Physics with magnetized branes

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Strings 2009 Rome, 22-26 June 2009

1. main questions in string phenomenology
   general issues of high string scale

2. framework of magnetized branes
   moduli stabilization, model building, Yukawa couplings
   SUSY breaking and D-term gauge mediation
- Are there low energy string predictions testable at LHC?
- What can we hope from LHC on string phenomenology?
Very different answers depending mainly on the value of the string scale $M_s$

- arbitrary parameter: Planck mass $M_P \rightarrow$ TeV

- physical motivations $\Rightarrow$ favored energy regions:

  - **High**:
    \[
    \begin{align*}
    M_P^* &\approx 10^{18} \text{ GeV} \quad \text{Heterotic scale} \\
    M_{\text{GUT}} &\approx 10^{16} \text{ GeV} \quad \text{Unification scale}
    \end{align*}
    \]

  - **Intermediate**: around $10^{11}$ GeV ($M_s^2/M_P \sim$ TeV)
    
    SUSY breaking, strong CP axion, see-saw scale

  - **Low**: TeV (hierarchy problem)
• Low string scale $\Rightarrow$ experimentally testable framework
  - spectacular model independent predictions
    - perturbative type I string setup
      see Lust’s talk for recent developments
  - radical change of high energy physics at the TeV scale
    - explicit model building is not necessary at this moment
      but unification has to be probably dropped

• Intermediate string scale:
  not directly testable but interesting possibility with several implications
  $\rightarrow$ ‘large volume’ compactifications
• High string scale:

perturbative heterotic string: the most natural for SUSY and unification prediction for GUT scale but off by almost 2 orders of magnitude

introduce large threshold corrections or strong coupling $\rightarrow M_s \simeq M_{GUT}$

$\Rightarrow$ other string theories:

- intersecting branes in extra dimensions: IIA, IIB, F-theory
- Heterotic M-theory
- internal magnetic fields in type I

Main problems: - gauge coupling unification is not automatic
different coupling for every brane stack, or incomplete GUT representations

- No top Yukawa coupling in D-brane GUT constructions
Maximal predictive power if there is common framework for:

- moduli stabilization
- model building (spectrum and couplings)
- SUSY breaking (calculable soft terms)
- computable radiative corrections (crucial for comparing models)

Possible candidate of such a framework: magnetized branes
Type I string theory with magnetic fluxes on 2-cycles of the compactification manifold

- Dirac quantization: \( H = \frac{m}{nA} \equiv \frac{p}{A} \) \[^{[10]}\] \( \Rightarrow \) moduli stabilization
  
  \( H \): constant magnetic field \hspace{1cm} m: units of magnetic flux
  
  \( n \): brane wrapping \hspace{1cm} A: area of the 2-cycle

- Spin-dependent mass shifts for charged states \( \Rightarrow \) SUSY breaking

- Exact open string description: \( \Rightarrow \) calculability

  \( qH \to \theta = \arctan qH\alpha' \) \hspace{1cm} weak field \( \Rightarrow \) field theory

- T-dual representation: branes at angles \( \Rightarrow \) model building

  \( (m, n)\): wrapping numbers around the 2-cycle directions
Magnetic fluxes can be used to stabilize moduli

I.A.-Maillard '04, I.A.-Kumar-Maillard '05, '06, Bianchi-Trevigne '05

e.g. $T^6$: 36 moduli \textit{(geometric deformations)}

\textbf{internal metric:} \quad 6 \times 7/2 = 21 = 9 + 2 \times 6

\textbf{type IIB RR 2-form:} \quad 6 \times 5/2 = 15 = 9 + 2 \times 3

\textbf{complexification:} \quad \begin{cases} 
\text{Kähler class} \quad J \\
\text{complex structure} \quad \tau 
\end{cases}

\textbf{magnetic flux:} \quad 6 \times 6 \text{ antisymmetric matrix } F \quad \text{complexification } \Rightarrow

$F_{(2,0)}$ on holomorphic 2-cycles: potential for $\tau$ \quad \text{superpotential}

$F_{(1,1)}$ on mixed (1,1)-cycles: potential for $J$ \quad \text{FI D-terms}
\[ N = 1 \text{ SUSY conditions} \implies \text{moduli stabilization} \]

1. \[ F_{(2,0)} = 0 \implies \tau \] matrix equation for every magnetized \( U(1) \)
   need ‘oblique’ (non-commuting) magnetic fields to fix off-diagonal components of the metric ← but can be made diagonal

2. \[ J \wedge J \wedge F_{(1,1)} = F_{(1,1)} \wedge F_{(1,1)} \wedge F_{(1,1)} \implies J \]
   vanishing of a Fayet-Iliopoulos term: \[ \xi \sim F \wedge F \wedge F - J \wedge J \wedge F \]
   magnetized \( U(1) \) → massive absorbs RR axion
   one condition \( \implies \) need at least 9 brane stacks

3. Tadpole cancellation conditions: introduce an extra brane(s)
   \( \implies \) dilaton potential from the FI D-term → two possibilities:
   - keep SUSY by turning on charged scalar VEVs
   - break SUSY in a dS or AdS vacuum \( d = \xi/\sqrt{1 + \xi^2} \) [11]

I.A.-Derendinger-Maillard '08
\[ F_{(2,0)} = 0 \ \Rightarrow \ \tau^T p_{xx} \tau - (\tau^T p_{xy} + p_{yx} \tau) + p_{yy} = 0 \] [7]

\[ T^6 \text{ parametrization: } (x^i, y^i) \quad i = 1, 2, 3 \quad z^i = x^i + \tau^{ij} y^i \]

Non-trivial VEVs \( \nu \) for charged brane scalars \( \Rightarrow \)

D-term condition is modified to:

\[ q \nu^2 (J \wedge J \wedge J - J \wedge F \wedge F) = -(F \wedge F \wedge F - F \wedge J \wedge J) \]
New gauge mediation mechanism

D-term SUSY breaking:

- problem with Majorana gaugino masses  lowest order R-symmetry
- broken at higher orders but suppressed by the string scale  
  I.A.-Taylor ’04, I.A.-Narain-Taylor ’05

- tachyonic squark masses

However in toroidal models gauge multiplets have extended SUSY  

- Dirac gauginos without $R$  $\Rightarrow m_{1/2} \sim d/M$
- Squark masses can arise dominantly from gauginos  $\Rightarrow m_0^2 \sim d^2/M^2$

Also non-chiral intersections have $N = 2$ SUSY  $\Rightarrow N = 2$ Higgs potential
oblique fluxes $\Rightarrow$ non-commuting boundary conditions

boundary CFT similar to non-abelian orbifolds

However spectrum involves only 2 branes: $a, b \leftrightarrow$ can be orientifold image

$\Rightarrow$ depends on relative flux: $R_a R_b^{-1}$

$$R_a \equiv (1 - F^a)(1 + F^a)^{-1}$$

Bianchi-Trevigne ’05

can go to a basis where $R_a R_b^{-1}$ is diagonal $\rightarrow$ mass eigenvalues

Multiplicities: ‘intersection’ matrix $N^{ab} = F^a - F^b$

$$\text{gives no of fermion 0-modes in all } (1, 1)\text{-cycles}$$

$\Rightarrow$ total multiplicity: $I^{ab} = \det N^{ab}$

Non-commutativity shows in interactions e.g. 3-pt functions

Yukawa couplings $\equiv$ overlap integral of 3 wave functions

$$\lambda_{ijk} = g \sigma_{ijk} \int_{\mathcal{T}^6} \psi_i^{N^{ab}} \psi_j^{N^{bc}} \psi_k^{N^{ca}}$$

$$N^{ab} + N^{bc} + N^{ca} = 0$$
commuting case in factorized $T^6 = (T^2)^3 \Rightarrow \lambda$’s products over 3 $T^2$’s

on a $T^2$: chirality $\rightarrow$ analyticity

$$\psi_i^N \propto \begin{cases} \theta_i(N\tau, Nz) & N > 0 \quad \text{+ ve helicity} \\ \theta_i^*(N\bar{\tau}, N\bar{z}) & N < 0 \quad \text{− ve helicity} \end{cases}$$

fusion of 2 wave functions $\rightarrow$ orthogonality: Riemann theta identity

$T^2 \rightarrow T^6$ with oblique fluxes $\rightarrow$ 2 main problems:

1. wave function: analyticity vs general helicity
   $$N: \text{eigenvalues of different sign}$$

2. fusion generalization $\rightarrow$ express Yukawa’s in a closed form

special case: $N\text{Im}\tau$ orthogonal and positive definite

$\Rightarrow$ generalized $\theta$-functions $\theta_i(N\tau, N\bar{z})$ Cremades-Ibanez-Marchesano ’04
wave functions and Yukawa’s for oblique fluxes

General solution: I.A.-Panda-Kumar ’09

1 wave function: relax extra conditions
   (i) fluxes: general hermitian matrices
   (ii) relax positivity \( \Rightarrow \) general helicity

map from all positive helicities to sign flip of one eigenvalue

\[ \tau \rightarrow \hat{\tau} \quad \text{where} \quad \hat{\tau}[N^{ab}] \]

2 Yukawa couplings: generalize Riemann \( \theta \)-function identity

new mathematical identities not given in Mumford Tata lectures
Full string embedding with all geometric moduli stabilized:

- all extra $U(1)$’s broken $\Rightarrow$ gauge group just susy $SU(5)$
- gauge non-singlet chiral spectrum: 3 generations of quarks + leptons
- SUSY can be broken in an extra $U(1)$ factor by D-term [18]
SUSY $SU(5)$ with stabilized moduli

12 brane-stacks: $U_5, U_1, O_1, \ldots, O_8, A, B$

$U(5) \times U(1) \times U(1)^{10}$

winding matrix $W = \mathbb{1}$, $B$-field $B_{x_iy_i} = \frac{1}{2}$

- $I_{U_5^*U_5^*} = I_{U_5^*U_1} = 3 \Rightarrow 3$ generations $(10 + \bar{5})$
- $I_{U_5^*U_1} = 0 \Rightarrow$ Higgs pairs $(5 + \bar{5})$
- $I_{U_5a} + I_{U_5a^*} = 0, \forall a \neq U_5, U_1 \Rightarrow$ no other $SU(5)$ chiral states
- $O_1, \ldots, O_8$: set of oblique fluxes for $B \neq 0$

with diagonal induced 5-brane tadpoles
• SUSY conditions on $U_5, O_1, \ldots, O_8$ ⇒
  fix all geometric moduli to diagonal metric
  $U(1)^9$ massive (absorb the RR Kähler moduli)

• Tadpole cancellation ⇒ add branes $A, B$

• SUSY D-flatness on $U_1, A, B$ ⇒
  charged scalar VEVs $\neq 0$ on their intersections:
  - satisfy perturbativity constraint
  - break $U(1)^3$

⇒ leftover gauge group: $SU(5)$
  gauge non-singlet chiral spectrum: 3 generations of quarks + leptons
Problem common in all D-brane GUTs: absence of top Yukawa coupling can be avoided in a $U(3) \times U(2) \times U(1)$ 3-stack model

\[ \Rightarrow HQu^c, H'Qd^c \neq 0 \quad \text{all Yukawa's exist} \]

but unification is not guaranteed although not excluded

e.g. $\alpha_2 = \alpha_3$ at 1% is guaranteed by:

(i) the correct SM spectrum: no chiral color sextets, weak triplets and antiquark doublets

(ii) weak magnetic fields $\Rightarrow M_{\text{GUT/comp}} \sim M_s/3$
Conclusions

Internal magnetic fields:

simple framework, exact string description,

$N = 1$ SUSY with chiral fermions

Moduli stabilization: ‘oblique’ magnetic fluxes

general: Kähler $\Rightarrow$ complementary to 3-form fluxes

toroidal: all geometric $+$ eventually the dilaton

Model building

natural implementation in intersecting branes

D-term SUSY breaking $\Rightarrow$ new mechanism of gauge mediation

Dirac gauginos, $N = 2$ Higgs potential