ATTRACTORS
IN EXTENDED SUPERGRAVITY

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STRINGS 2009, Rome

Outline of the Talk

1. **The Attractor Mechanism**
2. **The Black Hole Potential : examples**
3. **Attractor Equations in $N \geqslant 2$ extended ungauged $d = 4$ Supergravity**
4. **Critical Points for $N = 8$: BPS and non-BPS**
5. **Charge Orbits and Flat Directions (*Moduli Spaces*)**
6. **Scalar Flow Equations : First Order Formalism**
7. **Recent Developments (see also Talks at this Conference)**

A fascinating aspect of black hole physics is on thermodynamic properties that seem to encode fundamental insights of a so far not established final theory of Quantum Gravity

In this context a central role is played by the Bekenstein-Hawking entropy-area formula

$$S_{BH} = \frac{k_B}{l_p^2}\frac{1}{4}A_H = \pi R_{H}^{2} \quad (k_B = l_p = c = 1)$$

(k_B Boltzmann constant, $l_p^2 = \frac{G\hbar}{c^2}$ is the Planck length square, H denotes the horizon)

R_{H^+} is the radius of a sphere which encircles the outer horizon (effective radius)

Another thermodynamic property is the B-H temperature T_H also given by a geometric quantity

$$T_{BH} = \frac{\kappa}{2\pi}$$

where κ is the "surface gravity"

In terms of the "extremality parameter"

$$c = 2 T_{BH} S_{BH} = \frac{1}{2}(r_+ - r_-)$$

r_+ event horizon

r_- Cauchy horizon

For "extremal" black holes

$$c = 0 \Rightarrow T_{BH} = 0 \qquad r_+ = r_-$$

(F. Kallosh, Strominger)

The "Attractor Mechanism" applies to a class of "Extremal Black Holes" where "scalar field trajectories" behave as dynamical systems.

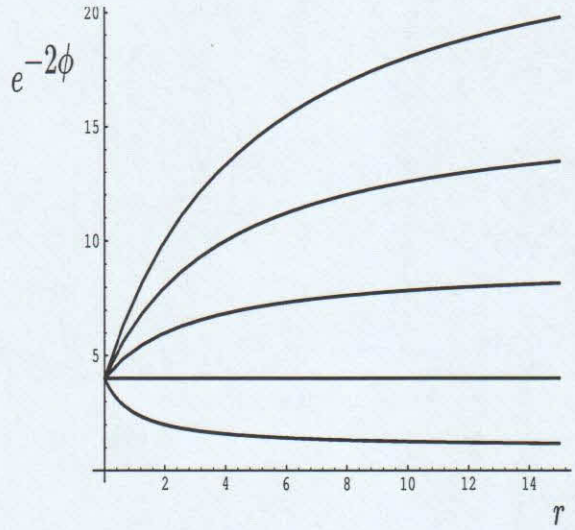
As the BH approaches the Horizon the scalar field tends to a "fixed point" (zero velocity)

$$\dot{\phi}^a(r) \to 0 \quad , \quad \phi^a(r) \to \phi_h(r_H)$$

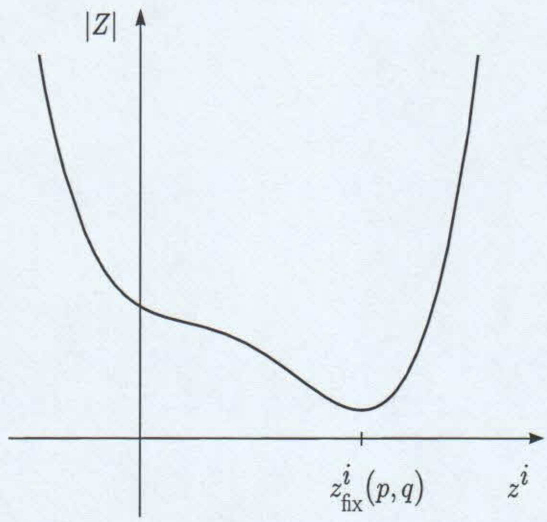
$$r \to r_H \qquad\qquad r \to r_H$$

whose value does depend on the original electric and magnetic charges (p^Λ, q_Λ) $\Lambda = 1 \dots n_v$ but not on their asymptotic value $\lim_{r \to \infty} \phi^a(r) \to \phi^a_\infty$ $\qquad \phi^a_\infty \in \mathcal{M}$

moduli space



Evolution of the dilaton from various initial conditions at infinity to a commom fixed point at $r = 0$



Minimization of the absolute value of the "central charge" function Z in $\mathcal{M}$.

The explanation of this phenomenon is due to the fact that in order

$\Phi^a(r) \underset{r \to r_h}{\longrightarrow} \Phi^a_h$ finite, then

at $r = r_H$

$$\left.\frac{\partial V_{BH}}{\partial \phi}\right|_{\phi=\phi_h} = 0 \qquad \phi_h = \phi_h(p^\Lambda, q_\Lambda)$$

when V_{BH} is an "effective black-hole potential" which appear in an "almost" geodesic (one dimensional) action describing the B-H dynamics.

$$V_{BH}(p, q, \phi) = -\frac{1}{2} Q^T M Q$$

where $Q = (p^\Lambda, q_\Lambda)$ is a symplectic vector (as dictated by electric-magnetic duality at $D=4$) and M is a $2n_V \times 2n_V$ real symplectic matrix

$$M^T\Omega M = M\Omega M = \Omega \qquad \left(\Omega = \begin{pmatrix} 0 & \mathbb{1} \\ -\mathbb{1} & 0 \end{pmatrix}\right)$$

which is symmetric and negative definite.

M is scalar field dependent and is related to the field-dependent normalization of the kinetic (abelian) vector fields as follows

$$M = \begin{pmatrix} \mathrm{Im}\mathcal{N} + \mathrm{Re}\mathcal{N}\,\mathrm{Im}\mathcal{N}^{-1}\mathrm{Re}\mathcal{N} & -\mathrm{Re}\mathcal{N}\,\mathrm{Im}\mathcal{N}^{-1} \\ -\mathrm{Im}\mathcal{N}^{-1}\mathrm{Re}\mathcal{N} & \mathrm{Im}\mathcal{N}^{-1} \end{pmatrix}$$

$$\frac{1}{\sqrt{g}}\mathcal{L}_{D=4} = \cdots\; \frac{1}{4}\mathrm{Im}\mathcal{N}_{\Lambda\Sigma}F^\Lambda F^\Sigma + \frac{1}{4}\mathrm{Re}\mathcal{N}_{\Lambda\Sigma}F^\Lambda\tilde{F}^\Sigma + \dots$$

$$\mathcal{N}_{\Lambda\Sigma}(\phi) = \mathcal{N}_{\Sigma\Lambda}(\phi)\,, \quad \mathrm{Im}\mathcal{N}_{\Lambda\Sigma}(\phi) < 0$$

Under $Sp(2n_v, \mathbb{R})$ transformations

$$\begin{cases} \mathcal{N} \to (C + D\mathcal{N})(A + B\mathcal{N})^{-1}\,, \quad S = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \\ Q \to SQ\,, \quad (S^T\Omega S = \Omega \to A^TC,\ B^TD \text{ sym},\ A^TD - C^TB = \mathbb{1}) \end{cases}$$

For a general class of metrics

$$ds^2 = -e^{2U}dt^2 + e^{-2U}\gamma_{mn}dx^m dx^n$$

$$= -e^{2U}dt^2 + e^{-2U}\left(\frac{c^4}{\sinh^4 c\tau}d\tau^2 + \frac{c^2}{\sinh^2 c\tau}d\Omega^2\right)$$

The following action can be derived

$$\mathcal{L}(U(\tau), \phi^a(\tau), p^\Lambda, q_\Lambda) = (\dot{U})^2 + \frac{1}{2}G_{ab}(\phi)\dot{\phi}^a\dot{\phi}^b$$

$$+ e^{2U}V_{BH}(\phi^a(\tau), p^\Lambda, q_\Lambda)$$

along with the constraints

$$\left(\frac{dU}{d\tau}\right)^2 + \frac{1}{2}G_{ab}\dot{\phi}^a\dot{\phi}^b - e^{2U}V_{BH} = c^2$$

for "extremal" black holes $c=0$

and the metric becomes

$$ds^2 = -e^{2U}dt^2 + e^{-2U}\frac{1}{\tau^2}\left(\frac{1}{\tau^2}d\tau^2 + d\Omega^2\right)$$

for the radial variable $r = -c\coth c\tau + M$

we have $r = -\frac{1}{\tau} + M = \rho + M$ in the extremal case

Evaluating the geodesic constraint

$$(\dot{U})^2 + \frac{1}{2} G_{ab} \dot{\phi}^a \dot{\phi}^b - e^{2U} V(\phi^a(\tau), p, q) = 0$$

in the asymptotic limits toward $r \to \infty$ and $r \to r_H$ one gets two fundamental relations relating thermodynamical quantities to the black-hole potential V_{BH}

$$\lim_{\substack{\tau \to 0^- \\ (r \to \infty)}} U(\tau) = M_{ADM}\, \tau \quad , \quad \lim \dot{\phi}^a = \Sigma^a$$

$$M^2_{ADM} = V(\phi_\infty, p^\Lambda, q_\Lambda) - \frac{1}{2} G_{ab} \Sigma^a \Sigma^b (\phi_\infty, p^\Lambda, q_\Lambda)$$

on the other hand in the near horizon limit

$$\lim_{\tau \to -\infty} e^{-2U} = R_H^2 \tau^2$$

$$\frac{1}{4} A_H = \pi V_{BH}(\phi_h, p, q) = \pi V(\phi, p, q) \Big|_{\frac{\partial V}{\partial \phi} = 0}$$

(S.F., Gibbons, Kallosh)

In the near horizon limit the extremal black hole enjoys a universal geometry $AdS_2 \times S_2$ (Gibbons Townsend)

$$ds^2 \underset{\tau \to -\infty}{=} -\frac{1}{R_H^2 \tau^2} dt^2 + \frac{R_H^2}{\tau^2}\left(d\tau^2 + \tau^2 d\Omega^2\right)$$

$$= R_H^2 \frac{1}{\tau^2}\left[dt^2 + d\tau^2 + \tau^2 d\Omega^2\right]$$

Conformally flat metric (Bertotti-Robinson)

Extreme black holes as solitons interpolating between solutions with higher symmetry.

For $\tau \to 0^-$, $\tau \to -\infty$ doubling of Killing spinors (4 → 8 supersymmetries)

$$R_H^2 = V(\phi_h, p, q) = V(\phi(p,q), p, q) = V\Big|_{\partial V = 0}$$

Examples of V_{BH} and Entropy

1) Reissner-Nordstrom $V_{BH} = p^2 + q^2$, $S = \pi(p^2+q^2)$

2) Dilaton-Maxwell $V_{BH} = p^2 e^{-2\phi} + q^2 e^{2\phi}$, $S = 2\pi|pq|$

3) $N=4$ supergravity (Dilaton-Axion-Maxwell)

$$V_{BH} = e^{2\phi}[S p_\Lambda - q_\Lambda][\bar{S} p^\Lambda - q^\Lambda]$$

$$S = a + i e^{-2\phi}$$

$$V_{BH} = (e^{2\phi} a^2 + e^{-2\phi}) p^2 + e^{2\phi} q^2 - 2a e^{2\phi} p \cdot q$$

$$\frac{\partial V}{\partial a} = 0 \quad a = \frac{p \cdot q}{p^2} \, , \quad \frac{\partial V}{\partial \phi} = 0 \quad e^{-2\phi} = \frac{\sqrt{|I_4|}}{p^2}$$

$$V_{BH}\Big|_{crit} = 2\sqrt{|p^2 q^2 - (p \cdot q)^2|} = |p^1 q_2 - p_2 q_1|$$

$(\Lambda = 1, 2)$

4) K-K dyon

$$V = \frac{1}{2}[p^{0\,2} V + q_0^2 V^{-1}] \, , \quad S_{BH} = \pi |p^0 q_0|$$

The ATTRACTOR MECHANISM IN SUPERGRAVITY

$$\{ Q_{\alpha A}, Q_{\beta B} \} = \epsilon_{\alpha\beta} Z_{AB}(\phi_\infty, p^1, q_1)$$

For given initial data ϕ_∞, p^1, q_1 of the black-hole background.

Dyonic black-holes transform in a symplectic representation of the e-m. duality symmetry.

At the classical level, for $N > 2$ extended supergravity the duality group (U-duality in the sense of Hull and Townsend) are known and the Moduli space is a Symmetric Space with metric $g_{ab}(\phi)$

ATTRACTOR EQUATIONS IN

$\mathcal{N}$-Extended Supergravity

$$V_{BH} = -\frac{1}{2} Q^T M Q = \frac{1}{2} Z_{AB} \bar{Z}^{AB} + Z_I \bar{Z}^I$$

$Z_{AB} = -Z_{BA}$ central charges

Z_I matter charges

For $N=2$ $Z_{AB} = \epsilon_{AB} Z$, $Z_i = \mathcal{D}_i Z$

and then

$$V_{BH}(z, \bar{z}) = |Z|^2 + g^{i\bar{j}} \mathcal{D}_i Z \mathcal{D}_{\bar{j}} \bar{Z}$$

$$\frac{\partial V}{\partial z^i} = 0 \Rightarrow 2|Z| \mathcal{D}_i Z + i C_{ijk} g^{j\bar{j}} g^{k\bar{k}} \mathcal{D}_{\bar{j}} \bar{Z} \mathcal{D}_{\bar{k}} \bar{Z} = 0$$

BPS: $\mathcal{D}_i Z = 0$

at the critical point $\partial_i \partial_j V = 0$, $\partial_i \partial_{\bar{j}} V = 2 g_{i\bar{j}} V$

(attractive nature of BPS attractors)

NBPS attractors $\mathcal{D}_i Z \neq 0$ Bellucci, S.F. Marrani, Gunaydin Trivedi, Tripathy, et al.

$\partial_i V = 0$ two cases:

$$Z = 0 \Rightarrow C_{ijk} g^{j\bar{j}} g^{k\bar{k}} \bar{\mathcal{D}}_{\bar{j}} \bar{Z} \bar{\mathcal{D}}_{\bar{k}} \bar{Z} = 0$$

$$Z \neq 0 \Rightarrow 2|Z| \mathcal{D}_i Z = -i C_{ijk} g^{j\bar{j}} g^{k\bar{k}} \bar{\mathcal{D}}_{\bar{j}} \bar{Z} \bar{\mathcal{D}}_{\bar{k}} \bar{Z}$$

(first solution is unique if $C_{ijk} = 0$)

For NBPS solutions the stability (positivity of the Hessian matrix) is not automatic.

However for all special geometries based on symmetric spaces the Hessian matrix is semipositive with some zero eigenvalues (there are actually flat directions of the attractor points!) ($\mathcal{D}_i C_{klm} = 0$)

	$\frac{1}{2}$-BPS orbits $\mathcal{O}_{\frac{1}{2}-BPS} = \frac{G}{H_0}$	non-BPS, $Z \neq 0$ orbits $\mathcal{O}_{non-BPS,Z\neq 0} = \frac{G}{\widehat{H}}$	non-BPS, $Z = 0$ orbits $\mathcal{O}_{non-BPS,Z=0} = \frac{G}{\widetilde{H}}$
I	$\frac{SU(1,n+1)}{SU(n+1)}$	$-$	$\frac{SU(1,n+1)}{SU(1,n)}$
II	$SU(1,1) \times \frac{SO(2,2+n)}{SO(2)\times SO(2+n)}$	$SU(1,1) \times \frac{SO(2,2+n)}{SO(1,1)\times SO(1,1+n)}$	$SU(1,1) \times \frac{SO(2,2+n)}{SO(2)\times SO(2,n)}$
III	$\frac{E_{7(-25)}}{E_6}$	$\frac{E_{7(-25)}}{E_{6(-26)}}$	$\frac{E_{7(-25)}}{E_{6(-14)}}$
IV	$\frac{SO^*(12)}{SU(6)}$	$\frac{SO^*(12)}{SU^*(6)}$	$\frac{SO^*(12)}{SU(4,2)}$
V	$\frac{SU(3,3)}{SU(3)\times SU(3)}$	$\frac{SU(3,3)}{SL(3,\mathbb{C})}$	$\frac{SU(3,3)}{SU(2,1)\times SU(1,2)}$
VI	$\frac{Sp(6,\mathbb{R})}{SU(3)}$	$\frac{Sp(6,\mathbb{R})}{SL(3,\mathbb{R})}$	$\frac{Sp(6,\mathbb{R})}{SU(2,1)}$

Table 1: *"Large"* **orbits of $N = 2$, $d = 4$ symmetric Maxwell-Einstein *ungauged* supergravities**

	$\frac{\widehat{H}}{\widehat{h}}$	r	$dim_{\mathbb{R}}$
$II : \mathbb{R} \oplus \Gamma_{n+2}$ $(n = n_V - 3 \in \mathbb{N} \cup \{0, -1\})$	$SO(1,1) \times \frac{SO(1,n+1)}{SO(n+1)}$	$1(n = -1)$ $2(n \geqslant 0)$	$n + 2$
$III : J_3^{\mathbb{O}}$	$\frac{E_{6(-26)}}{F_{4(-52)}}$	2	6
$IV : J_3^{\mathbb{H}}$	$\frac{SU^*(6)}{USp(6)}$	2	14
$V : J_3^{\mathbb{C}}$	$\frac{SL(3,C)}{SU(3)}$	2	8
$VI : J_3^{\mathbb{R}}$	$\frac{SL(3,\mathbb{R})}{SO(3)}$	2	5

Table 2: ***Moduli spaces*** **of non-BPS** $Z \neq 0$ **critical points of** $V_{BH,N=2}$ **in** $N = 2, d = 4$ **symmetric Maxwell-Einstein *ungauged* supergravities.** $\widehat{h}$ **is the maximal compact subgroup of** $\widehat{H}$**. They are the** $N = 2, d = 5$ **symmetric real special manifolds**

	$\frac{\widetilde{H}}{\widetilde{h}} = \frac{\widetilde{H}}{\widetilde{h}' \times U(1)}$	r	$dim_{\mathbb{C}}$
$I: Quadratic\ \ Sequence$ $(n = n_V - 1 \in \mathbb{N} \cup \{0\})$	$\frac{SU(1,n)}{U(1)\times SU(n)}$	1	n
$II: \mathbb{R} \oplus \Gamma_{n+2}$ $(n = n_V - 3 \in \mathbb{N} \cup \{0, -1\})$	$\frac{SO(2,n)}{SO(2)\times SO(n)}, n \geqslant 1$	$1(n = 1)$ $2(n \geqslant 2)$	n
$III: J_3^{\mathbb{O}}$	$\frac{E_{6(-14)}}{SO(10)\times U(1)}$	2	16
$IV: J_3^{\mathbb{H}}$	$\frac{SU(4,2)}{SU(4)\times SU(2)\times U(1)}$	2	8
$V: J_3^{\mathbb{C}}$	$\frac{SU(2,1)}{SU(2)\times U(1)} \times \frac{SU(1,2)}{SU(2)\times U(1)}$	2	4
$VI: J_3^{\mathbb{R}}$	$\frac{SU(2,1)}{SU(2)\times U(1)}$	1	2

Table 3: ***Moduli spaces*** **of non-BPS $Z = 0$ critical points of $V_{BH,N=2}$ in $N = 2, d = 4$ symmetric Maxwell-Einstein *ungauged* supergravities . $\widetilde{h}$ is the maximal compact subgroup of $\widetilde{H}$. They are (non-special) symmetric Kähler manifolds**

$N\geq 2$ CRITICAL POINTS:

One make use of Maurer-Cartan eqs.
(ex. $N=8$)

$$D_i Z_{AB} = \frac{1}{4!} P_{i,ABCD} \bar{Z}^{CD}$$

$\downarrow$

H connection, P Vielbein one form.

Extremize V_{BH} by using M-C eqs.

$$\partial_i V_{BH} = 0 \rightarrow Z_{[AB} Z_{CD]} + \frac{1}{4!} \epsilon_{ABCDEFGH} \bar{Z}^{EF} \bar{Z}^{GH} = 0$$

Similar for all other cases.

BPS Solution $Z_{AB} = \begin{pmatrix} z_1\epsilon & & & \\ & 0 & & \\ & & 0 & \\ & & & 0 \end{pmatrix}$ $SU(2)\times SU(6)$ Symmetry

Orbit $E_{7(7)}/E_{6(2)}$

nBPS Solution $Z_{AB} = e^{i\pi/4} \rho\, \Omega$ $USp(8)$ Symmetry

Orbit $\cancel{E_{7(7)}/USp(8)}$ $E_{7(7)}/E_{6(6)}$

Classical Regular attractors exist for all N-extended supergravity provided the solution is $\frac{1}{N}$ BPS or non BPS.

Contrary to the N=2 case flat direction exist for both BPS and non BPS solutions.

Since the moduli space of the theory is a symmetric space G/H with the charge vector $Q = (p, q)$ is a symplectic repr. R_s of G, they are classified by orbits of R_s of the type G/H^0 where H^0 is the stabilizer of the orbit.

(S.F. Gunaydin, Stelle, Roper, Yu, ...)

Unlike the case of $\frac{1}{2}$ BPS attractors for $N=2$, in all other cases H^0 is non-compact and H^0/h^0 (where h^0 is the maximal compact subgroup) classifies the moduli space (flat directions) of the critical points.

For $N>2$ all $\frac{1}{N}$ BPS solutions have the space H^0/h^0 quaternionic.

Similarly to the $N=2$ theory points, when they exist, either correspond to non BPS attractors with $Z_{AB}=0$ or to non-BPS attractors with $Z_{AB}\neq 0$ and in the latter case the moduli space is the one of five-dimensional supergravity-

	$\frac{1}{N}$-BPS orbits $\frac{G}{\mathcal{H}}$	nBPS, $Z_{AB} \neq 0$ orbits $\frac{G}{\mathcal{H}}$	nBPS, $Z_{AB} = 0$ orbits $\frac{G}{\mathcal{H}}$
$N = 3$	$\frac{SU(3,n)}{SU(2,n)}$	–	$\frac{SU(3,n)}{SU(3,n-1)}$
$N = 4$	$SU(1,1) \times \frac{SO(6,n)}{SO(2)\times SO(4,n)}$	$SU(1,1) \times \frac{SO(6,n)}{SO(1,1)\times SO(5,n-1)}$	$SU(1,1) \times \frac{SO(6,n)}{SO(2)\times SO(6,n-2)}$
$N = 5$	$\frac{SU(1,5)}{SU(3)\times SU(2,1)}$	–	–
$N = 6$	$\frac{SO^*(12)}{SU(4,2)}$	$\frac{SO^*(12)}{SU^*(6)}$	$\frac{SO^*(12)}{SU(6)}$
$N = 8$	$\frac{E_{7(7)}}{E_{6(2)}}$	$\frac{E_{7(7)}}{E_{6(6)}}$	–

Table 4: ***"Large"* charge orbits of $N \geqslant 3$-extended, $d = 4$ *ungauged* supergravities . n is the number of matter multiplets**

	$\frac{1}{N}$-BPS moduli space $\frac{\mathcal{H}}{\mathfrak{h}}$	non-BPS, $Z_{AB} \neq 0$ moduli space $\frac{\widehat{\mathcal{H}}}{\widehat{\mathfrak{h}}}$	non-BPS, $Z_{AB} = 0$ moduli space $\frac{\widetilde{\mathcal{H}}}{\widetilde{\mathfrak{h}}}$
$N = 3$	$\frac{SU(2,n)}{SU(2)\times SU(n)\times U(1)}$	–	$\frac{SU(3,n-1)}{SU(3)\times SU(n-1)\times U(1)}$
$N = 4$	$\frac{SO(4,n)}{SO(4)\times SO(n)}$	$SO(1,1) \times \frac{SO(5,n-1)}{SO(5)\times SO(n-1)}$	$\frac{SO(6,n-2)}{SO(6)\times SO(n-2)}$
$N = 5$	$\frac{SU(2,1)}{SU(2)\times U(1)}$	–	–
$N = 6$	$\frac{SU(4,2)}{SU(4)\times SU(2)\times U(1)}$	$\frac{SU^*(6)}{USp(6)}$	–
$N = 8$	$\frac{E_{6(2)}}{SU(6)\times SU(2)}$	$\frac{E_{6(6)}}{USp(8)}$	–

Table 5: ***Moduli spaces* of BH attractors with non-vanishing entropy in $N \geqslant 3$-extended, $d = 4$ *ungauged* supergravities . $\mathfrak{h}$, $\widehat{\mathfrak{h}}$ and $\widetilde{\mathfrak{h}}$ are maximal compact subgroups of $\mathcal{H}$, $\widehat{\mathcal{H}}$ and $\widetilde{\mathcal{H}}$, respectively, and n is the number of matter multiplets**

In the supersymmetric case (BPS)

$$M^2_{ADM} = |Z_1|^2_{HIGHEST} \text{ and } V_{BH} = \sum_{i=1}^{[N/2]} |Z_N|^2 + M_{MATTER}$$

$$\sum_{i\neq 1} |Z_i|^2 = \frac{1}{2} S_{ab} \Sigma^a \Sigma^b$$

$$\frac{1}{4\pi} A_H = R^2_{+H} = |Z_1|^2_{Horizon} \qquad Z_{iH} = \dot{\phi}^a = 0$$

BPS flow $(N=2)$

$$\frac{dU}{d\tau} = e^U |Z_1|$$

$$\frac{dz^i}{d\tau} = e^U g^{i\bar{j}} \partial_{\bar{k}} |Z_1|$$

Freedman, Townsend
Ceresole, Dall'Agata
Andrianopoli, D'Auria
Orazi, Trigiante

First order NBPS flow

$$\frac{dU}{d\tau} = e^U W, \quad \frac{dz^i}{d\tau} = e^U g^{i\bar{J}} \partial_{\bar{J}} W$$

$$M_{ADM} = W_\infty, \quad R_H^2 = W_{horizon}$$

For all symmetric manifolds of the

scalar field vector multiplets

there exist e.m. duality invariants

$I_k(Q)$ of the rep-Q (Kallosh, Kol

for $N=8$, Cvetic Youm for $N=4$ --)

$$\frac{1}{4} A_H = \pi \left| I_k(Q) \right|^{2/k}$$

if I_k is of order k.

Actually $k=2$ for $N=3$ and $k=4$ for $N>3$.

For $N=2$ $k=2$ for $C_{ijk}=0$, $k=4$

for all other cases.

In the stu model ($n=0$ member of

the infinite series with cubic norm)

I_4 is the Cayley hyperdeterminant

considered (Duff et al) in the application

of B.H. entropy to quantum information

A particular elegant form is taken

by the quartic invariant of

N=2 special geometry (S.F., Cerchiai, Marrani, Zumino)

$$I_4(\phi,\varphi) = I_4(\varphi) = (Z\bar{Z} - Z_i\bar{Z}^i)^2$$

$$+ \frac{2}{3} i\left(Z N_3(\bar{Z}) - \bar{Z} N_3(Z)\right)$$

$$- g^{i\bar{\imath}} C_{ijk} C_{\bar{\imath}\bar{\ell}\bar{m}} \bar{Z}^j \bar{Z}^k Z^{\bar{\ell}} Z^{\bar{m}}$$

$$\left(\bar{Z}^i = g^{i\bar{\imath}} \overline{\mathcal{D}_{\bar{\imath}} Z}\right)$$

Note that $\partial_\phi I_4 = 0$ as a consequence of the identities:

$$\mathcal{D}_i C_{k\ell m} = 0$$

$$C_{j(\ell m} C_{pq)k} \bar{C}_{\bar{\imath}\bar{j}\bar{k}} g^{i\bar{j}} g^{k\bar{k}} = \frac{4}{3} C_{(\ell m p} g_{q)\bar{\imath}}$$

$$I_4^{N=4} = S_1^2 - S_2 \bar{S}_2$$

$$S_1 = \frac{1}{2} Z_{AB} \bar{Z}^{AB} - Z_I \bar{Z}^I$$

$$S_2 = \frac{1}{4} \epsilon^{ABCD} Z_{AB} Z_{CD} - \bar{Z}^I \bar{Z}^I$$

$$I_4^{N=8} = \frac{1}{4}\Big[4\, \mathrm{Tr}\big(Z_{AC} \bar{Z}^{BC}\big)^2 - \big(\mathrm{Tr}(Z_{AC} \bar{Z}^{BC})\big)^2$$
$$+ 32\, \mathrm{Re}\, \mathrm{Pf}\, Z_{AB} \Big]$$

$$\mathrm{Pf}\, Z_{AB} = \frac{1}{2^4 4!} \epsilon^{ABCDEFGH} Z_{AB} Z_{CD} Z_{EF} Z_{GH}$$

$$\partial_\phi I_4 = 0 \quad \Rightarrow \quad I_4(\phi, Q) = I_4(Q)$$

Recent Work on Attractors and Black Holes

- **Split Attractor Flow and Marginal Stability; Wall Crossing (Soibelman-Kontsevich formula and developments)** [Bates, Denef, Moore, Neitzke, Gaiotto, Li, Padi, David, Sen, *et al.*]
- **Higher Order Corrections** [Mohaupt, de Wit, *et al.*, Kraus, Larsen, Prester, Terzič, Sen *et al.*]
- **Black Hole Precision Counting** [Dabholkar, Moore, Pioline, Denef, Sen *et al.*]
- **Entropy Function (Wald) Formalism / Entropy Function** [Sen *et al.*, Trivedi *et al.*]
- $d = 4 \Leftrightarrow d = 5$ **Relations for (Rotating) Black Holes/Black Rings** [Gaiotto, Strominger *et al.*]
- **Symmetric Scalar Manifolds :** $d = 3$ **Geodesic Flow Approach** [Günaydin, Neitzke, Pioline, Waldron, Gaiotto, Li, Padi, Bergshoeff, Chemissany, Ploegh, Trigiante, Van Riet, Rosseel, Fré, Sorin, Andrianopoli, D'Auria, Orazi, Trigiante, *et al.*]
- **First Order Formalism (for non-BPS)** [Ceresole, Dall'Agata, Andrianopoli, D'Auria, Orazi, Trigiante, Perz, Smyth, *et al.*]
- **Classification of static, spherically symmetric solutions of supergravity through nilpotent orbits in** $d = 3$ **para-quaternionic manifolds** [Nicolai, Stelle, Bossard]
- **Quantum Information Theory and Black Holes (mathematical analogy between black hole entropy and entanglement in QIT)** [Duff, Ferrara, Borsten, Dahnayake, Rubens, Ebrahim, Lévay, Saniga, Pracna, Vrana, *et al.*]
- **Black Holes Deconstruction and Elementary Black Hole Constituents** [Denef, Gaiotto, Strominger, Van den Bleeken, Yin, Gimon, Larsen, Simon, Castro, de Boer, El-Showk, Messamah, *et al.*]

School on Attractor Mechanism

29 June - 3 July 2009

This is the fourth school in a series of lectures and discussions held in 2005, 2006 and 2007 at the INFN – Laboratori Nazionali di Frascati on the general theme of supersymmetry, supergravity, black holes and the attractor mechanism

INFN-Laboratori Nazionali di Frascati, Via E. Fermi 40, Frascati, Italy

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INVITED SCIENTISTS:

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LECTURERS:

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G. Dall'Agata (Università di Padova, Italy)
S. Ferrara (CERN, Switzerland)
J.F. Morales (Università di Roma Tor Vergata, Italy)
G. Moore (Rutgers University, Piscataway, USA): to be confirmed
A. Sen (Harish-Chandra Research Institute, Allahabad, India)
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