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Holographic Neutron Stars

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Work with

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Why study Neutron Stars?

- Black hole precursors.
- Most extreme form of matter.
- Holographic description of gravitational collapse:

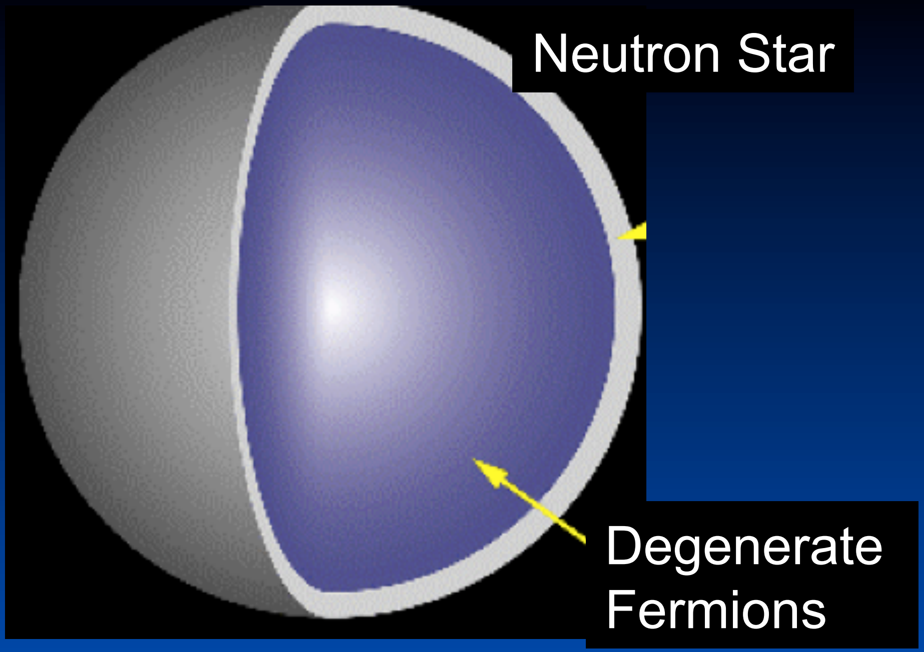
What is Space Time made of ?

(David Gross: 1st new question)

- ....**and it is fun!**

Neutron Stars

in a Nutshell

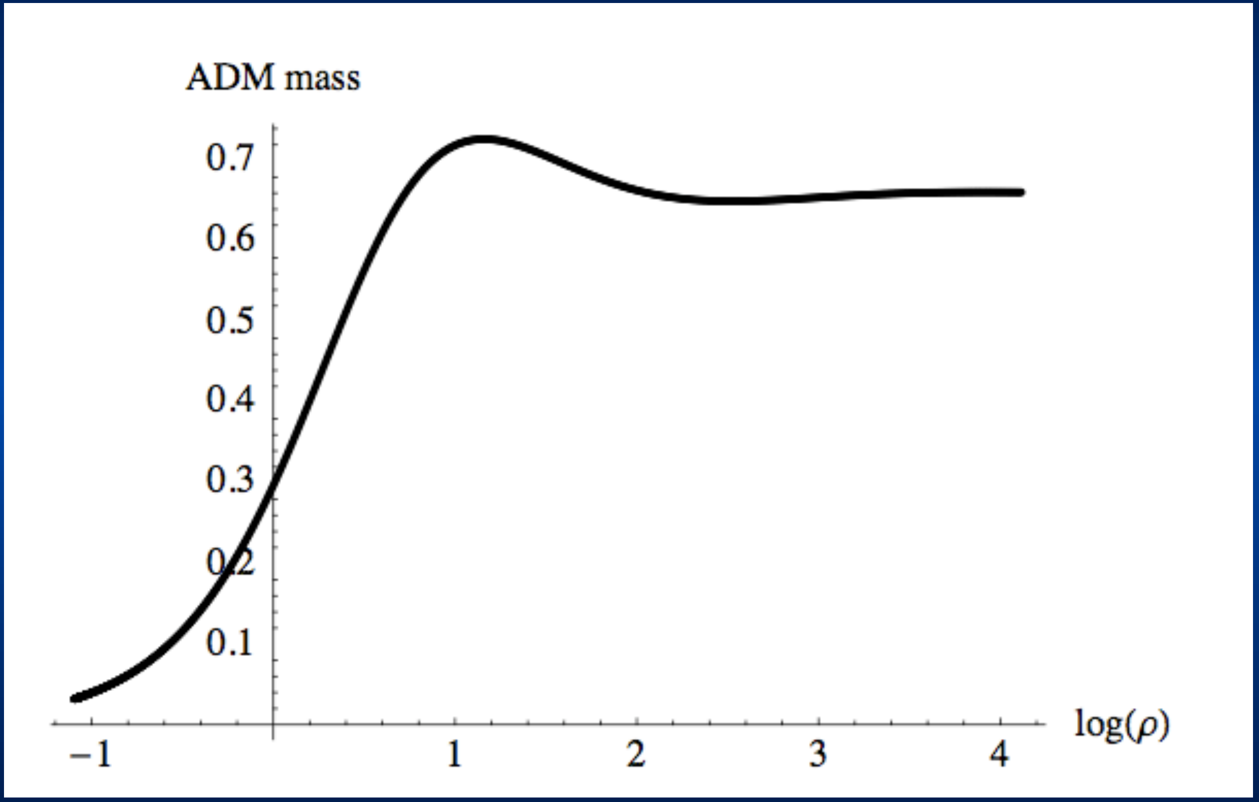


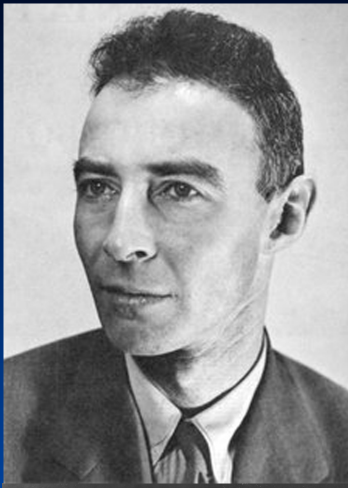
Typical mass

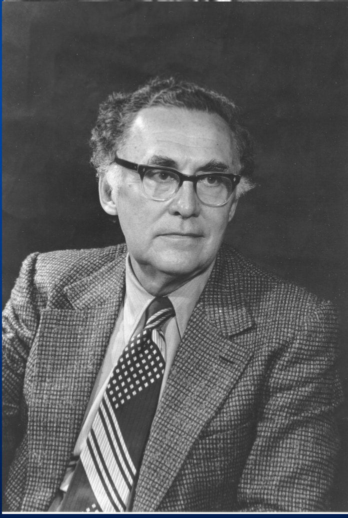
$$M = \frac{m_{pl}^3}{m_n^2}$$

“astrophysical
see saw formula”

Oppenheimer-Volkoff Limit

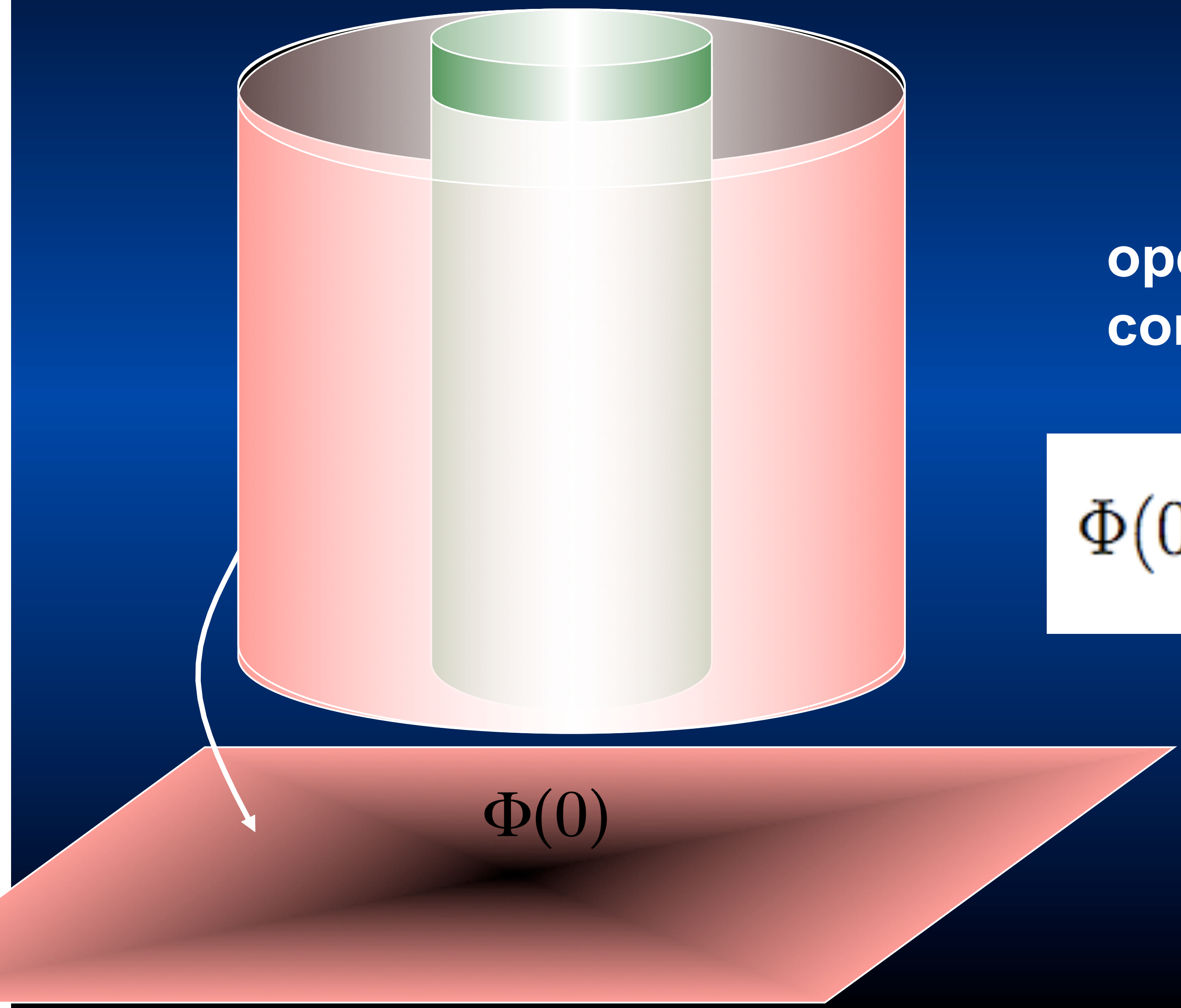






AdS/CFT :

$$ds^2 = -\left(1+\frac{r^2}{\ell^2}\right)dt^2 + \left(1+\frac{r^2}{\ell^2}\right)^{-1}dr^2 + r^2 d\Omega_{d-1}^2$$



operator-state correspondence

$$\Phi(0)\left|\text{vac}\right\rangle = \left|\Phi\right\rangle$$

Fermionic single trace operators

like “ $$\Psi = \mathrm{tr}\Big(\prod_{i=1}^{r} \lambda \phi^{k_i}\Big)$$ ” r = odd

=> Strong coupling

obey fermi statistics

$$\Psi(x)\Psi(y) = -\Psi(y)\Psi(x)$$

behave as free fields for $N \rightarrow \infty$

$$\langle \Psi(x)\Psi(y)\Phi(z)\rangle = \mathcal{O}\left(N^{-1}\right)$$

and correspond to a fermionic particles in the bulk

$$\left\langle \prod_{i=1}^{\mathcal{N}} \Psi(x_i) \prod_{i=j}^{\mathcal{N}} \overline{\Psi}(y_j) \right\rangle$$

What is the operator with the lowest dimension Δ found in the factorization $|x_i| \longrightarrow 0$ *and* $|y_j| \longrightarrow \infty$?

Note that for $N \longrightarrow \infty$ Wick's theorem gives:

$$\det{}_{ij} \langle \Psi(x_i) \overline{\Psi}(y_j) \rangle$$

Composite Operators

N-particle states correspond to composites defined by

$$\Psi(x)\Phi_{\mathcal{N}}(0) \sim x^{\delta}\,\Phi_{\mathcal{N}+1}(0) + \ldots$$

In each step the total mass $M = \dfrac{\Delta_{\mathcal{N}}}{\ell}$

increases by the amount $\epsilon_F = \dfrac{\delta + \Delta_1}{\ell}$ $\geq$ $m = \dfrac{\Delta_1}{\ell}$

=> fermi energy: increasing function of particle #

Composite Operators at infinite N

out of two fields

$$\Psi \partial_i \Psi(x) = \lim_{y \to x} \frac{x_i - y_i}{(x-y)^2} \Psi(x)\Psi(y)$$

out of d+1 fields

$$\Psi \prod_i \partial_i \Psi$$

or (d+1)(d+2)/2 fields

$$\Psi \prod_i \partial_i \Psi \prod_{\{i,j\}} \partial_i \partial_j \Psi$$

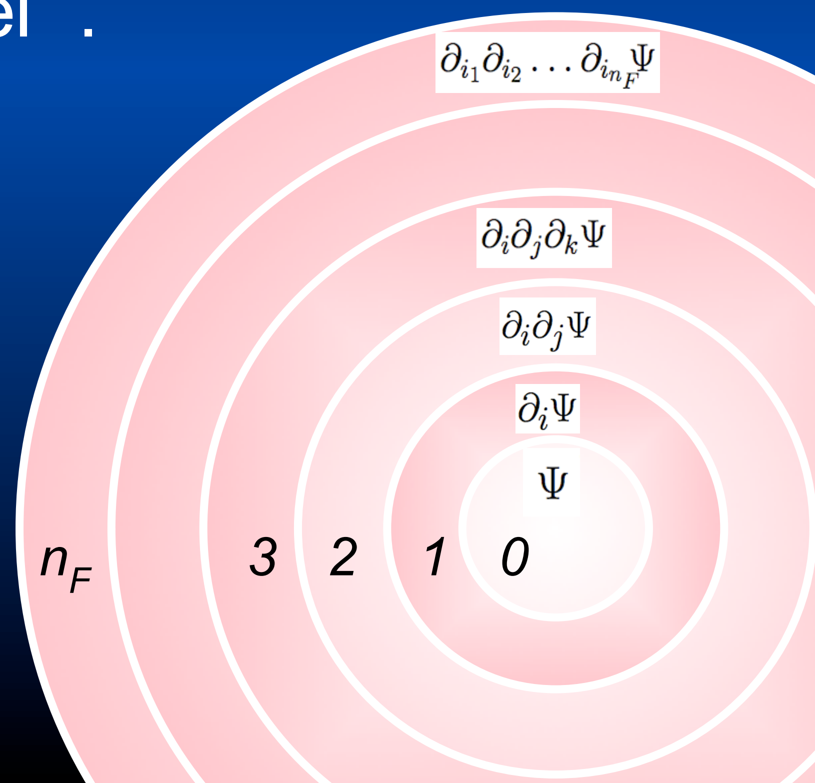
Degenerate Composite Operators

$$\Psi \prod_{i} \partial_i \Psi \prod_{\{i,j\}} \partial_i \partial_j \Psi \prod_{\{i,j,k\}} \partial_i \partial_j \partial_k \Psi \ldots\ldots \prod_{\{i_1,i_2,\ldots i_{n_F}\}} \partial_{i_1} \partial_{i_2} \ldots \partial_{i_{n_F}} \Psi$$

n_F = # of "shells"= "Fermi level" .

Total # of particles:

$$\mathcal{N} \sim \frac{n_F^d}{d!}$$



AdS description at infinite N

$$\partial_{i_1}\partial_{i_2}\ldots\partial_{i_n}\Psi$$

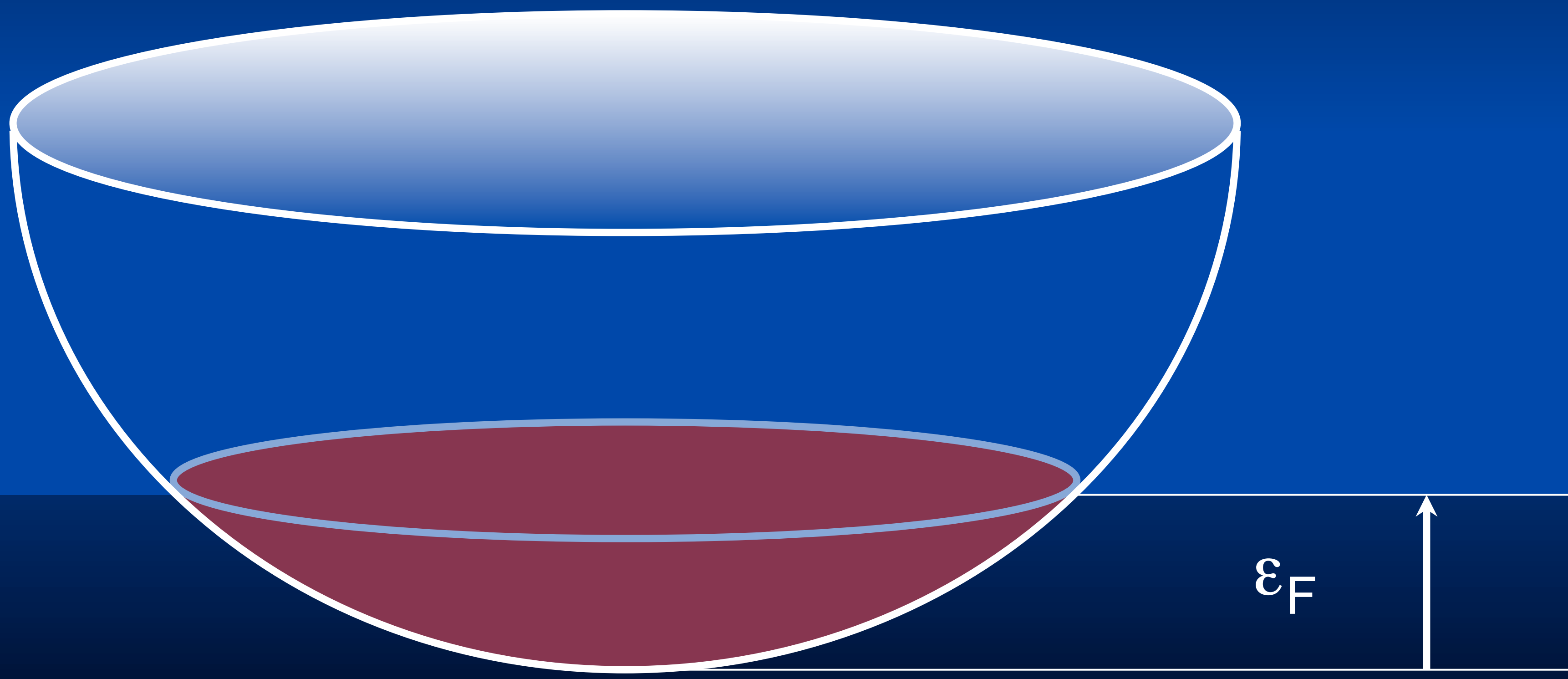
= one particle state with wave function

$$\psi_{n,l,\mathbf{m}}(r,\Omega) = f_{n,l}(r)\,Y_{n-2l,\mathbf{m}}(\Omega)$$

and energy $\epsilon = (n+\Delta_1)/\ell$

Multi-trace operator = Slater determinant

Trapped Cold Fermi Gas in AdS



Hydrodynamic (or “astrophysical”) limit

$$\Delta = \frac{4}{5} n_F \mathcal{N} + \Delta_1 \mathcal{N}$$

$$\mathcal{N} = \frac{1}{24} n_F^4$$

Together with $N \to \infty$

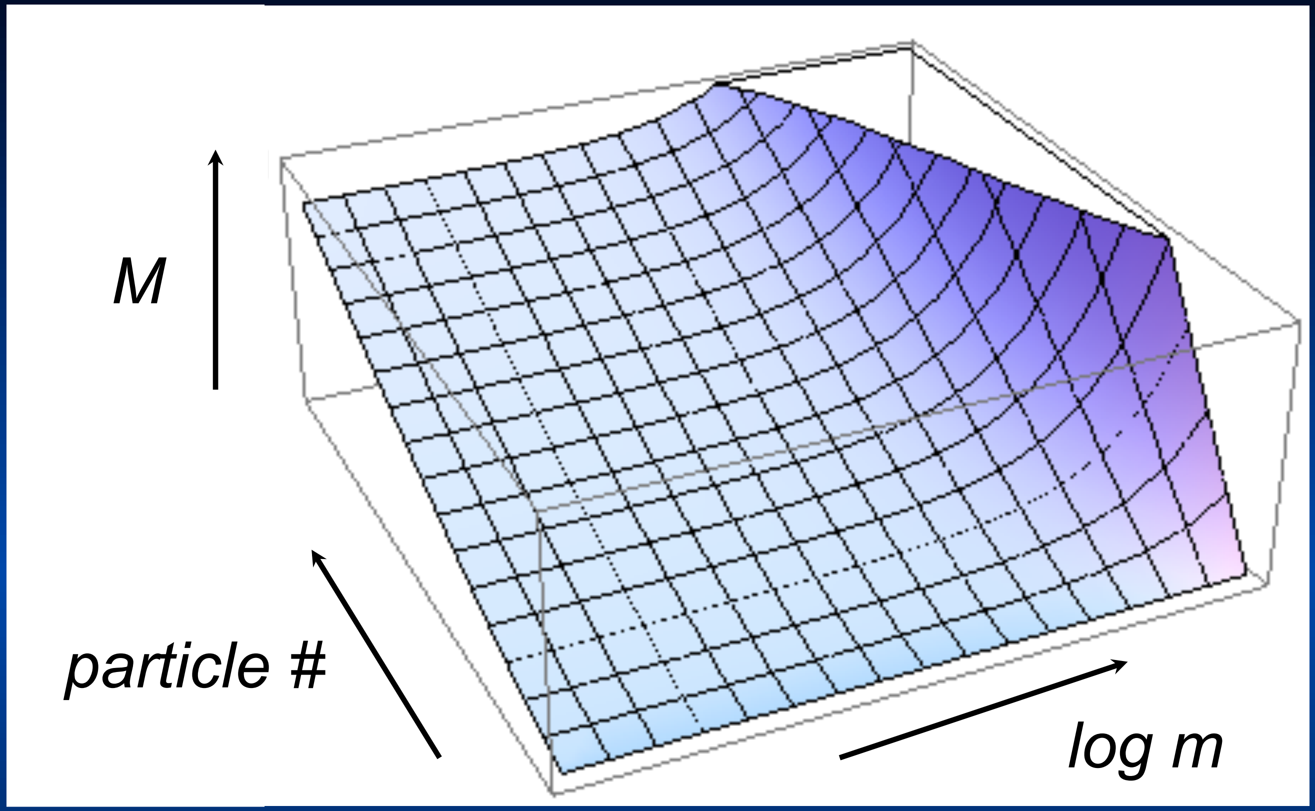
we take $$n_F \to \infty \quad \text{and} \quad \Delta_1 \to \infty$$

keeping $$\frac{\Delta}{N^2} \quad \text{and} \quad \frac{n_F}{\Delta_1} \quad \text{fixed}$$

Partial ‘t Hooft suppression: n-point functions vanish, except those that are sufficiently enhanced by combinatorical factors.

Most bulk interactions are still suppressed, except gravity.

Note: # of particles is infinite => classical fluid.



$$M = \frac{1}{5}(4\epsilon_F + m)\mathcal{N}$$

$$\mathcal{N} = \frac{\ell^4}{24}(\epsilon_F - m)^4$$

Hydrodynamic description

$$T_{\mu\nu} = (\rho + p) u_\mu u_\nu + p g_{\mu\nu}$$

Free degenerate Fermi gas:

$$\mu_F = \sqrt{k_F^2 + m^2}$$

Particle & energy densities

$$\mathbf{n} = \frac{V_{d-1} k_F^d}{d(2\pi)^d}$$

Determined by chemical potential

$$\rho = \int_m^{\mu_F} \mu \frac{d\mathbf{n}}{d\mu} d\mu$$

For a metric of the form

$$ds^2 = -A(r)^2 dt^2 + B(r)^2 dr^2 + r^2 d\Omega_{d-1}^2$$

conservation of energy momentum implies

$$\frac{d\mu_F}{dr} + \frac{1}{A}\frac{dA}{dr}\mu_F = 0$$

=> $$\mu_F(r) = \frac{\epsilon_F}{A(r)}$$

Tolman

The edge of the star $r = R$ is reached when

$$A(R) = \frac{\epsilon_F}{m}$$

← Fermi energy of boundary CFT

Mass and Particle Number

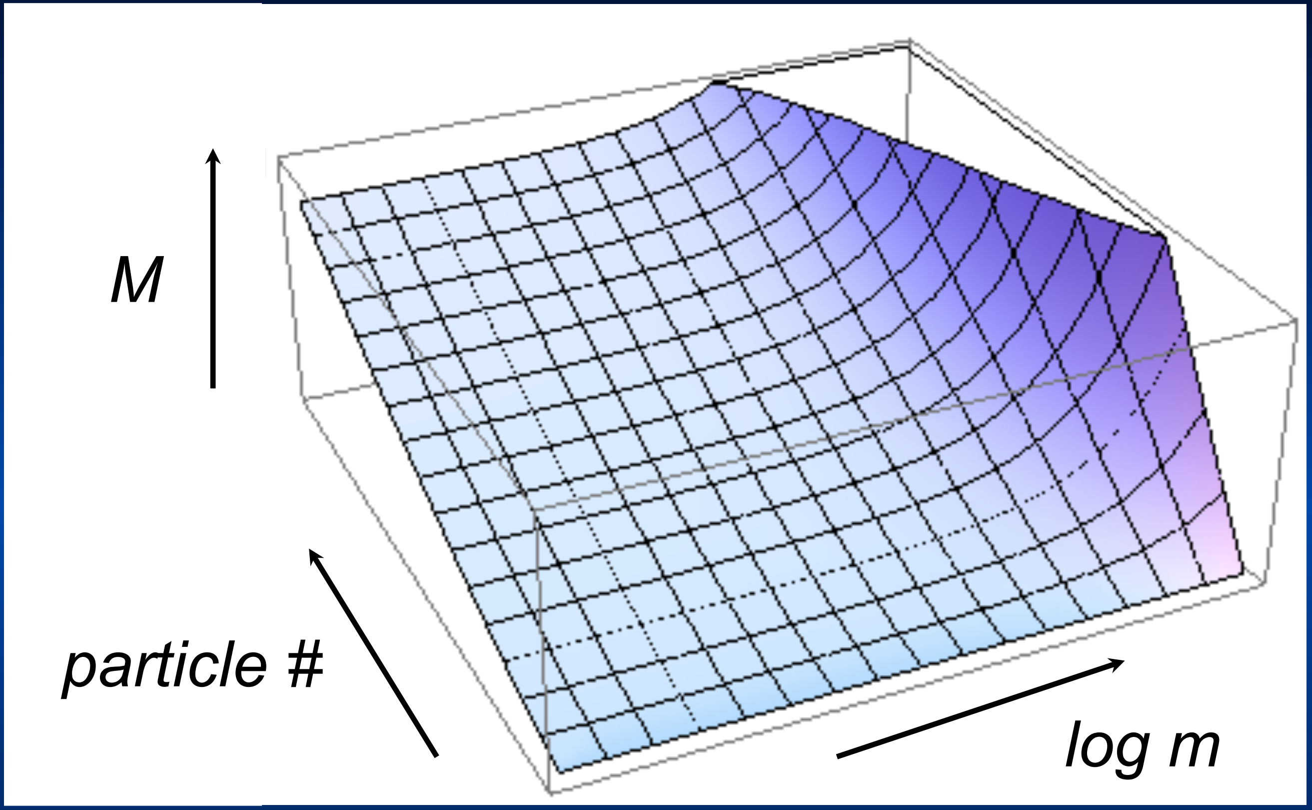
$$\mathcal{N} = V_{d-1} \int_0^R dr\, r^{d-1} B(r)\, \mathbf{n}(r)$$

$$M = V_{d-1} \int_0^R dr\, r^{d-1} \rho(r)$$

= ADM mass

$$M_{free} = V_{d-1} \int_0^R dr\, r^{d-1} A(r) B(r) \rho(r)$$

Pure AdS exactly matches CFT



$$M = \frac{1}{5}(4\epsilon_F + m)\mathcal{N}$$

$$\mathcal{N} = \frac{\ell^4}{24}(\epsilon_F - m)^4$$

Self Gravitating Neutron Star

Metric Ansatz + Einstein equations

$$ds^2 = -A(r)^2 dt^2 + B(r)^2 dr^2 + r^2 d\Omega_{d-1}^2$$

$$A^2(r) = e^{2\chi(r)} \left(1 - \frac{C_d M(r)}{r^{d-2}} + \frac{r^2}{\ell^2}\right)$$

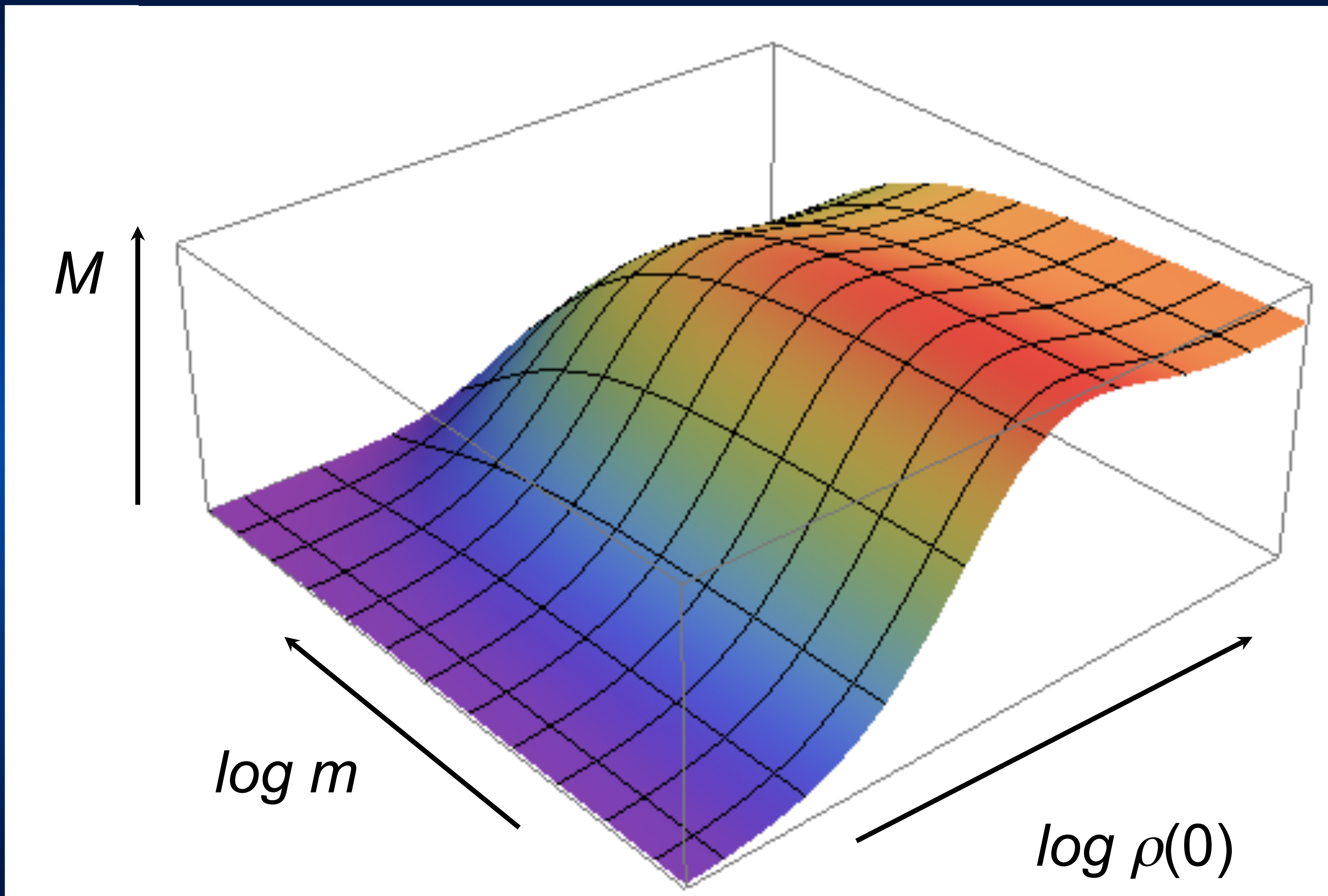
$$\frac{dM(r)}{dr} = V_{d-1}\, r^{d-1}\, \rho(r)$$

$$B^2(r) = \left(1 - \frac{C_d M(r)}{r^{d-2}} + \frac{r^2}{\ell^2}\right)^{-1}$$

$$\frac{d\chi}{dr} = V_{d-1} \frac{C_d}{2} B^2 r \left(\rho + p\right)$$

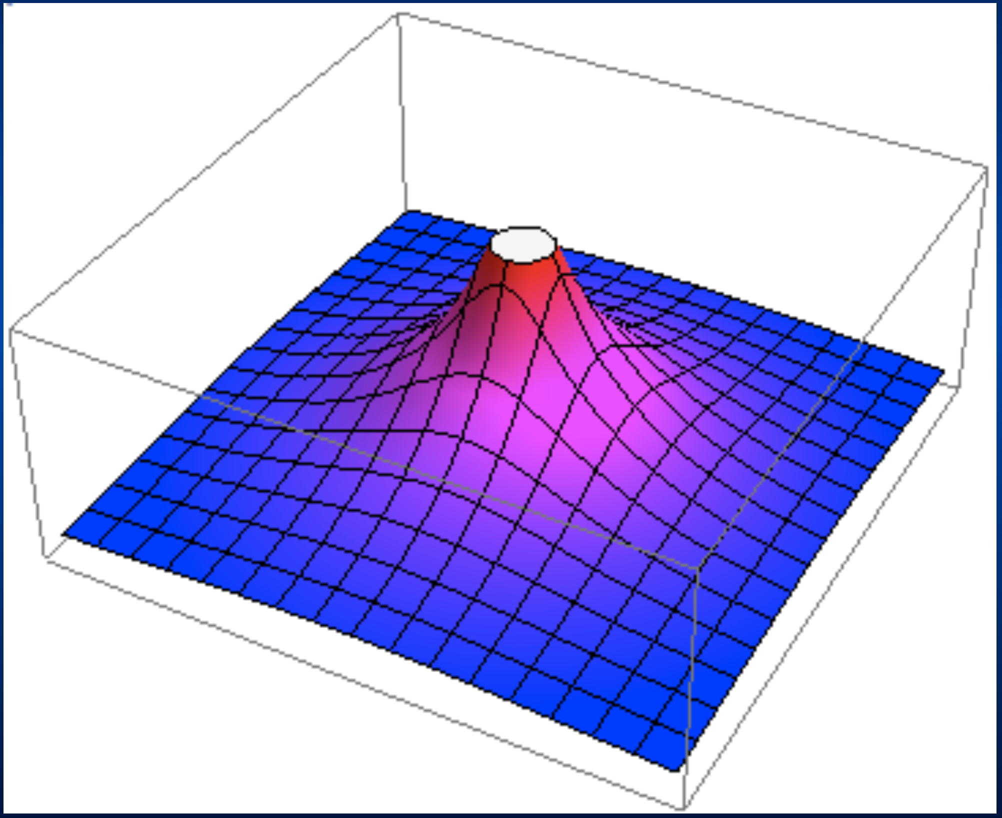
+ Stress energy conservation

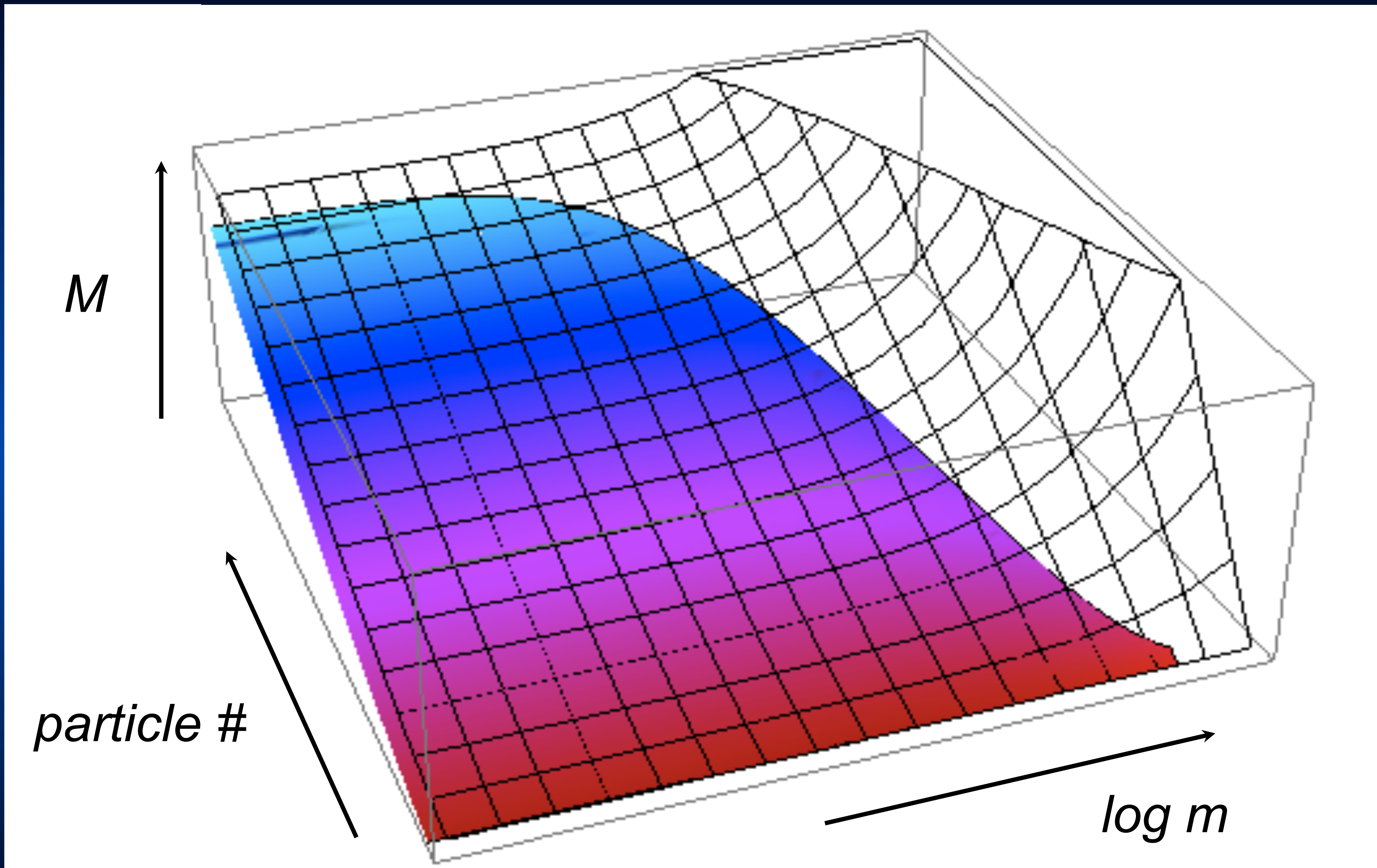
"TOV equations"



M
log m
log ρ(0)

Neutron Star near OV limit




M
particle #
log m

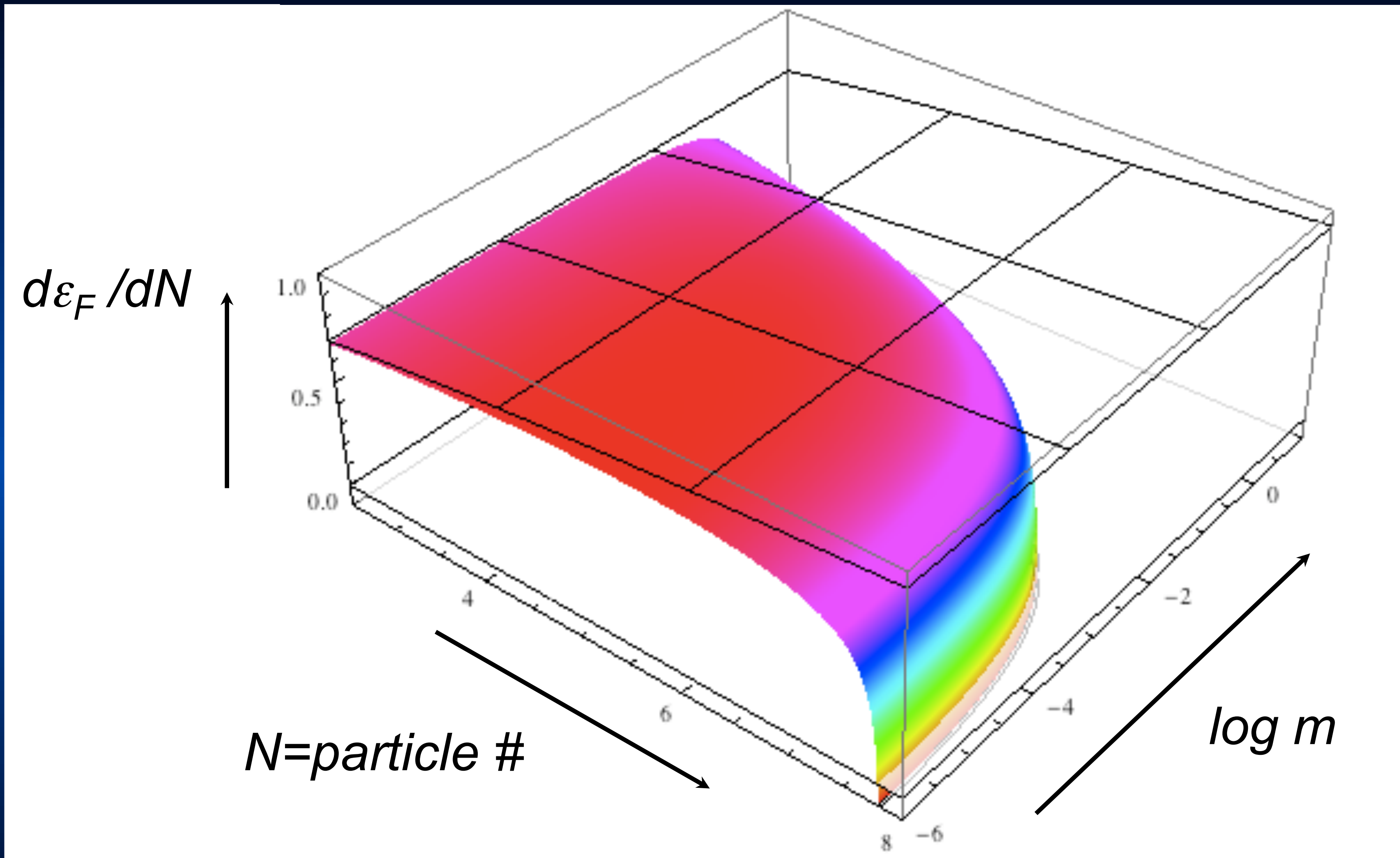
Thermodynamic relations in the CFT

$$M(\epsilon_F) = \int_m^{\epsilon_F} \epsilon \frac{dN(\epsilon)}{d\epsilon} d\epsilon$$

At OV limit # of states at Fermi surface diverges.
Assuming extensivity the CFT pressure obeys:

$$\frac{dP}{dN} = \frac{N}{V} \frac{d\epsilon_F}{dN} \rightarrow 0$$

Clear sign of an instability: phase transition.



$d\varepsilon_F / dN$
1.0
0.5
0.0
4
6
8
N=particle #
−6
−4
−2
0
log m

Concluding

- What happens beyond the OV limit?

A tachyonic mode develops leading to gravitational collapse.

Essential phenomenon happens at center => infrared in CFT.

- Signal of a high density phase transition in the CFT.

- Can one find more precise bounds on the other interactions.

- Do they play some role before the collapse?

- Can one study the dynamics of the collapse in the CFT