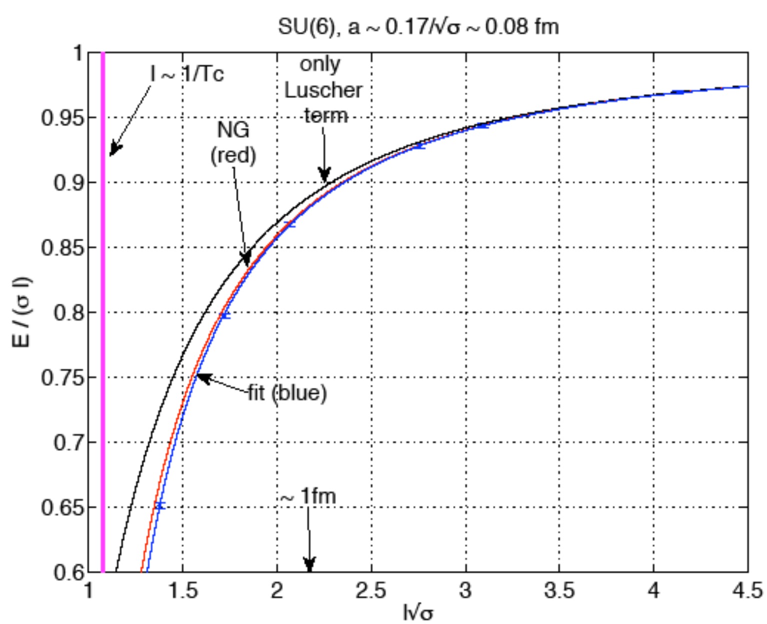
$$S_{NG} = -T\int d^2\sigma \sqrt{-\det(\partial_a X^\mu \partial_b X_\mu)} = -T\int d^2\sigma \sqrt{-\det(\eta_{ab} + \partial_a X^i \partial_b X^i)}$$

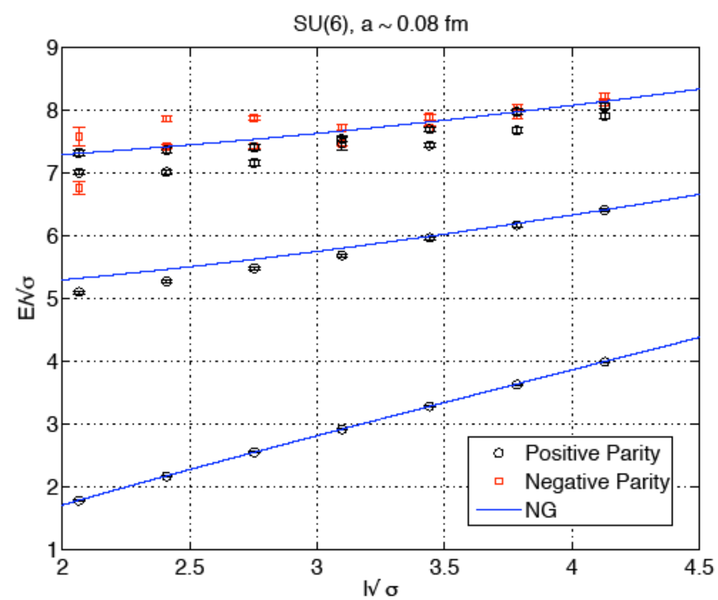
$$E_n(L) = \sqrt{(TL)^2 + 8\pi T\left(n - \frac{D-2}{24}\right)} = TL + \frac{4\pi}{L}\left(n - \frac{D-2}{24}\right) + O\left(\frac{1}{L^3}\right)$$

- This should not be exact, since its quantization is not consistent (for $D<26$). However, lattice results show it is a very good approximation. Why ?

Some lattice results

- The best lattice results for pure Yang-Mills theory in 2+1 dimensions are (for gauge group SU(6)) : (Athenodorou, Bringoltz, Teper)





(Thanks to Bringoltz for figures)

The general effective action

- Using only the obvious symmetries SO(1,1)xSO (D-2) and translations, the general effective Lagrangian density takes the form :

$$L = -T - \frac{1}{2}\partial^a X \bullet \partial_a X + c_2(\partial^a X \bullet \partial_a X)^2 + c_3(\partial^a X \bullet \partial^b X)(\partial_a X \bullet \partial_b X)$$
$$+ c_5(\partial^a X \bullet \partial_a X)^3 + c_6(\partial^a X \bullet \partial^b X)(\partial_a X \bullet \partial_b X)(\partial_c X \bullet \partial^c X)$$
$$+ c_4(\partial_a \partial_b X \bullet \partial^a \partial^b X)(\partial_c X \bullet \partial^c X) + O(\partial^8 X^4, \partial^8 X^6, \partial^8 X^8)$$

(up to EOM), and deviations from Nambu-Goto occur already at 4-derivative order (not seen).

- Some terms vanish in the special case of D=3.
- Terms can be classified by $d^n X^m$, or by $d^{m+t} X^m$ where t is the “twist”. All NG terms have twist 0.

Constraints from Lorentz invariance

- Adding Lorentz invariance imposes many constraints on this action. In particular, it turns out that all twist 0 operators must be equal to their values in the Nambu-Goto action, so that the leading deviation occurs at order d^6X^4.
- There are 3 different ways to see this :

1) Take ground state $X^i=0$ and Lorentz-transform it to get a rotated straight string $X^i=c^i_a s^a$. Action is known from Lorentz, and only $(dX)^n$ terms are non-zero; comparing to known answer determines all of them.

Constraints from Lorentz invariance

2) Write down the form of the non-linearly realized Lorentz transformation, and check invariance of the action. This relates the coefficients of all twist 0 terms to the tension, and seems to determine all twist 2t operators in terms of the lowest order twist 2t terms : (written in light-cone coordinates)

$$(\partial_+^{t+1} X \bullet \partial_-^{t+1} X)(\partial_+ X \bullet \partial_- X)$$

and for even t also

$$(\partial_+^{t+1} X \bullet \partial_+ X)(\partial_-^{t+1} X \bullet \partial_- X).$$

Constraints from Lorentz invariance

3) (Luscher+Weisz,Meyer) Compute the partition function of the effective action on the annulus (torus), and compare it with the sum over closed string states with energies $E_n(L)$ propagating on an interval (circle), which gives in a Lorentz-invariant theory :

$$Z^{annulus}(L,R) = \sum_n |v_n(L)|^2 \, 2R \left(\frac{E_n(L)}{2\pi R} \right)^{(D-1)/2} K_{(D-3)/2}(E_n(L)R),$$

$$Z^{torus}(L,R) = \sum_n R \left(\frac{E_n(L)}{2\pi R} \right)^{(D-1)/2} K_{(D-1)/2}(E_n(L)R).$$

The (perturbative) comparison gives constraints which determine some coefficients (c_2,c_3,c_5,c_6), and gives the corrections to some energy levels.

Constraints from Lorentz invariance

- To summarize, Lorentz invariance (for any string-like object) constrains its effective action to take the form :

$$
\begin{aligned}
L = &-T\sqrt{-\det(\eta_{ab} + \partial_a X^i \partial_b X^i)} + \\
&+ d_2[-32(\partial_+^2 X \bullet \partial_-^2 X)(\partial_+ X \bullet \partial_- X) + O(\partial^8 X^6)] + \\
&+ d_4[(\partial_+^3 X \bullet \partial_-^3 X)(\partial_+ X \bullet \partial_- X) + O(\partial^{10} X^6)] + \\
&+ \tilde{d}_4[(\partial_+^3 X \bullet \partial_+ X)(\partial_-^3 X \bullet \partial_- X) + O(\partial^{10} X^6)] + O(\partial^{10}).
\end{aligned}
$$

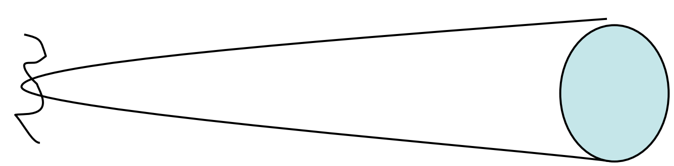
with some constants. Surprisingly, d_2 does not affect the partition function on the torus, and thus the ground state energy (though it does affect the annulus partition function).

Constraints from Lorentz invariance

- Thus, the deviation of the ground state energy from the Nambu-Goto value starts from order $1/L^7$, while for other states the deviation can start from order $1/L^5$ when $D>3$, but it also starts from order $1/L^7$ for $D=3$. Such corrections are rather hard to measure, but hopefully it can be done on the lattice (at least for the ground state).
- Are the coefficients d_n further constrained ? Would like to compute them in some example. Luckily, we can compute them for weakly curved holographic confining theories !

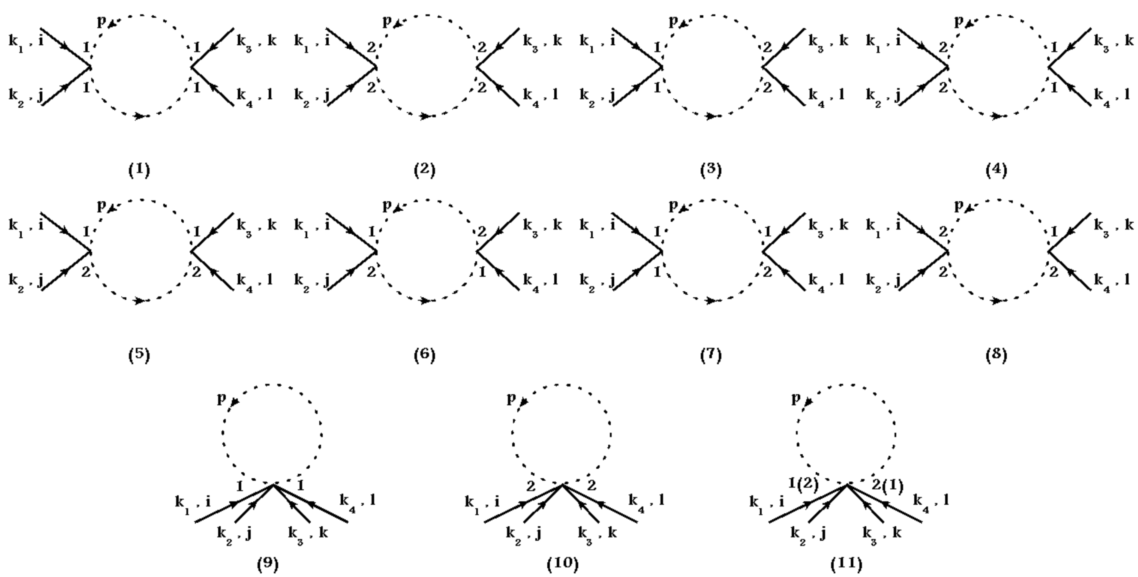
Holographic confining strings

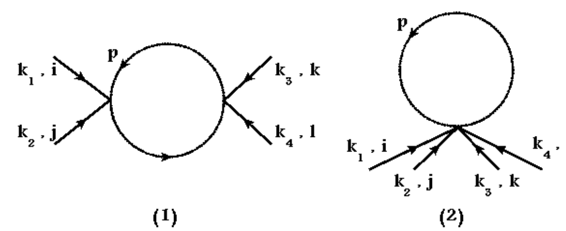
- We have many examples of holographic confining backgrounds that are weakly curved and weakly coupled in some limit (Witten,MN,KS). The confining string sits in the IR region of the background; expanding its action (in the Green-Schwarz formalism) in static gauge, we find that some of the bosonic fields and all unprotected fermionic fields are massive. At small curvature the theory is weakly coupled, so we can integrate out these massive fields at one-loop, and obtain corrections to Nambu-Goto.



Holographic confining strings

- We compute these corrections by looking at scattering amplitudes of the massless X^i fields on the worldsheet. For instance, for scattering four X's, the following diagrams appear (using the interactions coming from the NG action) :





Holographic confining strings

- At first sight it seems that we need to do a different computation in every confining background. However, it turns out that to second order in the massive fields (all we need at one-loop), all these backgrounds have the same action, just with different values for the boson and fermion masses. Thus, a single computation captures the corrections in all known weakly coupled holographic confining backgrounds.
- In the weakly curved limit, the massive fields on the worldsheet have a mass $m \sim 1/R << \sqrt{T}.$
- Corrections go as $m^2/T, \partial^2/T.$

The full action to order $O((\partial X)^6 Y^2, (\partial X)^4\theta^2)$ is then

$$
\begin{aligned}
S = & -\int d^2\sigma\{T + \frac{1}{2}\partial_\alpha X\cdot\partial^\alpha X + \frac{1}{2}\partial_\alpha Y\cdot\partial^\alpha Y + \frac{1}{2}m_B^2 Y_B^2 \\
& + \frac{i}{2}(\theta^1\partial_+\theta^1 + \theta^2\partial_-\theta^2) + \frac{im_F}{4}(\theta^1\tilde{\gamma}_F\theta^2 - \theta^2\tilde{\gamma}_F^T\theta^1) \\
& + \frac{1}{4T}\partial_\alpha X\cdot\partial^\alpha X[\partial_\beta Y\cdot\partial^\beta Y + m_B^2 Y_B^2 + \frac{i}{2}(\theta^1\partial_+\theta^1 + \theta^2\partial_-\theta^2) + \frac{im_F}{4}(\theta^1\tilde{\gamma}_F\theta^2 - \theta^2\tilde{\gamma}_F^T\theta^1)] \\
& - \frac{1}{2T}\partial_\alpha X\cdot\partial_\beta X\partial^\alpha Y\cdot\partial^\beta Y - \frac{i}{4T}\partial_\alpha X\cdot\partial_+ X\theta^1\partial^\alpha\theta^1 - \frac{i}{4T}\partial_\alpha X\cdot\partial_- X\theta^2\partial^\alpha\theta^2 \\
& + \frac{1}{4T^2}\partial^\gamma X\cdot\partial_\gamma X\partial_\alpha X\cdot\partial_\beta X\partial^\alpha Y\cdot\partial^\beta Y \\
& + \frac{i}{8T^2}\partial^\gamma X\cdot\partial_\gamma X[\partial_\alpha X\cdot\partial_+ X\theta^1\partial^\alpha\theta^1 + \partial_\alpha X\cdot\partial_- X\theta^2\partial^\alpha\theta^2] \\
& + \frac{1}{T^2}(\partial^\alpha X\cdot\partial_\alpha X)^2[\frac{T}{8} - \frac{3}{16}\partial_\beta Y\cdot\partial^\beta Y + \frac{1}{16}m_B^2 Y_B^2 - \frac{3i}{32}(\theta^1\partial_+\theta^1 + \theta^2\partial_-\theta^2) \\
& + \frac{im_F}{64}(\theta^1\tilde{\gamma}_F\theta^2 - \theta^2\tilde{\gamma}_F^T\theta^1)] \\
& + \frac{1}{T^2}(\partial_\alpha X\cdot\partial_\beta X\partial^\alpha X\cdot\partial^\beta X)[-\frac{T}{4} + \frac{1}{8}\partial_\gamma Y\cdot\partial^\gamma Y - \frac{1}{8}m_B^2 Y_B^2 + \frac{i}{16}(\theta^1\partial_+\theta^1 + \theta^2\partial_-\theta^2) \\
& - \frac{im_F}{32}(\theta^1\tilde{\gamma}_F\theta^2 - \theta^2\tilde{\gamma}_F^T\theta^1)] + \frac{im_F}{32T}\epsilon^{\alpha\beta}\partial_\alpha X^i\partial_\beta X^j(\theta^1[\gamma_i,\gamma_j]\tilde{\gamma}_F\theta^2 + \theta^2[\gamma_i,\gamma_j]\tilde{\gamma}_F^T\theta^1) \\
& + \frac{1}{32T^3}(\partial_\alpha X\cdot\partial^\alpha X)^3(-2T + 3\partial^\beta Y\cdot\partial_\beta Y - m_B^2 Y_B^2) - \frac{1}{16T^3}(\partial_\alpha X\cdot\partial^\alpha X)^2\partial_\beta X\cdot\partial_\gamma X\partial^\beta Y\cdot\partial^\gamma Y \\
& + \frac{1}{16T^3}\partial_\alpha X\cdot\partial^\alpha X\partial_\beta X\cdot\partial_\gamma X\partial^\beta X\cdot\partial^\gamma X(2T - \partial_\delta Y\partial^\delta Y + m_B^2 Y_B^2) \\
& - \frac{1}{8T^3}\partial_\alpha X\cdot\partial_\beta X\partial^\alpha X\cdot\partial^\beta X\partial_\gamma X\cdot\partial_\delta X\partial^\gamma Y\cdot\partial^\delta Y\} \quad .
\end{aligned}
\tag{4.23}
$$

Holographic confining strings

- The naïve computation leads to corrections already at four-derivative order, but these can be swallowed by a (known) correction to the tension:

$$\Delta T = \frac{1}{8\pi}\left\{\sum_F m_F^2 \log(m_F^2) - \sum_B m_B^2 \log(m_B^2)\right\}$$

and by rescaling the kinetic terms. This then cancels the $(dX)^6$ deviations as well. The first deviation from Nambu-Goto is found in d^6X^4 terms, as expected; it turns out to be given by c_4=constant/T^2, even though it gets also log(m) contributions (which exactly cancel). Higher order corrections go as negative powers of m, e.g. $(c/m^2T^2)\partial^8 X^4$. All consistent with Lorentz.

Constraints from string theory ?

- Are there additional constraints in the large N limit, coming from the fact that the worldsheet theory should describe a weakly coupled fundamental string ? (Maybe for $c_4=d_2$?)
- Polchinski+Strominger (1991) analyzed this question in a conformal gauge, where they wrote the effective action (Drummond) :

$$S = \frac{1}{4\pi}\int d^2\sigma[\partial_+ X^\mu \partial_- X_\mu + \beta \frac{\partial_+^2 X^\mu \partial_-^2 X_\mu}{\partial_+ X^\nu \partial_- X_\nu} + \cdots]$$

This is singular, but not when expanding around a long string. They showed that having c=26 on the worldsheet fixes $\beta = (26-D)/12.$

Constraints from string theory ?

- Translated into the static gauge, this determines the coefficient $c_4=d_2=(26-D)/(192pT^2)$, if the confining string action can be written as a c=26 CFT coupled to worldsheet gravity; this is not obvious in RR backgrounds.
- In the one-loop computation using the Green-Schwarz formalism, we reproduce precisely the expected coefficient (for effective D). It would be interesting to measure d_2 for a QCD string on the lattice, and to see if the coefficient agrees with the Polchinski-Strominger prediction or not.

Conclusions

- The effective action on a confining string can be measured on the lattice, and gives us information about the worldsheet theory of the QCD string. It can also be computed (perturbatively) in weakly curved backgrounds.
- Lorentz invariance places strong constraints on this action, with the leading deviation at six- or eight-derivative order. Measuring such deviations on the lattice is challenging but interesting.
- There may be one additional constraint on the effective action, which is there at least for some weakly coupled strings, and maybe more generally. It would be interesting to test it on the lattice.

Some generalizations

- Open strings – what boundary terms are allowed in the effective action ? Which ones arise for Wilson line computations (in general and in holographic backgrounds) ?
- How do k-strings behave ? Also studied on lattice. However, effective action is subtle at large N since binding energy vanishes.
- Additional massless fields on the worldsheet (e.g. confining strings in supersymmetric gauge theories; again holographic examples are known).

How to construct the QCD string ?

- Studying the low-energy effective action should give some hints, but will not reveal massive states – information needs to be combined with other approaches.
- Derive action from spectrum of glueballs ? Not clear how to do systematically…
- We could start from a known duality (Witten, KS, MN, PS) and take a limit (possibly with a deformation) where it goes over to QCD. This gives an “in principle” construction, but in practice it is very hard to do this since high-curvature RR backgrounds arise.

How to construct the QCD string ?

- We could try to understand first the UV region (an almost free gauge theory) and then deform it by the gauge coupling to flow to the IR. But so far string dual of free gauge theories is not explicitly known. (Gopakumar,…,Berkovits)
- We could try a bottom-up approach in space-time, trying to find a two-derivative effective action whose solutions would describe QCD; this works quite well, at least for some questions, but space-time approach is suspicious when curvatures are of order the string scale. (Gursoy,Kiritsis,Mazzanti,Nitti)